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Understanding the behavior of multivariable functions is essential in calculus, especially when analyzing their domains and evaluating specific points. The function in question, $G(x, Y, Z) = \ln(42 x^2 y^2 z^2)$, combines the properties of logarithmic and polynomial functions, making it a rich subject for exploration. This article aims to guide you through evaluating the function at a specific point and determining its domain, providing clarity on the underlying concepts.

Understanding the Function: $G(x, Y, Z) = \ln(42 x^2 y^2 z^2)$

Before diving into calculations and domain analysis, it's crucial to grasp the structure and properties of the function.

Breakdown of the Function Components

The given function is:

$G(x, Y, Z) = \ln(42 x^2 y^2 z^2)$

This can be viewed as the natural logarithm of a product involving a constant and variables raised to powers:

- Constant: 42
- Variables: x, y, z
- Exponents: 2

The function essentially measures the natural logarithm of the product of 42 and the squares of the variables.

Properties of the Logarithmic Function

The natural logarithm function, $\ln(t)$, has the following important properties:

- Defined only for t > 0
- ln(t) is increasing for t > 0
- ln(ab) = ln(a) + ln(b)
- ln(t^k) = k ln(t)

Applying these properties helps simplify and analyze the function.

Part A: Evaluating G(3, 4, 4)

Let's proceed to evaluate the function at the specific point where x=3, y=4, and z=4.

Step 1: Substitute the values into the function

$G(3, 4, 4) = \ln(42\ 3^2\ 4^2\ 4^2)$

First, calculate each squared term:

- $3^2 = 9$
- $4^2 = 16$

Since z=4, $z^2 = 16$ as well.

Now, the expression inside the logarithm becomes:

42 9 16 16

Step 2: Simplify the product

Calculate the product step-by-step:

- 42 9 = 378
- 16 16 = 256

Now, multiply 378 by 256:

378 256

To compute this:

378 256 = (378 200) + (378 56)

- 378 200 = 75,600
- 378 56 = 378 (50 + 6) = (378 50) + (378 6) = 18,900 + 2,268 = 21,168

Adding these:

75,600 + 21,168 = 96,768

Therefore, the value inside the logarithm is 96,768.

Step 3: Final calculation

G(3, 4, 4) = ln(96,768)

Using a calculator or logarithm table:

ln(96,768) ≈ 11.477

Answer:

G(3, 4, 4) ≈ 11.477

This numerical evaluation provides insight into the magnitude of the function at the specified point.

Part B: Finding the Domain of G(x, Y, Z)

The domain of a multivariable function is the set of all points (x, y, z) where the function is defined. For G(x, y, z) = ln(42 x^2 y^2 z^2), the primary consideration is the argument of the natural logarithm.

Step 1: Identify the restrictions from the logarithm

Since ln(t) is defined only for t > 0, the argument:

$42 x^2 y^2 z^2 > 0$

Note that 42 is positive, so the inequality simplifies to:

$x^2 y^2 z^2 > 0$

Because squares of real numbers are always non-negative, and zero squared is zero, the product $x^2 y^2 z^2$ is zero if any of x, y, or z is zero.

Therefore, for the argument to be strictly greater than zero:

- None of x, y, or z can be zero.

Thus, the domain is:

$x \neq 0, y \neq 0, z \neq 0$

Step 2: Express the domain mathematically

The domain D can be written as:

$$D = \{ (x, y, z) \in \mathbb{R}^3 \mid x \neq 0, y \neq 0, z \neq 0 \}$$

This means the domain encompasses all real-valued triplets where each variable is strictly non-zero.

Step 3: Visual and conceptual understanding

Visually, the domain is the entire three-dimensional space excluding the coordinate planes where any variable equals zero. In other words:

- The x-axis, y-axis, and z-axis are omitted in the domain.
- The domain consists of four quadrants in each variable's positive and negative regions, excluding the axes.

Additional Considerations and Applications

Understanding both the evaluation and the domain of this function has practical implications across various fields, including engineering, physics, and economics.

Applications in Real-World Contexts

- Modeling Growth Processes: Logarithmic functions often model exponential growth or decay, such as population dynamics or radioactive decay.
- Information Theory: The logarithm function is fundamental in entropy calculations and data compression.
- Financial Mathematics: Logarithms are used in calculating continuous compound interest and return rates.

Why Domain Matters

Knowing the domain ensures that calculations are valid and that functions are applied within their proper scope, avoiding mathematical errors or undefined expressions.

Summary and Key Takeaways

- The function $G(x, y, z) = \ln(42\ x^2\ y^2\ z^2)$ combines polynomial and logarithmic components.
- Evaluating G at a specific point involves substituting values, simplifying the product inside the logarithm, and calculating the natural logarithm.
- The domain of G encompasses all points in $\mathbb{R}^3$ where none of the variables are zero, due to the properties of the logarithm function.

Final Remarks

Mastering the evaluation and domain analysis of multivariable logarithmic functions equips students and professionals with essential tools for advanced calculus and mathematical modeling. By understanding the restrictions imposed by the logarithm and simplifying complex expressions, one can analyze diverse functions accurately and effectively.

Remember: Always check the argument's positivity before evaluating a logarithmic function, and carefully analyze the variables' constraints to determine the domain.

Frequently Asked Questions

What is the value of G(3, 4, 4) for the function G(x, y, z) = ln(42 x^2 y^2 z^2)?

$G(3, 4, 4) = \ln(42\ 3^2\ 4^2\ 4^2) = \ln(42\ 9\ 16\ 16) = \ln(42\ 9\ 256) = \ln(42\ 2304) = \ln(96768) \approx 11.479$.

How do you evaluate G(3, 4, 4) for the given function?

Substitute $x=3$, $y=4$, $z=4$ into $G(x, y, z) = \ln(42\ x^2\ y^2\ z^2)$, then compute the product inside the natural logarithm and take the natural logarithm of the result.

What is the domain of the function G(x, y, z) = ln(42 x^2 y^2 z^2)?

The domain consists of all real numbers x, y, z such that $42\ x^2\ y^2\ z^2 > 0$, which simplifies to all $x, y, z \neq 0$.

Are there any restrictions on the variables for the function G(x, y, z)?

Yes, since the natural logarithm is only defined for positive arguments, all variables must be non-zero; thus, $x \neq 0$, $y \neq 0$, $z \neq 0$.

Can the function G(x, y, z) be evaluated if any of x, y, or z is zero?

No, because ln(0) is undefined and the expression inside the logarithm would be zero if any variable is zero, making G undefined.

How does the square of variables affect the domain of G(x, y, z)?

Since squares are always non-negative and zero only when the variable is zero, the squaring ensures the inside of the log is positive only when none of the variables are zero, maintaining the domain as all non-zero real numbers.

Additional Resources

Consider The Function Below. G(x, Y, Z) = Ln(42 X2 Y2 Z2) (a) Evaluate G(3, 4, 4). (b) Find The Domain

Understanding mathematical functions is fundamental to numerous fields, from engineering to data science. They serve as the building blocks for modeling real-world phenomena, enabling analysts and scientists to interpret complex relationships between variables. In this article, we will explore a specific multivariable function, analyze its properties, evaluate it at a particular point, and determine its domain. Our goal is to present this information in a clear, accessible manner that balances technical precision with readability.

Introduction to the Function G(x, Y, Z) = Ln(42 X2 Y2 Z2)

Before delving into calculations and domain analysis, it's essential to interpret the given function accurately. The function is defined as:

G(x, Y, Z) = Ln(42 X2 Y2 Z2)

At first glance, the notation may seem somewhat ambiguous, but with careful interpretation, we can clarify its structure.

Deciphering the Notation

The notation appears to involve a natural logarithm function (Ln), applied to a product of constants and variables raised to powers. The key components are:

- The constant 42
- Variables x, Y, Z
- Their respective exponents, which are 2 for each variable

The expression "42 X2 Y2 Z2" is most likely a shorthand for:

$G(x, Y, Z) = Ln(42 x^2 Y^2 Z^2)$

In other words, the function involves the natural logarithm of the product of 42 and the squares of x, Y, and Z.

Expressing the Function Clearly

Rewriting in a more standard mathematical form:

$G(x, Y, Z) = \ln [42 x^2 Y^2 Z^2]$

This expression can be further simplified using properties of logarithms:

$G(x, Y, Z) = \ln(42) + \ln(x^2) + \ln(Y^2) + \ln(Z^2)$

Using the power rule for logarithms:

$\ln(x^2) = 2 \ln(x)$, and similarly for the others.

Therefore,

$G(x, Y, Z) = \ln(42) + 2 \ln(x) + 2 \ln(Y) + 2 \ln(Z)$

This simplified form makes analysis and calculation more straightforward.

Part A: Evaluating G(3, 4, 4)

Now, let's perform the specific calculation for the point (x=3, Y=4, Z=4).

Step 1: Write the expression explicitly

Using the simplified form:

$G(3, 4, 4) = \ln(42) + 2 \ln(3) + 2 \ln(4) + 2 \ln(4)$

Recall that ln(42) is a constant, and the other terms involve natural logarithms of the variables.

Step 2: Calculate individual logarithms

- $\ln(42) \approx 3.7377$ (using a calculator)
- $\ln(3) \approx 1.0986$
- $\ln(4) \approx 1.3863$

Then,

$G(3, 4, 4) \approx 3.7377 + 2(1.0986) + 2(1.3863) + 2(1.3863)$

Calculate each term:

- $2(1.0986) \approx 2.1972$
- $2(1.3863) \approx 2.7726$

Sum all:

$G(3, 4, 4) \approx 3.7377 + 2.1972 + 2.7726 + 2.7726$

Adding step-by-step:

- $3.7377 + 2.1972 \approx 5.9349$
- $5.9349 + 2.7726 \approx 8.7075$
- $8.7075 + 2.7726 \approx 11.4801$

Final result:

$G(3, 4, 4) \approx 11.48$ (to two decimal places)

This numerical evaluation demonstrates how the function behaves at specific points, which is useful for applications where precise values are necessary.

Part B: Finding the Domain of G(x, Y, Z)

Understanding the domain of a function involves identifying all the input values (x, Y, Z) for which the function is well-defined and produces real outputs.

1. Conditions Imposed by the Logarithm

Since $G(x, Y, Z)$ involves the natural logarithm, certain conditions must be met:

- The argument of the logarithm must be positive: $42 x^2 Y^2 Z^2 > 0$

Given that 42 is positive, the inequality reduces to:

$x^2 Y^2 Z^2 > 0$

- The squares of real numbers are always non-negative, and are zero only when the variable itself is zero.

- To ensure the product is strictly positive, none of the factors can be zero.

Thus:

$x \neq 0, Y \neq 0, Z \neq 0$

2. Final Domain Description

- The variables x, Y, Z can be any real numbers except zero because:

- If any variable equals zero, the entire product becomes zero, and the ln(0) is undefined (approaches negative infinity).

- Therefore, the domain is:

Domain $D = \{ (x, Y, Z) \in \mathbb{R}^3 \mid x \neq 0, Y \neq 0, Z \neq 0 \}$

This description encompasses all triplets of real numbers where none of the variables are zero, ensuring the function $G(x, Y, Z)$ yields real, finite values.

3. Additional Considerations

- The domain excludes points where variables are zero because the logarithm is undefined at zero and negative values.

- The domain includes all other real numbers, both positive and negative, since their squares are positive, maintaining the positivity of the argument.

- For practical purposes, the function is often used in contexts where variables represent quantities that cannot be negative or zero, such as concentrations, populations, or other positive measures. However, mathematically, negative values are permissible as long as variables are non-zero.

Summary and Practical Implications

The analysis of the function G(x, Y, Z) reveals a straightforward structure once expressed properly. The key takeaways are:

- The function involves the natural logarithm of a positive product, which simplifies to a sum involving the logarithm of a constant and scaled logarithms of variables.
- Evaluating the function at specific points involves calculating logarithms of the variables and summing accordingly.
- The domain of the function is all triplets of real numbers where none of the variables are zero, ensuring the argument of the logarithm remains positive.

Understanding the domain is crucial when applying the function to real-world problems. For example, in fields like physics or economics, variables such as density or price must be positive, aligning perfectly with the mathematical domain. Conversely, in purely theoretical contexts, negative values might be permissible, provided they do not violate the domain restrictions.

In conclusion, $G(x, Y, Z) = \ln(42\ x^2\ Y^2\ Z^2)$ offers an elegant example of multivariable calculus, combining properties of logarithms with domain considerations. Mastering such functions is foundational for more advanced topics like differentiation, optimization, and modeling complex systems. Whether used in academic research or practical applications, understanding their structure and constraints empowers analysts to make informed decisions and accurate calculations.

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