Consider The Random Experiment Of Rolling A Pair Of Fair Dice. What Is The Probability That One Of The

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When exploring the fascinating world of probability, one of the most classic and instructive experiments involves rolling a pair of fair dice. This simple yet rich experiment provides foundational insights into probability theory, combinatorics, and the calculation of likelihoods in random events. Whether you're a student, educator, or simply a curious mind, understanding the probability associated with rolling dice helps develop a deeper appreciation of how chance works in everyday situations.

In this article, we will delve into the details of rolling a pair of fair dice, explore the various outcomes, and specifically analyze the probability that a certain event occurs—such as "one of the dice shows a specific number" or "the sum of the dice falls within a range." We will examine the fundamental concepts, calculations, and practical applications of probability in this context, providing a comprehensive guide for beginners and advanced learners alike.

Understanding the Basics of Dice and Probability

Before diving into specific probability calculations, it’s essential to understand the fundamental concepts related to dice and probability.

What Are Fair Dice?

Fair dice are cubes with faces numbered from 1 to 6, with each face equally likely to land face-up when rolled. This symmetry ensures that the probability of any particular number appearing on a single roll is:

$$
P(\text{specific number}) = \frac{1}{6}
$$

when the die is fair.

Sample Space of Rolling Two Dice

When rolling two fair dice simultaneously, each die's outcome is independent of the other. The total number of possible outcomes (the sample space) is:

$$
6 \times 6 = 36
$$

since each die has 6 possible outcomes, and the outcomes combine in all possible pairs.

Each outcome can be represented as an ordered pair:

$$
(\text{Die 1 result}, \text{Die 2 result})
$$

for example, (3, 5) indicates die 1 shows 3, and die 2 shows 5.

Event Definitions and Probability Calculations

In probability, an event is a subset of the sample space. To compute the probability of an event, we consider the number of favorable outcomes over the total number of outcomes.

$$
P(\text{event}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}}
$$

Let's explore some common events related to rolling a pair of dice.

Event Types of Interest

- Event A: The sum of the two dice equals a specific number (e.g., 7).
- Event B: One die shows a specific number (e.g., a 3), regardless of the other die.
- Event C: Both dice show the same number (doubles).
- Event D: The sum of the dice falls within a certain range.
- Event E: Exactly one die shows a specific number.

Our focus here is on the probability that "one of the" specific events occurs, especially events like "one die shows a particular number" or "the sum is within a range."

Calculating the Probability That One Die Shows a Specific Number

Suppose we are interested in finding the probability that at least one of the dice shows a specific number, for example, the number 3.

Event Definition: At Least One Die Shows 3

This event includes all outcomes where:

- Die 1 shows 3, regardless of die 2.
- Die 2 shows 3, regardless of die 1.
- Both dice show 3.

In terms of outcomes, these are:

- All pairs where die 1 = 3: (3,1), (3,2), (3,3), (3,4), (3,5), (3,6)
- All pairs where die 2 = 3: (1,3), (2,3), (4,3), (5,3), (6,3)

Note that the pair (3,3) appears in both sets, so we must avoid double-counting.

Calculating the Probability

Number of outcomes where at least one die shows 3:

\[
\text{Favorable outcomes} = 6 + 5 = 11
\]

(6 outcomes with die 1 = 3, plus 5 additional outcomes with die 2 = 3, excluding the double count (3,3)).

Alternatively, one can use the complement rule:

- The complement event is neither die showing 3.
- Outcomes where neither die shows 3:

\[
5 \times 5 = 25
\]

since each die can show any number from 1 to 6 except 3, which is 5 options.

Thus,

\[

P(\text{at least one die shows 3}) = 1 - P(\text{neither die shows 3}) = 1 - \frac{25}{36} = \frac{11}{36}
\]

Final probability:

\[
\boxed{
P(\text{at least one die shows 3}) = \frac{11}{36} \approx 0.3056
}
\]

This means there is approximately a 30.56% chance that rolling two fair dice results in at least one die showing a 3.

Probability That Exactly One Die Shows a Specific Number

Building upon this, another common question is: What is the probability that exactly one die shows a specific number, say 3?

Calculation Steps:

- Outcomes where die 1 shows 3, and die 2 does not:

\[
1 \text{ (for die 1)} \times 5 \text{ (for die 2 ≠ 3)} = 5
\]

- Outcomes where die 2 shows 3, and die 1 does not:

\[
5 \text{ (for die 1 ≠ 3)} \times 1 \text{ (for die 2)} = 5
\]

- Total outcomes where exactly one die shows 3:

\[
5 + 5 = 10
\]

- Outcomes where both dice show 3 are excluded because we're only interested in exactly one die showing 3.

Final probability:

\[
P(\text{exactly one die shows 3}) = \frac{10}{36} = \frac{5}{18} \approx 0.2778
\]

Thus, there is approximately a 27.78% chance that exactly one die shows the number 3 when rolling a pair of fair dice.

Probability That The Sum of Two Dice Falls Within a Range

Another interesting event involves the sum of the dice. For example, what is the probability that the sum is greater than or equal to 8?

Identifying Favorable Outcomes

The possible sums when rolling two dice range from 2 (1+1) to 12 (6+6). The sums greater than or equal to 8 are 8, 9, 10, 11, and 12.

Let's list the outcomes for each sum:

- Sum = 8:

\[
(2,6), (3,5), (4,4), (5,3), (6,2)
\]

Number of outcomes: 5

- Sum = 9:

\[
(3,6), (4,5), (5,4), (6,3)
\]

Number of outcomes: 4

- Sum = 10:

\[
(4,6), (5,5), (6,4)
\]

Number of outcomes: 3

- Sum = 11:

$$
(5,6), (6,5)
$$

Number of outcomes: 2

- Sum = 12:

$$
(6,6)
$$

Number of outcomes: 1

Total favorable outcomes:

$$
5 + 4 + 3 + 2 + 1 = 15
$$

Probability Calculation:

$$
P(\text{sum} \geq 8) = \frac{15}{36} = \frac{5}{12} \approx 0.4167
$$

Therefore, there is approximately a 41.67% chance that the sum of two dice is at least 8.

Probability of Specific Events Involving Multiple Conditions

In probability theory, it's often necessary to compute the likelihood of complex events involving multiple conditions. Let's explore a couple of such scenarios related to rolling a pair of fair dice.

Event: The Sum Is 7, and One Die Shows a 4

We want to find the probability that the sum of the dice is 7, and at least one die shows a 4.

Step 1: List Outcomes Where Sum = 7

Possible outcomes:

\[
(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)
\]

Total: 6 outcomes.

Step 2: Identify Outcomes with a Die Showing 4

From the list, outcomes with at least one 4:

\[
(3,4),

Frequently Asked Questions

What is the probability that exactly one die shows a six when rolling a pair of fair dice?

The probability is 10/36 or 5/18 because there are 5 favorable outcomes where one die shows a six and the other does not, out of 36 total possible outcomes.

How do you calculate the probability that at least one die shows a specific number, such as a 4?

Calculate the probability that neither die shows a 4 (which is 25/36), then subtract from 1: 1 - 25/36 = 11/36.

What is the probability that one die shows a 2 and the other die shows a different number?

There are 10 favorable outcomes where exactly one die shows a 2 and the other shows any of the other 5 numbers, totaling 10/36 or 5/18.

If you roll two dice, what is the probability that only one die lands on an even number?

The probability is 12/36 or 1/3, calculated by considering scenarios where exactly one die is even and the other is odd.

What is the probability that one of the dice shows a prime number (2, 3, or 5)?

The probability is 27/36 or 3/4 because each die has 3 prime numbers out of 6, and we consider cases where only one die shows a prime number.

How do you find the probability that exactly one die shows a number greater than 4?

Numbers greater than 4 are 5 and 6. The probability that exactly one die shows >4 is 8/36 or 2/9, considering scenarios where one die is 5 or 6, and the other is not.

What is the probability that in rolling two dice, only one die shows an odd number?

The probability is 12/36 or 1/3, accounting for cases where exactly one die is odd and the other is even.

When rolling a pair of dice, what is the probability that one die shows a 1 and the other die shows any number?

The probability is 10/36 or 5/18 because there are 5 outcomes where one die is 1 and the other is any of 2-6, excluding the case where both are 1.

How do you determine the probability that exactly one die shows a number less than 3?

Numbers less than 3 are 1 and 2. The probability is 8/36 or 2/9, considering scenarios where only one die shows 1 or 2, and the other shows a number 3-6.

What is the probability that one of the dice shows a number divisible by 3?

The probability is 12/36 or 1/3, calculated by considering cases where exactly one die shows 3 or 6, and the other shows any number.

Additional Resources

Consider The Random Experiment Of Rolling A Pair Of Fair Dice. What Is The Probability That One Of The

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Introduction

In the realm of probability and statistics, simple experiments often serve as fundamental building blocks for understanding complex phenomena. One such classic experiment is rolling a pair of fair dice—a scenario that has captivated mathematicians, gamblers, and educators alike for centuries. At its core, this experiment involves randomness, symmetry, and combinatorial analysis, making it an ideal case study for exploring probability principles.

In this article, we will delve into the detailed analysis of rolling two fair dice, focusing specifically on the probability that one of the dice shows a particular outcome or that a specific pattern occurs. The phrase "What is the probability that one of the" hints at focusing on events involving individual dice, such as the probability that "one die shows a 4" or "exactly one die shows a certain value." We will dissect the problem systematically, covering foundational concepts, probability calculations, and implications.

Setting the Stage: The Basics of Rolling Two Fair Dice

The Nature of Fair Dice

A single fair die is a cube with six faces, each numbered distinctly from 1 to 6. When rolled, each face has an equal probability (1/6) of landing face-up, assuming the die is unbiased and the roll is random.

The Sample Space

When considering two dice rolled simultaneously, the total number of possible outcomes can be enumerated as follows:

- Each die has 6 outcomes.
- The outcomes are independent of each other.

So, the total sample space contains:

$$6 \times 6 = 36$$

possible outcomes, each represented as an ordered pair: (d_1, d_2), where d_1 is the result of the first die, and d_2 is the result of the second die.

Example: $(3, 5)$ indicates the first die shows 3, and the second shows 5.

Each of these 36 outcomes is equally likely, with probability $1/36$.

Defining the Event of Interest

The core question revolves around understanding the probability that one of the dice shows a specific outcome. For clarity, consider the general case:

"What is the probability that exactly one of the two dice shows a particular number k?"

This framing captures the essence of the phrase "the probability that one of the" in the prompt.

Analytical Breakdown of the Event

1. Event Definition: Exactly One Die Shows a Specific Number k

Suppose we select a specific number k between 1 and 6. The event E_k is:

> E$_k$: "Exactly one of the two dice shows the number k."

This event can occur in two mutually exclusive ways:

- The first die shows k, and the second die does not show k.
- The second die shows k, and the first die does not show k.

2. Enumerating Favorable Outcomes

Let's analyze each case:

- Case A: First die shows k, second die shows any number except k.

Number of outcomes:

$$ 1 \times 5 = 5 $$

since the first die is fixed at k, and the second die can be any of the remaining 5 numbers (1-6 excluding k).

- Case B: Second die shows k, first die shows any number except k.

Similarly, outcomes:

$$ 5 $$

because the second die is fixed at k, and the first die can be any of the remaining 5 options.

Since these events are mutually exclusive (both cannot happen simultaneously because that would mean both dice show k), the total favorable outcomes are:

$$ 5 + 5 = 10 $$

Calculating the Probability

Given the total sample space size of 36, the probability $P(E_k)$ of exactly one die showing the number k is:

$$
P(E_k) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{10}{36} = \frac{5}{18}
$$

Thus, the probability that exactly one die shows a specific number k is $\frac{5}{18}$.

Extending the Analysis: The Probability That One Die Shows a Number, Regardless of Which Die

The above calculation considers a fixed number k. But what if the question is more general, such as:

> "What is the probability that one of the two dice shows a number in a specific subset of outcomes?"

For example:

- The probability that at least one die shows a number in $\{1, 2, 3\}$.

This introduces combinatorial considerations for different subsets.

Broader Scenarios and Their Probabilities

1. Probability that at least one die shows a particular number k

This can be approached using the complement rule:

$$
P(\text{at least one die shows }k) = 1 - P(\text{neither die shows }k)
$$

- The probability that a specific die does not show k is $\frac{5}{6}$.
- Since the dice are independent:

$$
P(\text{neither die shows }k) = \left(\frac{5}{6}\right) \times \left(\frac{5}{6}\right) = \frac{25}{36}
$$

Therefore:

$$
P(\text{at least one die shows }k) = 1 - \frac{25}{36} = \frac{11}{36}
$$

Note: This aligns with the previous calculation for "exactly one die shows k", which was $10/36$. The total probability that at least one shows k includes the case where both dice show k:

- Probability both dice show k:

$$
P(\text{both show }k) = \frac{1}{36}
$$

- Probability exactly one die shows k:

$$
\frac{10}{36}
$$

- Summing these:

$$
\frac{10}{36} + \frac{1}{36} = \frac{11}{36}
$$

which matches the previous complement calculation.

2. Probability that exactly one die shows a number in a subset S

Suppose S is a subset of $\{1, 2, 3, 4, 5, 6\}$, and $|S| = s$.

- The probability that exactly one die shows a number in S:

$$
P(\text{exactly one die shows } S) = 2 \times \left(\frac{s}{6} \times \frac{6 - s}{6} \right)
$$

- Explanation:

- The factor of 2 accounts for the two cases: first die shows a number in S and second does not, or vice versa.

- The probability that, say, the first die shows a number in S:

$$
\frac{s}{6}
$$

- The probability that the other die shows a number not in S:

$$
\frac{6 - s}{6}
$$

Multiplying and doubling gives the total probability for exactly one die in S.

Implications and Practical Applications

Understanding these probabilities is not just an academic exercise; it has real-world relevance in various fields:

- Gambling and Game Design: Knowing the likelihood of certain outcomes influences game strategies and betting odds.

- Quality Control: Random sampling processes often model inspections as dice rolls, where outcomes determine acceptance or rejection.

- Education and Teaching: Demonstrating probability concepts through dice rolling helps students grasp fundamental principles such as independence, complementarity, and combinatorics.

- Simulation and Modeling: Random experiments like dice rolls are fundamental for Monte Carlo simulations, risk analysis, and decision-making algorithms.

Advanced Considerations: Conditional and Joint Probabilities

While the previous sections focused on straightforward probability calculations, more advanced analyses involve:

- Conditional probability: e.g., “Given that the sum of the dice is 7, what is the probability that exactly one die shows a 4?”

- Joint probability distributions: understanding the full probability matrix of outcomes.

For example, the probability that the sum of two dice is 7:

$$
\text{Number of outcomes with sum 7} = 6 \quad \text{(pairs: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1))}
$$

$$
P(\text{sum} = 7) = \frac{6}{36} = \frac{1}{6}
$$

Conditional probability involves more intricate calculations but is vital for comprehensive probabilistic modeling.

Final Thoughts: The Power of Simplicity in Probability

The experiment of rolling a pair of fair dice exemplifies how simple setups can unveil complex probabilistic phenomena. The question of "what is the probability that one of the dice shows a particular outcome" opens pathways to understanding concepts like

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