Describe Geometrically (line, Plane, Or All Of R^3) All Linear Combinations Of (a) [1 2 3] And [3 6 9]

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Understanding the geometric interpretation of linear combinations is fundamental in the study of vector spaces and linear algebra. When dealing with two vectors, such as $v_1 = [1, 2, 3]$ and $v_2 = [3, 6, 9]$, exploring the set of all their linear combinations reveals whether these vectors span a line, a plane, or the entire three-dimensional space $\mathbb{R}^3$. This article aims to thoroughly analyze and describe geometrically the set of all linear combinations of v_1 and v_2, clarifying whether they form a line, a plane, or fill $\mathbb{R}^3$.

Understanding Linear Combinations

A linear combination of vectors involves multiplying each vector by a scalar and then adding the results. Formally, for vectors v_1 and v_2, and scalars α and β, the linear combination is:

$$ \mathbf{L} = \alpha \mathbf{v}_1 + \beta \mathbf{v}_2 $$

The set of all such combinations as α and β vary over all real numbers is called the span of v_1 and v_2, denoted as:

$$ \text{span}\{\mathbf{v}_1, \mathbf{v}_2\} $$

This span is a subset of $\mathbb{R}^3$ and may form a line, a plane, or the entire space depending on the linear independence of the vectors.

Given Vectors: v_1 and v_2

Let's explicitly state our vectors:

$$
\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}
\quad \text{and} \quad
$$

$$
\mathbf{v}_2 = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}
$$

Notice that v_2 appears to be a scalar multiple of v_1:

$$
\mathbf{v}_2 = 3 \times \mathbf{v}_1
$$

which indicates that v_1 and v_2 are linearly dependent.

Analyzing Linear Dependence and Span

Linear Dependence of v_1 and v_2

Since $v_2 = 3 \times v_1$, any linear combination of v_1 and v_2 can be expressed solely in terms of v_1:

$$
\alpha \mathbf{v}_1 + \beta \mathbf{v}_2 = \alpha \mathbf{v}_1 + \beta (3 \mathbf{v}_1) = (\alpha + 3 \beta) \mathbf{v}_1
$$

This simplifies the set of all linear combinations to:

$$
\left\{ \gamma \mathbf{v}_1 : \gamma \in \mathbb{R} \right\}
$$

meaning that the span of v_1 and v_2 is the same as the span of v_1 alone.

Geometric Interpretation of the Span

Given that v_1 is a single vector in $\mathbb{R}^3$, its span is a line passing through the origin, directed along v_1.

- Line through the origin: All scalar multiples of v_1 form a line in $\mathbb{R}^3$.
- Direction vector: The vector v_1 = [1, 2, 3] indicates the direction of this line.

Because v_2 is a scalar multiple of v_1, the set of all linear combinations does not extend beyond this line.

Summary: Geometric Description

Aspect	Description
Vectors	$v_1 = [1, 2, 3]$, $v_2 = [3, 6, 9]$
Linear dependence	Yes; v_2 is a scalar multiple of v_1
Span	All scalar multiples of v_1
Geometric shape	A line passing through the origin in $\mathbb{R}^3$

Visualizing the Geometric Structure

To better understand the geometric structure, consider the following:

Visual Representation of the Line

- The line passes through the origin (0, 0, 0).
- It extends infinitely in both directions.
- Any point P on this line can be written as:

$$
P = \lambda \times \mathbf{v}_1 = \lambda \times [1, 2, 3], \quad \lambda \in \mathbb{R}
$$

Visualizing Dependence

- Because v_2 is just $3 \times v_1$, every point generated by v_2 is also on this line.
- The set of all linear combinations is therefore a single line in $\mathbb{R}^3$.

Implications for Subspace Classification

Based on the above analysis, we classify the subspace spanned by v_1 and v_2 as follows:

- Line: Since the vectors are linearly dependent, their span is a one-dimensional subspace.
- Not a plane: Because the vectors are not linearly independent, they do not

span a two-dimensional plane.
- Not all of $\mathbb{R}^3$: The span does not fill the entire three-dimensional space.

Additional Examples and Variations

To reinforce the concept, consider different scenarios involving vectors:

1. Two linearly independent vectors: Their span is a plane.
2. Two identical vectors: Their span is a single line.
3. Vectors spanning $\mathbb{R}^3$: Require three linearly independent vectors.

Summary and Key Takeaways

- The vectors [1, 2, 3] and [3, 6, 9] are linearly dependent because v_2 is a scalar multiple of v_1.
- The set of all linear combinations of these vectors is the set of scalar multiples of v_1.
- Geometrically, this set forms a line passing through the origin in $\mathbb{R}^3$.
- Therefore, the span of v_1 and v_2 is a line, not a plane or the entire space.

Understanding the geometric interpretation of linear combinations is crucial for visualizing subspaces and their dimensions in vector spaces. Recognizing linear dependence or independence among vectors allows one to determine the dimension of the subspace they generate, which is foundational in linear algebra applications across science and engineering.

Conclusion

In conclusion, the linear combinations of [1, 2, 3] and [3, 6, 9] form a line in $\mathbb{R}^3$. Since one vector is a scalar multiple of the other, their span does not extend to a plane or fill the entire space. This example neatly illustrates how linear dependence restricts the span to a lower-dimensional subset of $\mathbb{R}^3$. Recognizing such relationships is essential for understanding the structure of vector spaces and for solving systems of linear equations, optimizing in high-dimensional spaces, and many other applications in mathematics, physics, and engineering.

Word count: approximately 1050 words

Frequently Asked Questions

What is the geometric interpretation of all linear combinations of vectors [1, 2, 3] and [3, 6, 9] in R^3?

All linear combinations of these vectors lie along the line passing through the origin and the points represented by the vectors, since the second vector is a scalar multiple of the first.

Are the vectors [1, 2, 3] and [3, 6, 9] linearly independent?

No, they are linearly dependent because [3, 6, 9] is a scalar multiple (by 3) of [1, 2, 3].

What is the span of the vectors [1, 2, 3] and [3, 6, 9]?

The span is the set of all scalar multiples of [1, 2, 3], which geometrically is a line through the origin in R^3.

Does the set of all linear combinations of [1, 2, 3] and [3, 6, 9] form a plane in R^3?

No, since the vectors are linearly dependent, their linear combinations form only a line, not a plane.

What is the geometric description of the subset of R^3 generated by [1, 2, 3] and [3, 6, 9]?

It is a one-dimensional subspace (a line) passing through the origin, aligned with the vector [1, 2, 3].

Can all linear combinations of [1, 2, 3] and [3, 6, 9] be represented as a single vector multiple of [1, 2, 3]?

Yes, because [3, 6, 9] is just 3 times [1, 2, 3], so all combinations are scalar multiples of [1, 2, 3].

Is the set of all linear combinations of [1, 2, 3] and [3, 6, 9] equal to R^3?

No, because the set only spans a line, not the entire three-dimensional space.

How can you verify that [3, 6, 9] is a linear combination of [1, 2, 3]?

By checking if there exists a scalar c such that c[1, 2, 3] = [3, 6, 9]; in this case, c=3.

What is the significance of the linear dependence between [1, 2, 3] and [3, 6, 9] in terms of their span?

Their linear dependence implies they do not generate a plane but only a line in R^3, limiting the span to a one-dimensional subspace.

Additional Resources

Describe Geometrically (line, Plane, Or All Of R^3) All Linear Combinations Of (a) [1 2 3] And [3 6 9]

In the realm of linear algebra, understanding the geometric interpretation of linear combinations is fundamental to grasping the structure of vector spaces. When we analyze particular vectors and their combinations, we unveil the shapes and subspaces they form within the ambient space. This article delves into the geometric nature of all linear combinations of two specific vectors in $\mathbb{R}^3$: $[1, 2, 3]$ and $[3, 6, 9]$. Our goal is to determine whether these combinations form a line, a plane, or encompass the entire three-dimensional space, and to understand the underlying reasons for these geometric configurations.

Understanding Linear Combinations and Their Geometric Significance

Before examining the specific vectors, it’s essential to clarify what a linear combination entails and how it relates to the geometry of $\mathbb{R}^3$.

Definition:

Given vectors \(\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k \in
\mathbb{R}^3\), a linear combination is any vector of the form:
\[
\mathbf{w} = c_1 \mathbf{v}_1 + c_2 \mathbf{v}_2 + \ldots + c_k \mathbf{v}_k,
\]
where \(c_1, c_2, \ldots, c_k \in \mathbb{R}\).

Geometric Interpretation:
- The set of all linear combinations of a single non-zero vector
\(\mathbf{v}\) forms a line passing through the origin, called the span of
\(\mathbf{v}\).
- The set of all linear combinations of two vectors \(\mathbf{v}_1\) and
\(\mathbf{v}_2\) can be either a line, a plane, or the entire
\(\mathbb{R}^3\) depending on their linear independence.
- When considering multiple vectors, the span of those vectors indicates the
subspace they generate, which can be a line, a plane, or the whole space.

Examining the Given Vectors: \([1, 2, 3]\) and \([3, 6, 9]\)

The vectors in question are:

\[
\mathbf{v}_1 = [1, 2, 3]
\]
\[
\mathbf{v}_2 = [3, 6, 9]
\]

At first glance, these vectors seem related, and understanding their
relationship is key to determining the geometric nature of their linear
combinations.

Step 1: Checking for Linear Dependence

A critical step in analyzing the span of these vectors is to verify whether
they are linearly independent or dependent.

Linear dependence test:
\[
\mathbf{v}_2 = \lambda \mathbf{v}_1
\]

for some scalar \(\lambda \in \mathbb{R}\).

Calculating:
\[
\lambda = \frac{\text{corresponding component of } \mathbf{v}_2}{\text{component of } \mathbf{v}_1}
\]

- For the first component:
\[
\lambda = \frac{3}{1} = 3
\]
- For the second component:
\[
\lambda = \frac{6}{2} = 3
\]
- For the third component:
\[
\lambda = \frac{9}{3} = 3
\]

Since all components yield the same \(\lambda = 3\), the vectors are scalar multiples of each other. This directly implies they are linearly dependent.

Implication:
Because \(\mathbf{v}_2\) is just 3 times \(\mathbf{v}_1\), the set \(\{\mathbf{v}_1, \mathbf{v}_2\}\) spans the same space as \(\mathbf{v}_1\) alone.

Geometric Interpretation of the Span of \(\mathbf{v}_1\) and \(\mathbf{v}_2\)

Given their linear dependence, the set of all linear combinations of \(\mathbf{v}_1\) and \(\mathbf{v}_2\) simplifies to the set of all scalar multiples of \(\mathbf{v}_1\).

Mathematically:
\[
\text{Span}\{\mathbf{v}_1, \mathbf{v}_2\} = \text{Span}\{\mathbf{v}_1\}
\]
which is:
\[
\left\{ c \cdot [1, 2, 3] \mid c \in \mathbb{R} \right\}
\]

Geometrically:

- This set forms a line passing through the origin in \(\mathbb{R}^3\).
- Every vector in this set points in the direction of \(\mathbf{v}_1\).
- The line is infinitely long and includes the origin, extending in both directions.

Visualizing the line:
Imagine a straight line passing through the origin, aligned with the vector \([1, 2, 3]\). Any point on this line can be reached by scaling \(\mathbf{v}_1\) by some real number \(c\).

Summary of the Geometric Configuration

Since \(\mathbf{v}_1\) and \(\mathbf{v}_2\) are linearly dependent, their linear combinations do not produce a plane or fill the entire \(\mathbb{R}^3\). Instead, they generate a single line through the origin, aligned with \([1, 2, 3]\).

Final conclusion:

> The set of all linear combinations of \([1, 2, 3]\) and \([3, 6, 9]\) forms a line passing through the origin in \(\mathbb{R}^3\).

Broader Implications and Related Concepts

Understanding the linear dependence of vectors is crucial in many areas of linear algebra, including solving systems of equations, understanding subspaces, and basis selection.

Key points to remember:

- Linear dependence means one vector can be expressed as a scalar multiple of another.
- The span of two vectors that are scalar multiples is a line, not a plane or the whole space.
- For vectors to span a plane in \(\mathbb{R}^3\), they must be linearly independent but not scalar multiples.
- To span all of \(\mathbb{R}^3\), three vectors must be linearly independent.

Extensions to More Complex Cases

While the current example involves simple scalar multiples, the concept extends to more complex situations — for example, when vectors are not scalar multiples but are linearly independent.

Scenario:
Suppose we have vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^3$ such that they are linearly independent. Their span would be a plane passing through the origin, containing all linear combinations of $\mathbf{u}$ and $\mathbf{v}$.
- This plane is a two-dimensional subspace of $\mathbb{R}^3$.

Further extension:
Adding a third vector $\mathbf{w}$, linearly independent of the first two, would generate the entire space $\mathbb{R}^3$ as their span.

Practical Applications of Geometric Understanding

Grasping the geometric nature of linear combinations has numerous applications:

- Computer Graphics:
Vectors define directions and positions; understanding their spans helps in rendering objects and transformations.

- Data Science:
Dimensionality reduction techniques rely on identifying the spans of data vectors to find underlying structures.

- Engineering:
Structural analysis often involves understanding the subspaces formed by force vectors or displacements.

- Robotics:
Movement and control systems use linear combinations of joint vectors to determine positions and orientations.

Conclusion

Analyzing the linear combinations of vectors in $\mathbb{R}^3$ reveals the underlying geometric structure they form. In the case of $[1, 2, 3]$ and $[3, 6, 9]$, their linear dependence simplifies the span to a line through the origin, directed along $[1, 2, 3]$. Recognizing such relationships not only clarifies the geometric picture but also informs practical problem-solving across various scientific and engineering disciplines.

This example underscores the importance of linear dependence and independence in understanding the fabric of vector spaces, guiding mathematicians and practitioners in navigating the multidimensional landscapes of modern science.

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