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Details Find The Derivative Of F(x) = 2e^{x} Sin(x). = F'(x) = Submit Question Which Is The Derivative Of

Understanding how to differentiate functions is a fundamental skill in calculus, especially when dealing with composite functions involving exponential and trigonometric components. In this article, we will explore in detail how to find the derivative of the function $F(x) = 2e^{x} \sin(x)$. This process involves applying the product rule, the chain rule, and understanding the properties of exponential and sine functions. By the end of this guide, you will be equipped with the knowledge to differentiate similar functions accurately and confidently.

Understanding the Function $F(x) = 2e^{x} \sin(x)$

Before diving into the differentiation process, it is essential to analyze the structure of the function:

- The function is a product of two functions:
1. $u(x) = 2e^{x}$
2. $v(x) = \sin(x)$

- The constant factor 2 multiplies the entire product.

This structure suggests that the product rule is the appropriate differentiation technique. The product rule states:

$$
\frac{d}{dx}[u(x) \cdot v(x)] = u'(x) \cdot v(x) + u(x) \cdot v'(x)
$$

Applying this rule requires computing the derivatives of $u(x)$ and $v(x)$ separately.

Step-by-Step Derivation of $F'(x)$

Let's now proceed step-by-step to find the derivative $F'(x)$.

1. Identify the functions involved

- $u(x) = 2e^{x}$
- $v(x) = \sin(x)$

2. Compute the derivatives of $u(x)$ and $v(x)$

- Derivative of $u(x) = 2e^{x}$:

$$
u'(x) = 2 \cdot \frac{d}{dx}(e^{x}) = 2e^{x}
$$

- Derivative of $v(x) = \sin(x)$:

$$
v'(x) = \cos(x)
$$

3. Apply the product rule

Using the product rule:

$$
F'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)
$$

Substituting the derivatives:

$$
F'(x) = (2e^{x}) \cdot \sin(x) + (2e^{x}) \cdot \cos(x)
$$

4. Factor the expression

Notice that both terms share a common factor $2e^{x}$:

$$
F'(x) = 2e^{x} (\sin(x) + \cos(x))
$$

Thus, the derivative of the function is:

$$
\boxed{F'(x) = 2e^{x} (\sin(x) + \cos(x))}
$$

Summary of the Differentiation Process

Step	Description	Result
1	Recognize the product structure	$F(x) = 2e^{x} \sin(x)$
2	Derive $u(x)$ and $v(x)$	$u'(x) = 2e^{x}$, $v'(x) = \cos(x)$
3	Apply product rule	$F'(x) = u'(x) v(x) + u(x) v'(x)$
4	Simplify	$F'(x) = 2e^{x} (\sin(x) + \cos(x))$

Additional Techniques and Considerations

While the above method effectively computes the derivative, there are some additional techniques and considerations that can help deepen understanding and handle more complex functions.

Using the Product Rule Efficiently

- Always identify the parts of the product clearly.
- Remember that constants can be factored out to simplify calculations.

Chain Rule in Context

Although the chain rule isn't directly necessary here, it plays a role in differentiating functions like e^{x}. Remember:

$$
\frac{d}{dx} [f(g(x))] = f'(g(x)) \cdot g'(x)
$$

In the case of e^{x}, the outer function $f(u) = e^{u}$, with $u = x$, so the derivative is simply e^{x}.

Handling Similar Functions

For functions such as:

- $F(x) = a \cdot e^{b x} \cdot \sin(c x)$,

the differentiation process involves:

- Applying the product rule to multiple factors.
- Using the chain rule for $e^{b x}$ and $\sin(c x)$.

Example:

$$
\frac{d}{dx} [a e^{b x} \sin(c x)] = a \left[b e^{b x} \sin(c x) + e^{b x} c \cos(c x) \right]
$$

which simplifies to:

$$
a e^{b x} \left(b \sin(c x) + c \cos(c x) \right)
$$

Common Mistakes to Avoid

When differentiating functions like $F(x) = 2e^{x} \sin(x)$, it's easy to make certain errors. Be mindful of the following:

- Forgetting the constant factor 2: Always include constants in derivatives.
- Incorrect application of the product rule: Remember to differentiate both factors separately.
- Misapplication of the chain rule: Ensure the chain rule is used when functions are composed, such as $e^{g(x)}$.
- Sign errors: When differentiating $\sin(x)$, the derivative is $\cos(x)$, not $-\cos(x)$.

Implications and Applications of the Derivative

Understanding the derivative of $F(x) = 2e^{x} \sin(x)$ has several practical implications:

- Analyzing the function's behavior: The derivative indicates where the function increases or decreases.
- Finding critical points: Set $F'(x) = 0$ to find potential maxima, minima, or points of inflection.
- Solving real-world problems: Functions involving exponential and sinusoidal components appear in physics, engineering, and economics.

Example: Critical Points

To find critical points:

$$
F'(x) = 2e^{x} (\sin(x) + \cos(x)) = 0
$$

Since $2e^{x} \neq 0$ for all real x, the critical points occur where:

$$
\sin(x) + \cos(x) = 0
$$

Solve for x:

$$
\sin(x) = -\cos(x)
$$

Dividing both sides by $\cos(x)$ (where $\cos(x) \neq 0$):

$$
\tan(x) = -1
$$

Therefore:

$$
x = -\frac{\pi}{4} + n\pi, \quad n \in \mathbb{Z}
$$

These are the points where the function's slope is zero, indicating potential local maxima or minima.

Conclusion

Differentiating functions involving exponential and trigonometric components is a vital aspect of calculus. The function $F(x) = 2e^{x} \sin(x)$ exemplifies such a case where applying the product rule simplifies the process. The derivative:

$$
F'(x) = 2e^{x} (\sin(x) + \cos(x))
$$

provides insight into the behavior of the function, including where it increases, decreases, and reaches critical points. Mastery of these differentiation techniques is essential for solving complex problems in mathematics and its applications across various scientific fields.

By understanding the principles outlined in this article, you can confidently approach similar problems involving products of exponential and trigonometric functions, enhancing your calculus proficiency and problem-solving skills.

Frequently Asked Questions

What is the derivative of the function $F(x) = 2e^x \sin(x)$?

The derivative is $F'(x) = 2e^x (\sin(x) + \cos(x))$.

How do you find the derivative of $F(x) = 2e^x \sin(x)$ using the product rule?

Apply the product rule: $F'(x) = 2 [d(e^x)/dx \sin(x) + e^x d(\sin(x))/dx] = 2 [e^x \sin(x) + e^x \cos(x)] = 2e^x (\sin(x) + \cos(x))$.

Is the derivative of $F(x) = 2e^x \sin(x)$ positive or negative for $x > 0$?

Since e^x, $\sin(x)$, and $\cos(x)$ are positive or oscillate, the derivative $2e^x (\sin(x) + \cos(x))$ varies, but for certain $x > 0$ where $\sin(x) + \cos(x) > 0$, the derivative is positive.

Can the derivative of $F(x) = 2e^x \sin(x)$ be simplified further?

No, the derivative $F'(x) = 2e^x (\sin(x) + \cos(x))$ is already simplified using standard derivatives and the product rule.

What is the importance of applying the product rule in finding the derivative of $F(x) = 2e^x \sin(x)$?

Because the function is a product of two functions, e^x and $\sin(x)$, the product rule is necessary to correctly compute its derivative, capturing the contributions of both functions.

Additional Resources

Details Find The Derivative Of $F(x) = 2e^x \sin(x)$. = F'(x) = Submit Question Which Is The Derivative Of is a common query encountered in calculus, especially when students and professionals seek to understand how to differentiate functions involving exponential and trigonometric components. This task demands a solid grasp of differentiation rules, including the product rule, chain rule, and the properties of exponential and sine functions. In this comprehensive review, we will explore the step-by-step process to find the derivative of the function $F(x) = 2e^x \sin(x)$, analyze the methods involved, and discuss the significance of mastering such derivatives in mathematical analysis and applied fields.

Understanding the Function $F(x) = 2e^x \sin(x)$

Before diving into the differentiation process, it is crucial to understand the structure of the function itself. The function combines an exponential term e^x and a trigonometric function $\sin(x)$, multiplied by a constant factor of 2. This type of function is typical in many areas of mathematics, physics, and engineering, where exponential growth or decay interacts with oscillatory behavior.

Key features:
- The exponential component e^x exhibits continuous growth as x increases.
- The sine component $\sin(x)$ oscillates between -1 and 1, creating periodic behavior.
- The constant 2 scales the entire function but does not affect the differentiation process directly.

Understanding these features helps in visualizing the function's behavior and anticipating the nature of its derivative.

Step-by-Step Derivation of $F'(x)$

The derivative of $F(x) = 2e^x \sin(x)$ involves the application of the product rule, as the function is a product of two functions: $u(x) = 2e^x$ and $v(x) = \sin(x)$.

Applying the Product Rule

The product rule states:

$$
\frac{d}{dx}[u(x) \cdot v(x)] = u'(x) v(x) + u(x) v'(x)
$$

Applying this to our function:

$$
F'(x) = \frac{d}{dx}[2e^x \sin(x)] = (2e^x)' \sin(x) + 2e^x (\sin(x))'
$$

Calculating each derivative:

- $(2e^x)' = 2e^x$, since the derivative of e^x is e^x.
- $(\sin(x))' = \cos(x)$.

Substituting these back in:

$$
F'(x) = 2e^x \sin(x) + 2e^x \cos(x)
$$

Factoring out common terms:

$$
F'(x) = 2e^x (\sin(x) + \cos(x))
$$

This is the simplified form of the derivative.

Final Expression for the Derivative

The derivative of $F(x) = 2e^x \sin(x)$ is:

$$
\boxed{
F'(x) = 2e^x (\sin(x) + \cos(x))
}
$$

This concise form clearly expresses the rate of change of $F(x)$ at any point x, combining the exponential growth with the sum of sine and cosine functions.

Features and Significance of the Derivative

Understanding the derivative's structure reveals a lot about the behavior of the original function.

Features:
- The exponential term e^x amplifies the oscillations of $\sin(x) + \cos(x)$ as x increases.
- The derivative contains both sine and cosine, reflecting the combined oscillatory and exponential growth characteristics.
- The sign and magnitude of $F'(x)$ inform about the increasing or decreasing nature of $F(x)$.

Significance:
- In physics, such derivatives can describe oscillatory systems with exponential damping or amplification.
- In engineering, they are relevant for analyzing signals and control systems.
- In mathematics, they serve as foundational examples illustrating the product rule and the behavior of composite functions.

Pros and Cons of Differentiating Functions Like $2e^x \sin(x)$

Pros:
- Demonstrates the power of the product rule in handling combined functions.
- Enhances understanding of how exponential and trigonometric functions interact.
- Facilitates solving real-world problems involving oscillations and growth.

Cons:
- The process can be error-prone without careful application of differentiation rules.
- Simplification might be overlooked, leading to more complicated expressions.
- Requires familiarity with multiple differentiation rules and their interplay.

Alternative Methods and Considerations

While the direct application of the product rule is straightforward here, other methods or considerations can sometimes simplify the process or provide additional insights.

Using Complex Exponentials

One alternative involves expressing $\sin(x)$ using Euler's formula:

$$
\sin(x) = \frac{e^{ix} - e^{-ix}}{2i}
$$

Rewriting $F(x)$ in terms of complex exponentials allows differentiation using exponential rules. However, this method often complicates the real-valued interpretation and is more suitable in advanced contexts like Fourier analysis.

Graphical Interpretation

Plotting $F(x)$ and $F'(x)$ can provide visual insights into how the function behaves and how its rate of change varies across different regions.

Practical Applications of the Derivative $F'(x)$

Knowing the derivative enables various practical applications:

- Finding critical points: Setting $F'(x) = 0$ helps locate maxima, minima, and points of inflection.
- Analyzing growth and decay: The sign of $F'(x)$ indicates whether $F(x)$ is increasing or decreasing.
- Modeling systems: In physics and engineering, derivatives like $F'(x)$ model how systems evolve over time, especially with exponential and oscillatory behaviors.

Conclusion

The process of finding the derivative of $F(x) = 2e^x \sin(x)$ exemplifies the elegant application of fundamental calculus rules. By leveraging the product rule and understanding the derivatives of exponential and sine functions, we arrive at a concise and meaningful expression:

$$
F'(x) = 2e^x (\sin(x) + \cos(x))
$$

This derivative encapsulates the combined effects of exponential growth and oscillation, serving as a foundational example in calculus. Mastery of such derivatives not only enhances mathematical proficiency but also empowers applications across scientific and engineering disciplines.

Understanding, deriving, and analyzing derivatives like this is essential for anyone delving into advanced calculus, differential equations, or applied mathematics, making it a vital skill in both academic and professional contexts.

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