Determine The Equation Of The Parabola That Opens To The Right, Has Vertex (8,4), And A Focal Diameter

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When working with parabolas in coordinate geometry, understanding how to derive their equations based on given properties is crucial. Whether you're a student preparing for exams or a professional involved in design and engineering, mastering this process enhances your analytical skills. In this article, we will explore the step-by-step method to determine the equation of a parabola that opens to the right, given its vertex at (8,4), and its focal diameter. We will cover the fundamental concepts, formulas, and practical examples to ensure clarity and comprehensive understanding.

Understanding Parabolas and Their Key Features

Before delving into the derivation process, it's essential to familiarize yourself with the basic properties of parabolas, especially those that open horizontally.

What Is a Parabola?

A parabola is a symmetric, U-shaped curve that is defined as the set of all points equidistant from a fixed point called the focus and a fixed line called the directrix. The standard forms of a parabola depend on its orientation:
- Opens upward or downward: $(x - h)^2 = 4p(y - k)$
- Opens left or right: $(y - k)^2 = 4p(x - h)$

Key Features of a Parabola

- Vertex: The turning point of the parabola (either maximum or minimum).
- Focus: A fixed point inside the parabola from which distances are measured.
- Directrix: A line perpendicular to the axis of symmetry, equidistant from the vertex as the focus.
- Axis of Symmetry: The line passing through the focus and vertex, dividing the parabola into mirror images.
- Focal Diameter: The length of the chord passing through the focus and perpendicular to the axis of symmetry. It measures how "wide" the parabola is at the focus.

Given Data and Its Implications

In our specific problem, we're told:
- The parabola opens to the right.
- The vertex is at $(8, 4)$.
- The focal diameter is known (though the exact length is not specified in the prompt, we will assume a general focal diameter D).

Understanding the implications:

- Since the parabola opens to the right, its axis of symmetry is horizontal.
- The vertex at $(8, 4)$ lies on the parabola, serving as a pivotal point for the equation.
- The focal diameter relates directly to the parabola's focus and the width of the parabola at that point.

Standard Form of a Horizontal Parabola

The general equation for a parabola that opens sideways (left or right) with vertex (h,k) is:

$$
(y - k)^2 = 4p(x - h)
$$

- p is the distance from the vertex to the focus (positive if the parabola opens to the right, negative if to the left).
- The focus is at $(h + p, k)$.
- The directrix is the vertical line $x = h - p$.

Note: The focal diameter is related to $4p$. Specifically:

$$
\text{Focal Diameter} = |4p|
$$

This is because the focal diameter is the length of the chord passing through the focus, perpendicular to the axis of symmetry.

Step-by-Step Process to Derive the Equation

Let's proceed systematically:

1. Identify Known Values

- Vertex: $(h, k) = (8, 4)$
- Focal Diameter: D (unknown, but related to p)
- Parabola opens to the right, so $p > 0$

2. Express the Equation with Unknown p

Using the standard form:

$$
(y - 4)^2 = 4p (x - 8)
$$

Our goal is to find p using the focal diameter.

3. Relate Focal Diameter to p

Since the focal diameter D equals $4p$:

$$
D = 4p \Rightarrow p = \frac{D}{4}
$$

Therefore, once D is known, p can be directly computed.

4. Determine the Focus Coordinates

The focus is at:

$$
(x_f, y_f) = (h + p, k) = (8 + p, 4)
$$

The directrix is:

$$
x = h - p = 8 - p
$$

Practical Example: Calculating the Equation with a Given Focal Diameter

Suppose the focal diameter D is given as 6 units.

Step 1: Calculate p:

$$
p = \frac{D}{4} = \frac{6}{4} = 1.5
$$

Step 2: Write the equation:

$$
(y - 4)^2 = 4 \times 1.5 (x - 8)
$$

Simplify:

$$
(y - 4)^2 = 6 (x - 8)
$$

Step 3: Final Equation:

$$
(y - 4)^2 = 6 (x - 8)
$$

This is the equation of the parabola that opens to the right, with vertex at $(8, 4)$, and a focal diameter of 6 units.

Additional Considerations and Variations

While the above example provides a clear methodology, real-world problems may involve different focal diameters or additional parameters. Here are some considerations:

Handling Unknown Focal Diameter

- If the focal diameter is not given, it must be provided or derived from other data.
- The focal diameter directly influences p, and thus the shape and width of the parabola.

Adjusting for Different Orientations

- For a parabola opening upward or downward, the standard form changes to $(x - h)^2 = 4p (y - k)$.
- The principles remain similar, with appropriate adjustments in formulas.

Graphing the Parabola

- Plot the vertex at $(8, 4)$.
- Mark the focus at $(8 + p, 4)$.
- Draw the directrix at $x = 8 - p$.
- Sketch the parabola passing through key points, ensuring the focus and directrix are correctly positioned.

Summary and Key Takeaways

- The equation of a parabola opening to the right with vertex $(8, 4)$ can be written as $(y - 4)^2 = 4p (x - 8)$.
- The focal diameter D relates to p via $D = 4p$.
- To find the specific equation, determine $p = D/4$, then substitute into the standard form.
- The focus is at $(8 + p, 4)$, and the directrix is at $x = 8 - p$.

By understanding these steps and relationships, you can confidently derive the equation of any parabola given its vertex and focal diameter, especially those opening to the right.

Conclusion

Determining the equation of a parabola with specified properties is a fundamental skill in coordinate geometry. Remember to identify the parabola's orientation, relate the focal diameter to the parameter p, and use the standard form accordingly. Whether working with real-world data or academic problems, this approach provides a clear pathway to understanding and deriving the parabola's equation efficiently.

SEO Keywords and Phrases

- Equation of a parabola
- Parabola opening to the right
- Vertex of a parabola

- Focal diameter of parabola
- Standard form of parabola equation
- Derive parabola equation from focus and vertex
- Horizontal parabola formula
- Coordinate geometry parabola
- Focus and directrix of parabola
- Parabola derivation step-by-step

If you have specific focal diameters or additional parameters, applying this methodology will help you accurately determine the parabola's equation tailored to your problem.

Frequently Asked Questions

How do you find the equation of a parabola that opens to the right with vertex (8, 4) and a given focal diameter?

For a parabola opening to the right with vertex (h, k), the standard form is $(y - k)^2 = 4p(x - h)$. Given the focal diameter (length of the latus rectum), which equals $4|p|$, you can find p and then write the equation accordingly.

What is the relationship between the focal diameter and the value of p in a parabola opening to the right?

The focal diameter is equal to 4 times the absolute value of p. So, if the focal diameter is known, p = focal diameter / 4. This value of p determines the width and shape of the parabola.

Given the vertex (8, 4) and focal diameter of 6, what is the equation of the parabola opening to the right?

First, calculate $p = 6 / 4 = 1.5$. Since it opens to the right, the equation is $(y - 4)^2 = 4(1.5)(x - 8)$, which simplifies to $(y - 4)^2 = 6(x - 8)$.

How do you determine the focus point of a parabola that opens to the right with a known vertex and p?

The focus is located at (h + p, k). If the vertex is (h, k) and p is known, then the focus is at (h + p, k). For example, with vertex (8, 4) and p = 1.5, the focus is at (8 + 1.5, 4) = (9.5, 4).

What are the key steps to derive the equation of a

parabola that opens to the right given its vertex and focal diameter?

First, determine p from the focal diameter (p = focal diameter / 4). Then, write the standard form (y - k)^2 = 4p(x - h) using the vertex (h, k). Substitute the values of p, h, and k to obtain the equation.

Additional Resources

Parabola Equation Derivation: Unlocking the Geometry of a Right-Opening Parabola with a Known Vertex and Focal Diameter

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When delving into the fascinating world of conic sections, the parabola stands out for its elegant structure and wide-ranging applications—from satellite dishes to satellite trajectories, and even in the design of headlights. Understanding how to precisely determine the equation of a parabola given specific features, such as its vertex and focal diameter, is fundamental in both theoretical mathematics and practical engineering.

In this comprehensive review, we explore the step-by-step process of deriving the equation of a parabola that opens to the right, with a vertex at (8, 4), and a specified focal diameter. Our goal is to clarify the underlying principles, illuminate the methods, and equip you with the tools to analyze similar problems confidently.

Understanding the Key Components of a Parabola

Before jumping into calculations, it’s essential to grasp the core features that define a parabola:

The Vertex

- The point where the parabola changes direction.
- For our case, Vertex (8, 4) indicates the parabola's tip is located at that coordinate.

The Focus

- A point located along the parabola's axis of symmetry such that the parabola is the set of points equidistant from the focus and the directrix.

The Directrix

- A fixed line perpendicular to the axis of symmetry, serving as a reference for the parabola's shape.

The Focal Diameter (Latus Rectum)

- The length of the chord passing through the focus and perpendicular to the axis of symmetry.
- It provides a measure of the parabola's "width" at the focus.

Determining the Parabola’s Orientation and Basic Form

Since the parabola opens to the right, its axis of symmetry is horizontal, heading from left to right. This orientation simplifies our coordinate considerations because:

- The parabola's vertex is at (8, 4).
- The focus is located somewhere to the right of the vertex, along a line $y = 4$.

This configuration aligns with the standard form of a parabola opening to the right:

$$(y - k)^2 = 4p(x - h)$$

Where:

- (h, k): Vertex coordinates.
- p: Distance from the vertex to the focus along the axis of symmetry.
- $4p$: The focal parameter, related directly to the parabola's focal diameter.

Step 1: Establishing the Standard Equation Framework

Given the parabola opens to the right and vertex at $(8, 4)$, the general form is:

$$(y - 4)^2 = 4p (x - 8)$$

The sign and magnitude of p determine the direction and steepness of the parabola:

- $p > 0$: Opens to the right.
- $p < 0$: Opens to the left.

Our goal is to find the value of p, which directly influences the parabola's focus and focal diameter.

Step 2: Connecting Focal Diameter to the Parabola

The focal diameter (also called the length of the latus rectum) is given as a specific value. It is directly related to p:

$$ \text{Focal Diameter} = |4p| $$

This relationship arises because:

- The focus is located at $(h + p, k)$.
- The latus rectum is a line segment passing through the focus, perpendicular to the axis of symmetry, with length $ |4p| $.

Therefore:

$$ \text{Focal Diameter} = |4p| $$

Suppose the focal diameter is known; then:

$$ p = \pm \frac{\text{Focal Diameter}}{4} $$

Since the parabola opens to the right, and the focus is to the right of the vertex, p must be positive:

$$ p = \frac{\text{Focal Diameter}}{4} $$

Note: The problem statement mentions "has a focal diameter" but does not specify the numerical value. For demonstration, let's assume the focal diameter is given as 8 units.

Step 3: Calculating p and the Focus Location

Given:

$$ \text{Focal Diameter} = 8 $$

Therefore:

$$ p = \frac{8}{4} = 2 $$

The focus is located at:

$$ (h + p, k) = (8 + 2, 4) = (10, 4) $$

The directrix, being perpendicular to the axis and equidistant from the vertex but in the opposite direction, is:

$$ x = h - p = 8 - 2 = 6 $$

Step 4: Writing the Equation of the Parabola

With all parameters established, we substitute into the standard parabola formula:

$$ (y - 4)^2 = 4p (x - 8) $$

Plugging in $p = 2$:

$$ (y - 4)^2 = 8(x - 8) $$

This is the explicit equation of the parabola that opens to the right, has vertex (8, 4), and focal diameter 8.

Additional Considerations and Variations

If the focal diameter differs

- For a focal diameter d, the equation becomes:

$$ (y - 4)^2 = d (x - 8) $$

- The focus is at $(8 + \frac{d}{4}, 4)$.
- The directrix is at $x = 8 - \frac{d}{4}$.

If the parabola opens to the left

- p would be negative.
- The equation would be:

$$ (y - 4)^2 = -|4p| (x - 8) $$

For other focal diameters or vertex positions

- Adjust p accordingly.
- Recalculate focus and directrix based on the new parameters.

Summary: The Complete Equation and Its Significance

Final Equation:

$$ (y - 4)^2 = 8 (x - 8) $$

This equation precisely characterizes the parabola in question, encapsulating its geometric properties:

- Vertex at (8, 4).
- Opens to the right.
- Focal diameter of 8 units.
- Focus at (10, 4).
- Directrix at $x = 6$.

Key Takeaways for Practitioners and Students

- The standard form of a parabola simplifies analysis when the orientation is known.
- The focal diameter directly relates to the parameter p via $d = |4p|$.
- Knowing the vertex and focal diameter allows for quick derivation of the focus and directrix, leading to the full equation.
- This process exemplifies the power of algebraic methods in geometric problem-solving.

Conclusion

Understanding how to determine the equation of a parabola from given features is fundamental for both theoretical exploration and real-world applications. By carefully analyzing the vertex, focal diameter, and orientation, one can derive precise mathematical representations that serve as foundational tools in various scientific and engineering fields.

This detailed walkthrough emphasizes the importance of connecting geometric intuition with algebraic formulation, ensuring that each step is grounded in the properties of conic sections. Whether you're designing a reflective surface or analyzing projectile motion, mastering these principles will significantly enhance your analytical toolkit.

Note: The specific numerical example assumes a focal diameter of 8 units for illustrative purposes. Adjustments should be made based on the actual focal diameter provided in any problem.

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