Determine Whether The Vector Field Is Conservative. F(x, Y) = 4y /x I + 4X/y2 J A. Conservative B. Not

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Introduction

In vector calculus, understanding whether a vector field is conservative is a fundamental concept that has significant implications in physics and engineering. A conservative vector field is one where the work done by the field along any path depends only on the endpoints, not on the specific path taken. This property is crucial when analyzing potential energy, gravitational fields, electrostatics, and fluid flow.

In this article, we will explore how to determine whether the given vector field:

$$
\mathbf{F}(x, y) = \frac{4y}{x} \mathbf{i} + \frac{4x}{y^2} \mathbf{j}
$$

is conservative or not. We will delve into the definitions, key criteria, calculation steps, and interpret the results thoroughly to help you understand the process.

Understanding Conservative Vector Fields

What Is a Conservative Vector Field?

A vector field $\mathbf{F}(x, y)$ in two dimensions is called conservative if there exists a scalar potential function $\phi(x, y)$ such that:

$$
\mathbf{F} = \nabla \phi = \left(\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y} \right)
$$

This means:

$$
F_x = \frac{\partial \phi}{\partial x} \quad \text{and} \quad F_y = \frac{\partial \phi}{\partial y}
$$

\]

Key features of conservative fields:

- The line integral of $\mathbf{F}$ between two points is path-independent.
- The curl (or in 2D, the scalar curl) of $\mathbf{F}$ is zero everywhere in simply connected regions.

Why is Determining Conservativeness Important?

- Physical interpretation: Conservative fields correspond to force fields where energy is conserved.
- Mathematical convenience: Simplifies calculations of work and potential energy.
- Application in physics: Gravitational, electrostatic, and elastic fields are often conservative.

Mathematical Criteria for Conservativeness

In two dimensions, a vector field $\mathbf{F} = P(x, y) \mathbf{i} + Q(x, y) \mathbf{j}$ is conservative if:

1. The domain is simply connected (no holes).
2. The curl of the field is zero:

$$
\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}
$$

This condition ensures the existence of a potential function.

Step-by-Step Analysis of $\mathbf{F}(x, y) = \frac{4 y}{x} \mathbf{i} + \frac{4 x}{y^2} \mathbf{j}$

1. Identify $P(x, y)$ and $Q(x, y)$

$$
P(x, y) = \frac{4 y}{x}
$$
$$
Q(x, y) = \frac{4 x}{y^2}
$$

Note: The domain restrictions are important; for P, $x \neq 0$, and for Q, $y \neq 0$.

2. Compute $\frac{\partial Q}{\partial x}$

$$
\frac{\partial Q}{\partial x} = \frac{\partial}{\partial x} \left(\frac{4 x}{y^2} \right) = \frac{4}{y^2}
$$

since y is treated as a constant.

3. Compute $\frac{\partial P}{\partial y}$

$$
\frac{\partial P}{\partial y} = \frac{\partial}{\partial y} \left(\frac{4 y}{x} \right) = \frac{4}{x}
$$

since x is a constant during differentiation with respect to y.

---

4. Compare the Mixed Partial Derivatives

Recall the condition for conservative fields in 2D:

$$
\frac{\partial Q}{\partial x} \stackrel{?}{=} \frac{\partial P}{\partial y}
$$

From calculations:

$$
\frac{\partial Q}{\partial x} = \frac{4}{y^2}
$$
$$
\frac{\partial P}{\partial y} = \frac{4}{x}
$$

These are not equal in general, unless specific conditions are met (e.g., at particular points where $x = y^2$, but this does not hold universally).

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5. Conclusion: Is $\mathbf{F}$ Conservative?

Since:

$$
\frac{\partial Q}{\partial x} \neq \frac{\partial P}{\partial y}
$$

the vector field $\mathbf{F}(x, y)$ is not conservative in the domain where both components are defined.

Important note: The domain restrictions ($x \neq 0$, $y \neq 0$) mean the field is defined on \hemph{punctured regions} such as $\mathbb{R}^2 \setminus \{x=0\} \cup \{y=0\}$, which are not simply connected. This further supports the conclusion that $\mathbf{F}$ is not conservative in its domain.

Additional Insights

Physical and Mathematical Implications

- Because $\mathbf{F}$ is not conservative, there is no scalar potential $\phi(x, y)$ satisfying $\nabla \phi = \mathbf{F}$.
- The work done moving along different paths between the same points may vary, indicating non-conservative behavior.
- The non-zero curl ($\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}$) confirms the non-conservativeness.

Visualizing the Vector Field

Understanding the behavior of $\mathbf{F}$ can be enhanced through visualization:

- The x-component $F_x = \frac{4 y}{x}$ suggests that for fixed y, the component varies inversely with x.
- The y-component $F_y = \frac{4 x}{y^2}$ indicates a dependence on x scaled inversely with the square of y.
- Plotting the vectors can reveal flow patterns and regions where the field's behavior changes drastically, especially near axes where the components become undefined.

Summary and Final Verdict

Criterion	Result	Explanation
Is the curl zero?	No	$\frac{\partial Q}{\partial x} = \frac{4}{y^2}$ and $\frac{\partial P}{\partial y} = \frac{4}{x}$ are not equal.
Is the domain simply connected?	No	Domain excludes axes $x=0$ and $y=0$, which creates holes.
Does a potential function exist?	No	Due to non-zero curl and domain topology.

Therefore, the vector field $\mathbf{F}(x, y)$ is not conservative.

Final Answer:

B. Not

Additional Tips for Determining Conservativeness

- Always check the partial derivatives $\frac{\partial Q}{\partial x}$ and $\frac{\partial P}{\partial y}$.
- Ensure the domain is simply connected; otherwise, the field might be conservative in a restricted region.
- For fields with variable coefficients, consider whether the line integral is path-independent.

Conclusion

Determining whether a vector field is conservative involves analyzing the derivatives of its components and understanding the domain's topology. In our specific case, the mismatch in the partial derivatives confirms that $\mathbf{F}(x, y) = \frac{4 y}{x} \mathbf{i} + \frac{4 x}{y^2} \mathbf{j}$ is not a conservative vector field. Recognizing this property is essential when solving related problems in physics, engineering, and calculus, particularly in potential theory and flow analysis.

Keywords for SEO Optimization

- Conservative vector field
- Non-conservative vector field
- Vector calculus
- Partial derivatives
- Curl in 2D
- Potential function
- Path independence
- Field analysis
- Mathematical criteria for conservativeness
- Vector field examples

Frequently Asked Questions

How can I determine if the vector field F(x, y) = (4y/x) i + (4x/y^2) j is conservative?

To determine if the vector field is conservative, check if its curl is zero in the domain. If curl F = 0 everywhere and the domain is simply connected, then F is conservative.

What is the curl of the vector field F(x, y) = (4y/x) i + (4x/y^2) j?

The curl in two dimensions is given by $\partial Q/\partial x - \partial P/\partial y$. For this field, curl F = $\partial(4x/y^2)/\partial x - \partial(4y/x)/\partial y$, which simplifies to zero, indicating the field is conservative in a simply connected domain.

Is the domain of F(x, y) = (4y/x) i + (4x/y^2) j simply connected?

The domain excludes points where division by zero occurs, such as x=0 or y=0. Since these points create discontinuities, the domain is not simply connected everywhere, which affects whether the field is conservative.

Given the calculations, is the vector field F(x, y) = (4y/x) i + (4x/y^2) j conservative?

Yes, in the domain where $x \neq 0$ and $y \neq 0$, the curl is zero, and the domain is simply connected, so the vector field is conservative.

What is the significance of the curl being zero for F(x, y) = (4y/x) i + (4x/y^2) j?

A zero curl indicates that the vector field is irrotational, which is a necessary condition for being conservative, provided the domain is simply connected.

Based on the analysis, should the vector field F(x, y) = (4y/x) i + (4x/y^2) j be classified as conservative?

Yes, since the curl is zero in the domain where the field is defined and the domain is simply connected (excluding points where x=0 or y=0), the field is conservative.

Additional Resources

Determine Whether the Vector Field Is Conservative: An In-Depth Analysis of F(x, y) = (4y/x) i + (4x/y^2) j

In the realm of vector calculus, understanding the nature of a vector field is fundamental, especially when distinguishing between conservative and non-conservative fields. This distinction has profound implications across physics and engineering, particularly in the study of potential functions, work done by forces, and the behavior of fields such as gravity and electromagnetism. In this article, we undertake a comprehensive examination of the vector field $F(x, y) = (4y/x)\ i + (4x/y^2)\ j$, aiming to determine whether it is conservative or not. Through meticulous analysis, including mathematical criteria and detailed calculations, we will shed light on the characteristics of this field and elucidate the underlying principles that govern such classifications.

Understanding Conservative Vector Fields

Definition and Significance

A vector field $F(x, y)$ in two dimensions is said to be conservative if there exists a scalar potential function $\varphi(x, y)$ such that:

$$
\nabla \phi = \mathbf{F}
$$

or equivalently,

$$
\frac{\partial \phi}{\partial x} = P(x, y) \quad \text{and} \quad
\frac{\partial \phi}{\partial y} = Q(x, y)
$$

where $F(x, y) = P(x, y)\ i + Q(x, y)\ j$.

The importance of identifying a conservative field lies in its properties:
- The line integral of F between two points is path-independent.
- The work done by the field in moving an object between two points depends only on the endpoints, not the path taken.
- The presence of a potential function simplifies the analysis of the field considerably.

Mathematical Criteria for Conservativeness in 2D

In two dimensions, a continuous vector field $F(x, y) = P(x, y)\ i + Q(x, y)\ j$ is conservative if and only if:
1. The partial derivatives $\partial P/\partial y$ and $\partial Q/\partial x$ are continuous on a simply connected domain.
2. The curl of F is zero, which in 2D reduces to:

\[

$$
\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}
$$

This criterion stems from Clairaut's theorem (symmetry of second derivatives) and ensures the existence of a potential function.

Analyzing the Vector Field $F(x, y) = (4y/x)\, i + (4x/y^2)\, j$

Component Functions

Given:

$$
P(x, y) = \frac{4y}{x}
$$

$$
Q(x, y) = \frac{4x}{y^2}
$$

Our goal is to determine whether F is conservative by analyzing the derivatives $\partial P/\partial y$ and $\partial Q/\partial x$.

Domain Considerations

Before proceeding, it is crucial to identify the domain of F:
- The function $P(x, y) = 4y/x$ is defined for all $x \neq 0$.
- The function $Q(x, y) = 4x/y^2$ is defined for $y \neq 0$.

Therefore, the domain D is:

$$
D = \left\{ (x, y) \in \mathbb{R}^2 \; | \; x \neq 0, y \neq 0 \right\}
$$

This domain is not simply connected because it excludes the axes x=0 and y=0. The connectivity of the domain is significant because conservative fields are typically studied in simply connected regions. The presence of holes (due to the missing axes) could influence whether the field is conservative.

Calculating the Necessary Derivatives

Partial derivative of P with respect to y

$$
\frac{\partial P}{\partial y} = \frac{\partial}{\partial y} \left(\frac{4y}{x} \right) = \frac{4}{x}
$$

Since $x \neq 0$, this derivative is well-defined on the domain.

Partial derivative of Q with respect to x

$$
\frac{\partial Q}{\partial x} = \frac{\partial}{\partial x} \left(\frac{4x}{y^2} \right) = \frac{4}{y^2}
$$

Again, $y \neq 0$, so this derivative is defined on the domain.

Comparison of Derivatives and Implications

The critical step in determining whether F is conservative involves comparing $\partial P/\partial y$ and $\partial Q/\partial x$:

$$
\frac{\partial P}{\partial y} = \frac{4}{x}
$$
$$
\frac{\partial Q}{\partial x} = \frac{4}{y^2}
$$

Since these two derivatives are not equal in general:

$$
\frac{4}{x} \neq \frac{4}{y^2}
$$

for most points in the domain, the curl of F is non-zero:

$$
\text{curl } \mathbf{F} = \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \neq 0
$$

This indicates that F is not conservative in the domain D.

Considering Domain Topology and Potential Exceptions

The analysis above suggests that F is not conservative due to the mismatch of derivatives. However, the domain's topology could influence this conclusion.

Key points:
- The domain excluding the axes means F is not defined on the entire plane, only on regions that avoid $x=0$ and $y=0$.
- Since the domain is not simply connected, the standard criterion for conservativeness (curl zero in a simply connected domain) does not automatically apply.

However, even if the domain were simply connected (say, considering only a quadrant or the entire plane minus the axes), the non-zero curl would still imply non-conservativeness.

Special cases:
- If we restrict ourselves to a domain where x and y are positive (e.g., $x > 0$, $y > 0$), the derivatives remain as above, and the field is still non-conservative because the derivatives are unequal.
- The existence of a potential function $\varphi(x, y)$ would require that $\partial\varphi/\partial x = P(x, y)$ and $\partial\varphi/\partial y = Q(x, y)$ are compatible, which they are not, given the derivative mismatch.

Conclusion: The field is not conservative on any domain where both x and y are allowed to vary freely (excluding the axes). The non-zero curl confirms this.

Additional Insights and Physical Interpretation

Understanding whether a vector field is conservative is not just a mathematical exercise; it has physical implications:

- Conservative fields often model forces where energy is conserved, such as gravitational or electrostatic fields. They allow for potential functions that simplify calculations.
- Non-conservative fields may represent phenomena like magnetic fields or certain velocity fields in fluid dynamics, where work depends on the path taken.

In the case of $F(x, y) = (4y/x)\ i + (4x/y^2)\ j$, the non-zero curl indicates it cannot be derived from a potential function, implying it does not conserve energy in the classical sense and exhibits properties consistent with non-conservative forces or flows.

Final Verdict and Summary

Based on rigorous mathematical analysis, the vector field $F(x, y) = (4y/x)\ i + (4x/y^2)\ j$ is not conservative.

Key points summarized:
- The partial derivatives $\partial P/\partial y$ and $\partial Q/\partial x$ are:

$$
\frac{\partial P}{\partial y} = \frac{4}{x} \quad \text{and} \quad
\frac{\partial Q}{\partial x} = \frac{4}{y^2}
$$

- These derivatives are not equal in general, indicating a non-zero curl.
- The domain's topology, while excluding axes, does not alter the fundamental conclusion.
- The field cannot be expressed as the gradient of a scalar potential function, confirming it is not conservative.

Therefore, the answer is:

B. Not conservative

Conclusion

The analysis of the vector field $F(x, y) = (4y/x)\ i + (4x/y^2)\ j$ underscores the importance of derivative comparisons in classifying vector fields. The non-zero curl and derivative mismatch firmly establish that F is not conservative over its domain. This insight is pivotal in various applications, guiding engineers and scientists in understanding the

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