# Diogo's Utility Function Is U= 100X0.9Z0.1 The Price Of X Is Px = $12, The Price Of Z Is Pz = $14, And

## Understanding Diogo's Utility Function and Consumer Choice Theory

**Diogo's Utility Function Is U= 100X0.9Z0.1 The Price Of X Is Px = $12, The Price Of Z Is Pz = $14, And** provides a foundational framework for analyzing how Diogo allocates his income between two goods, X and Z, to maximize his satisfaction. This utility function reveals his preferences and the relative importance he places on each good, while the given prices help determine his optimal consumption bundle. In this article, we will explore the intricacies of this utility function, how to compute Diogo's optimal consumption, and the broader implications in consumer choice theory.

## Decoding the Utility Function: The Basics

### What Is a Utility Function?

A utility function represents a consumer's preferences over a set of goods and services. It assigns a numerical value (utility) to different combinations of goods, with higher values indicating more preferred combinations. In Diogo's case, his utility function is:

- $U= 100X^{0.9}Z^{0.1}$

This form is a Cobb-Douglas utility function, characterized by its constant elasticity of substitution and specific exponents that reflect relative preferences.

### Interpreting the Exponents

- Exponent of X (0.9): Indicates Diogo's strong preference for good X, as it contributes more to his overall utility.
- Exponent of Z (0.1): Shows a lesser preference for Z, but it still plays a role in maximizing his satisfaction.

The sum of the exponents (0.9 + 0.1 = 1) confirms that this is a constant-elasticity-of-substitution (CES) utility function, typical of Cobb-Douglas forms.

# Calculating the Consumer's Budget Constraint

## Budget Equation

Diogo’s total income allows him to purchase goods X and Z at prices Px and Pz respectively. The budget constraint is:

- $P_x X + P_z Z = I$

Given:
- Px = $12
- Pz = $14
- Income (I): Assume Diogo has a certain income, say, $168 for illustration purposes.

Therefore, the budget constraint becomes:

- $12 X + $14 Z = $168

This equation defines all feasible combinations of X and Z that Diogo can purchase with his income.

## Implications of the Budget Constraint

- As X increases, Z must decrease if income remains constant.
- The slope of the budget line is determined by the ratio of prices: -Px / Pz = -12/14 ≈ -0.857.

Understanding this helps in identifying the point where Diogo maximizes his utility subject to his budget.

# Maximizing Utility: The Optimization Process

## Setting Up the Problem

Diogo aims to choose quantities of X and Z that maximize:

U= 100X0.9Z0.1

subject to the budget constraint:

12X + 14Z = 168

This is a constrained optimization problem, solvable using methods like Lagrange multipliers.

## Applying the Lagrangian Method

Define the Lagrangian:

L = 100X0.9Z0.1 + λ(168 - 12X - 14Z)

Where λ is the Lagrange multiplier.

## First-Order Conditions

To find the maximum, take partial derivatives:

1. ∂L/∂X = 100 0.9 X−0.1 Z0.1 - 12λ = 0
2. ∂L/∂Z = 100 0.1 X0.9 Z−0.9 - 14λ = 0
3. ∂L/∂λ = 168 - 12X - 14Z = 0

By solving these equations simultaneously, Diogo's optimal quantities of X and Z can be determined.

# Deriving the Optimal Consumption Bundle

## Step-by-Step Solution

- From the first two conditions, set the ratios of the derivatives to eliminate λ:

(100 0.9 X−0.1 Z0.1) / (100 0.1 X0.9 Z−0.9) = 12 / 14

- Simplify numerator and denominator:

(0.9 X−0.1 Z0.1) / (0.1 X0.9 Z−0.9) = 12 / 14

- Recognize that:

X−0.1 / X0.9 = X−1.0

Z0.1 / Z−0.9 = Z1.0

- Therefore:

(0.9 / 0.1) X−1.0 Z1.0 = 12 / 14

- Calculate:

9 (Z / X) = 12 / 14 ≈ 0.857

- Solving for Z / X:

$Z / X \approx 0.857 / 9 \approx 0.0952$

- This ratio indicates the relative quantities of Z and X Diogo will consume at optimality.

### Solving for X and Z

Using the budget constraint:

$12X + 14Z = 168$

Substitute $Z = 0.0952X$:

$12X + 14\ 0.0952X = 168$

$12X + 1.333X = 168$

$13.333X = 168$

$X \approx 12.6$ units

$Z \approx 0.0952\ 12.6 \approx 1.2$ units

Thus, Diogo’s optimal consumption bundle is approximately:

- $X \approx 12.6$ units
- $Z \approx 1.2$ units

## Interpreting the Results and Consumer Behavior

### Utility Maximization Outcome

Diogo will allocate his income to purchase roughly 12.6 units of good X and 1.2 units of good Z to maximize his utility, given the prices and his preferences.

### Implications of Price Changes

- If the price of X decreases, Diogo can afford more of X, likely increasing his utility.
- If the price of Z increases, Z becomes less attractive relative to X, and Diogo might reduce Z consumption.

### Elasticity of Demand

Given the Cobb-Douglas form, demand for each good typically has unit elasticity, meaning a percentage change in price results in an equal percentage change in demand, maintaining proportional utility levels.

# Broader Applications in Economics and Policy

## Consumer Choice Theory

Understanding utility functions like Diogo’s helps economists predict consumer responses to price changes, income variations, and policy interventions.

## Implications for Market Equilibrium

- Firms can use this analysis to set prices strategically.
- Policymakers can assess how taxes or subsidies influence consumer welfare.

## Limitations and Assumptions

- Assumes perfect rationality and consistent preferences.
- Presumes utility can be quantified and compared.
- Real-world preferences may be more complex and variable.

# Conclusion

Analyzing Diogo’s utility function and the associated budget constraint reveals the delicate balance consumers strike between preferences and affordability. By applying mathematical optimization techniques, we gain insight into real-world decision-making processes and the factors influencing consumer behavior. This understanding is vital for economists, policymakers, and businesses seeking to predict and influence market outcomes effectively.

Key Takeaways:
- Utility functions like U= 100X0.9Z0.1 represent consumer preferences.
- Budget constraints limit feasible consumption bundles.
- Optimization methods determine the best combination of goods for maximum utility.
- Price changes significantly impact consumer choices and demand elasticity.
- These concepts are fundamental in understanding market dynamics and designing effective economic policies.

# Frequently Asked Questions

## What is Diogo's utility function?

Diogo's utility function is $U = 100X^{0.9}Z^{0.1}$.

## What are the prices of goods X and Z?

The price of X is \$12, and the price of Z is \$14.

## How can Diogo maximize his utility given his budget constraint?

He can maximize utility by allocating his budget optimally between X and Z, considering their prices and the utility function, often using methods like the Lagrangian multiplier or marginal utility per dollar.

## What is the marginal utility of X and Z in Diogo's utility function?

Marginal utility of X is proportional to 0.9 100X^{-0.1}Z^{0.1}, and for Z it is proportional to 0.1 100X^{0.9}Z^{-0.9}.

## How does the elasticity of substitution affect Diogo's choice between X and Z?

Since the utility function is Cobb-Douglas, the elasticity of substitution is 1, meaning Diogo can substitute between X and Z at a constant rate depending on relative prices.

## What is the optimal consumption bundle for Diogo given his budget?

The optimal bundle can be found by setting the ratio of marginal utilities equal to the ratio of prices: (0.9/0.1) (Z/X) = Pz/Px, leading to Z/X = (Pz/Px) (0.1/0.9).

## How does a change in the prices of X or Z affect Diogo's consumption?

An increase in the price of a good typically decreases its consumption, while a decrease tends to increase it, assuming other factors remain constant, following the law of demand.

## What is the total budget needed for Diogo to purchase a certain quantity of X and Z?

The total budget is calculated as Budget = Px X + Pz Z. For specific quantities, substitute the values to determine the total cost.

## Can Diogo achieve higher utility by increasing consumption of X or Z independently?

In general, increasing consumption of either good without considering the budget constraint may not increase utility. Optimal utility is achieved by balancing both goods within the budget to maximize the utility function.

## Additional Resources

Diogo's Utility Function Is U= 100X0.9Z0.1 The Price Of X Is Px = $12, The Price Of Z Is Pz = $14, And

Understanding consumer choices and how individuals allocate their budgets among various goods is a cornerstone of microeconomic theory. For consumers like Diogo, who seek to maximize satisfaction within their financial constraints, analyzing their utility functions alongside market prices provides valuable insights into their purchasing behavior. In this article, we delve into the specifics of Diogo's utility function, explore how he makes decisions given the prices of goods X and Z, and examine the broader implications for understanding consumer preferences and market dynamics.

---

The Foundation: Deciphering Diogo's Utility Function

What Is a Utility Function?

At its core, a utility function represents a consumer's preference structure, assigning a numerical value—called utility—to different bundles of goods. The higher the utility, the more preferred or satisfying the bundle is to the consumer. Utility functions serve as mathematical tools that help economists analyze how consumers make choices to maximize their satisfaction given their budget constraints.

The Specific Utility Function: U= 100X0.9Z0.1

In Diogo's case, his utility function is expressed as:

$$ U = 100 \times X^{0.9} \times Z^{0.1} $$

where:

- $X$ is the quantity of good X consumed.
- $Z$ is the quantity of good Z consumed.

This function is a form of Cobb-Douglas utility, characterized by constant elasticities of substitution. The exponents, 0.9 for X and 0.1 for Z, reveal how Diogo values these goods relative to each other.

Interpreting the Exponents and Utility

- Exponent of X (0.9): Indicates that Diogo derives a significant portion of his satisfaction from good X. A higher exponent suggests a strong preference or reliance on good X.
- Exponent of Z (0.1): Indicates that Z contributes less to Diogo's overall utility, but still plays a role in his preferences.

The coefficient 100 acts as a scaling factor, amplifying the utility levels but not affecting the relative preferences.

Implications of the Utility Function

- Diminishing Marginal Utility: Because this is a Cobb-Douglas function, marginal utility diminishes as the quantity of each good increases, reflecting realistic consumer behavior.
- Substitutes and Complements: The exponents suggest that X is more substitutable or more heavily weighted than Z in Diogo’s preferences.

---

## Budget Constraints and Market Prices

### The Financial Landscape

Diogo faces the following market prices:

- Price of X ($P_x$) = $12
- Price of Z ($P_z$) = $14

His total budget, which is a critical factor in decision-making, constrains his choices. While the total budget isn’t explicitly specified in the initial data, understanding how to optimize utility given these prices is essential.

### The Budget Line

The budget constraint can be expressed as:

$$P_x \times X + P_z \times Z \leq M$$

where:

- $M$ is Diogo’s total available income or budget.

Given prices, the maximum quantities he can purchase depend on his income. The budget line represents all possible combinations of $X$ and $Z$ that Diogo can afford.

---

## Maximizing Utility: The Optimization Process

### The Objective

Diogo aims to choose quantities of $X$ and $Z$ that maximize his utility function while respecting his budget constraint:

$$
\text{Maximize } U = 100 \times X^{0.9} \times Z^{0.1} \\
\text{Subject to } 12X + 14Z \leq M
$$

### The Method: Lagrangian Optimization

To solve this constrained optimization problem, economists often use the Lagrangian method, which involves setting up the following:

$$
\mathcal{L} = 100 \times X^{0.9} \times Z^{0.1} + \lambda ( M - 12X - 14Z )
$$

where $\lambda$ is the Lagrange multiplier representing the marginal utility of income.

### Deriving the Optimal Bundle

By taking partial derivatives with respect to $X$ and $Z$, and setting them to zero, Diogo’s optimal consumption bundle can be found:

$$
\frac{\partial \mathcal{L}}{\partial X} = 100 \times 0.9 \times X^{-0.1} \times Z^{0.1} - 12 \lambda = 0
$$
$$
\frac{\partial \mathcal{L}}{\partial Z} = 100 \times 0.1 \times X^{0.9} \times Z^{-0.9} - 14 \lambda = 0
$$
$$
\frac{\partial \mathcal{L}}{\partial \lambda} = M - 12X - 14Z = 0
$$

Dividing the first by the second yields the ratio of marginal utilities per dollar:

$$
\frac{0.9 \times X^{-0.1} \times Z^{0.1}}{0.1 \times X^{0.9} \times Z^{-0.9}} = \frac{12}{14}
$$

Simplifying this expression provides the relationship between $X$ and $Z$ at the optimum, guiding Diogo’s choice.

---

## Practical Insights: How Diogo Makes Decisions

### The Marginal Rate of Substitution (MRS)

The MRS indicates how much of Z Diogo is willing to give up for an additional unit of X while maintaining the same utility level. For Cobb-Douglas functions, the MRS is:

$$
\text{MRS}_{X,Z} = \frac{\partial U / \partial X}{\partial U / \partial Z} = \frac{0.9}{0.1} \times \frac{Z}{X} = 9 \times \frac{Z}{X}
$$

At the optimal point, the MRS equals the ratio of prices:

$$
\frac{P_x}{P_z} = \frac{12}{14} \approx 0.857
$$

Thus, the optimal bundle occurs where:

$$
9 \times \frac{Z}{X} = 0.857
$$

which simplifies to:

$$
\frac{Z}{X} \approx \frac{0.857}{9} \approx 0.0952
$$

or

$$
Z \approx 0.0952 \times X
$$

This ratio suggests Diogo prefers to allocate his budget in a way that maintains this proportion between $Z$ and $X$.

Calculating the Exact Quantities (Given Budget)

Suppose Diogo has a specific budget $M$. Using the budget constraint:

$$
12X + 14Z = M
$$

and the ratio:

$$
Z = 0.0952 \times X
$$

we can substitute and solve for $X$:

$$
12X + 14 \times 0.0952 \times X = M
$$
$$
(12 + 14 \times 0.0952) X = M
$$
$$
(12 + 1.333) X = M
$$
$$
13.333 X = M
$$
$$
X = \frac{M}{13.333}
$$

\]

Similarly,

$$
Z = 0.0952 \times \frac{M}{13.333}
$$

This calculation shows how Diogo would allocate his income across the two goods for a given budget, emphasizing the importance of the marginal utilities and prices in decision-making.

---

Broader Implications and Market Considerations

Consumer Behavior and Market Demand

Understanding Diogo’s utility maximization process sheds light on broader market demand patterns. When many consumers have similar utility functions and face comparable prices, aggregate demand for goods X and Z can be predicted based on these optimization principles.

Price Changes and Consumer Response

- If Price of X Increases: Diogo might reduce his consumption of X, substituting toward Z if possible, depending on substitution elasticity.
- If Price of Z Decreases: Consumption of Z would likely increase, given Diogo’s marginal utility considerations.

Policy and Business Strategy Implications

Businesses can leverage such analyses to optimize pricing strategies, product placement, and marketing campaigns by understanding consumer preferences and how they respond to price fluctuations.

---

Conclusion

Diogo’s utility function, $U = 100X^{0.9}Z^{0.1}$, provides a nuanced window into his preferences and decision-making process. By analyzing the relative importance of goods X and Z, their prices, and Diogo’s budget constraints, we gain insights into how consumers maximize satisfaction in real-world markets. Such models are fundamental not only for individual consumers but also for firms and policymakers aiming to understand demand dynamics, craft effective strategies, and shape economic policies. As markets evolve and prices fluctuate, understanding the underlying utility maximization principles remains central to deciphering consumer behavior and ensuring efficient resource allocation.

# Diogo S Utility Function Is U 100x0 9z0 1 The Price Of X Is Px 12 The Price Of Z Is Pz 14 And

Diogo S Utility Function Is U 100x0 9z0 1 The Price Of X Is Px 12 The Price Of Z Is Pz 14 And

## Related Articles


- Dust Billowed Up From The Dirt Road As Big Red Raced Through The Country To The Family Farm. Dan Drove
- Contingent Liabilities Should Be Recorded In The Accounts When: Group Of Answer Choices It Is Probable
- Consider The Process Of Signal Transduction In The Case Of An Animal's Response To Noise. Which Of The

Back to Home