Element X Decays Radioactively With A Half Life Of 10 Minutes. If There Are 270 Gramsof Element X, How

Element X Decays Radioactively With A Half Life Of 10 Minutes. If There Are 270 Grams of Element X, How to Calculate the Remaining Quantity Over Time

Radioactive decay is a fascinating process that has intrigued scientists for centuries. When dealing with radioactive elements, understanding how their quantities diminish over time is crucial for applications in medicine, nuclear energy, environmental science, and physics research. One common question posed in these contexts is: given a certain amount of a radioactive substance and its half-life, how much of it remains after a specific period?

In this article, we will explore the problem of calculating the remaining mass of Element X, which decays radioactively with a half-life of 10 minutes, starting with an initial amount of 270 grams. We will delve into the fundamental concepts of radioactive decay, introduce relevant formulas, perform detailed calculations, and discuss practical implications.

Understanding Radioactive Decay and Half-Life

What Is Radioactive Decay?

Radioactive decay is a spontaneous process by which unstable atomic nuclei lose energy by emitting radiation in the form of particles or electromagnetic waves. This process transforms the original nucleus into a different element or a different isotope of the same element.

Key points about radioactive decay:
- It is a random process at the level of individual atoms.
- The decay rate of a large number of atoms is predictable and follows a specific statistical pattern.
- The decay is characterized by a parameter called the decay constant.

Defining Half-Life

The half-life (denoted as \(T_{1/2} \)) of a radioactive isotope is the time required for half of the radioactive nuclei in a sample to decay.

For Element X:
- $T_{1/2} = 10$ minutes

This means:
- After 10 minutes, only 50% of the original amount remains.
- After 20 minutes, 25% remains.
- After 30 minutes, 12.5% remains, and so on.

Mathematical Foundations of Radioactive Decay

The Decay Law

The decay of a radioactive substance can be modeled mathematically with the exponential decay law:

$$
N(t) = N_0 \times e^{-\lambda t}
$$

Where:
- $N(t)$ = amount of substance remaining at time t
- N_0 = initial amount of substance
- λ = decay constant (per unit time)
- t = elapsed time

Relating Half-Life to Decay Constant

The decay constant λ is linked to the half-life $T_{1/2}$ through the formula:

$$
\lambda = \frac{\ln 2}{T_{1/2}}
$$

Given:
- $T_{1/2} = 10$ minutes

Calculate:

$$
\lambda = \frac{\ln 2}{10} \approx \frac{0.6931}{10} = 0.06931 \text{ min}^{-1}
$$

\]

This decay constant indicates the probability per unit time that a single atom will decay.

Calculating Remaining Mass of Element X After a Given Time

Suppose we start with 270 grams of Element X. To find how much remains after a certain period, we apply the decay law.

General Formula for Remaining Mass

$$
N(t) = N_0 \times e^{-\lambda t}
$$

Where:
- $N_0 = 270$ grams
- t = time in minutes
- $\lambda \approx 0.06931 \text{ min}^{-1}$

The remaining mass after t minutes is:

$$
\text{Remaining mass} = 270 \times e^{-0.06931 \times t}
$$

Practical Examples and Calculations

1. Remaining Mass After 10 Minutes

Since 10 minutes is the half-life:

\[
N(10) = 270 \times e^{-0.06931 \times 10} = 270 \times e^{-0.6931}

\]

Calculate:

$$
e^{-0.6931} \approx 0.5
$$

Thus:

$$
N(10) = 270 \times 0.5 = 135 \text{ grams}
$$

Result: After 10 minutes, 135 grams of Element X remain, confirming the concept of half-life.

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2. Remaining Mass After 20 Minutes

20 minutes is twice the half-life:

$$
N(20) = 270 \times e^{-0.06931 \times 20} = 270 \times e^{-1.3862}
$$

Calculate:

$$
e^{-1.3862} \approx 0.25
$$

Therefore:

$$
N(20) = 270 \times 0.25 = 67.5 \text{ grams}
$$

Result: After 20 minutes, approximately 67.5 grams remain, which is a quarter of the original amount.

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3. Remaining Mass After 30 Minutes

30 minutes corresponds to three half-lives:

$$
N(30) = 270 \times e^{-0.06931 \times 30} = 270 \times e^{-2.0793}
$$

Calculate:

$$
e^{-2.0793} \approx 0.125
$$

Thus:

$$
N(30) = 270 \times 0.125 = 33.75 \text{ grams}
$$

Result: After 30 minutes, roughly 33.75 grams of Element X remain.

Determining Remaining Mass After Arbitrary Time

The decay formula allows us to find the remaining amount after any arbitrary time t.

Example: Remaining mass after 45 minutes

Calculate:

$$
N(45) = 270 \times e^{-0.06931 \times 45}
$$

$$
e^{-3.119} \approx 0.044
$$

So:

$$
N(45) \approx 270 \times 0.044 \approx 11.88 \text{ grams}
$$

This illustrates the rapid decay characteristic of elements with short half-lives.

Applications of Radioactive Decay Calculations

Understanding how to calculate the remaining quantity of a radioactive element over time has numerous practical applications:

- Medical Treatments: Determining dosage and timing for radionuclide therapies.
- Nuclear Power: Managing fuel decay and waste disposal.
- Radiocarbon Dating: Estimating ages of archaeological samples.
- Environmental Monitoring: Tracking radioactive contamination over time.
- Nuclear Safety: Planning storage durations for radioactive waste.

Additional Considerations and Real-World Factors

While the mathematical model provides a precise theoretical framework, real-world scenarios often involve:

- Measurement uncertainties in initial mass and decay rates.
- External factors affecting decay, such as temperature, pressure, or chemical environment (though decay rates are typically unaffected).
- Decay chains, where decay products themselves are radioactive and influence subsequent calculations.
- Safety protocols for handling radioactive materials.

Conclusion

Calculating the remaining amount of a radioactive element like Element X, which has a half-life of 10 minutes, involves understanding exponential decay laws and applying them through formulas involving the decay constant. Starting with an initial 270 grams, the amount of Element X diminishes by half every 10 minutes, following a predictable exponential pattern.

By applying the decay law:

$$
N(t) = N_0 \times e^{-\lambda t}
$$

and calculating the decay constant $\lambda = \frac{\ln 2}{T_{1/2}}$, we can accurately determine the remaining mass after any given period. This knowledge is vital across various fields such as medicine, environmental science, archaeology, and nuclear energy management.

Understanding the principles of radioactive decay not only helps answer specific questions like "How much of Element X remains after a certain time?" but also provides insights into the fundamental behavior of unstable nuclei, contributing to advancements in science and technology.

Keywords: radioactive decay, half-life, decay constant, exponential decay, Element X, nuclear physics, decay calculations, radioactivity, decay law, scientific applications

Frequently Asked Questions

How much of Element X remains after 30 minutes if it initially weighs 270 grams?

After 30 minutes, which is 3 half-lives, the remaining amount is $270 / 2^3 = 270 / 8 = 33.75$ grams.

What is the decay constant (λ) for Element X with a half-life of 10 minutes?

The decay constant $\lambda = \ln(2) / \text{half-life} = 0.693 / 10 = 0.0693$ per minute.

How long will it take for 90% of Element X to decay?

Since 90% decays, 10% remains. Using $N = N0 (1/2)^{(t/T)}$, solving for t: $t = T \log2(N0/N) = 10 \log2(1/0.1) \approx 10 \; 3.32 \approx 33.2$ minutes.

If you start with 270 grams of Element X, how much remains after 50 minutes?

After 50 minutes, which is 5 half-lives, remaining amount = $270 / 2^5 = 270 / 32 \approx 8.44$ grams.

What is the activity (in decays per minute) of 270 grams of Element X initially?

First, calculate the number of atoms: N = (mass / molar mass) Avogadro's number; then activity $A = \lambda N$. Without molar mass, exact activity can't be determined, but the decay rate is proportional to N and λ.

How do you determine the remaining mass of Element X after a certain time?

Use the exponential decay formula: $N = N_0 (1/2)^{(t / \text{half-life})}$. Multiply the initial mass by this fraction to find the remaining mass.

Is Element X safe to handle after 60 minutes?

After 6 half-lives (60 minutes), only about 1.56% of the original amount remains, significantly reducing radioactivity. However, safety depends on initial activity and containment; proper precautions are recommended until decay reduces radioactivity to safe levels.

How does the half-life of Element X influence its use in medical or industrial applications?

A 10-minute half-life allows for rapid decay, making Element X suitable for applications requiring quick clearance, but it also means it must be produced and used quickly due to its short-lived nature.

Additional Resources

Element X Decays Radioactively With A Half Life Of 10 Minutes. If There Are 270 Grams of Element X, How — this intriguing question opens the door to exploring the fascinating world of radioactive decay, half-lives, and how scientists quantify the transformations of unstable elements over time. Whether you're a student, a science enthusiast, or a professional seeking clarity on decay calculations, understanding how to approach this problem provides insight into fundamental principles of nuclear physics and radiochemistry. In this guide, we'll walk through the concepts step-by-step, unraveling the mathematical underpinnings and practical applications of radioactive decay analysis.

Understanding Radioactive Decay and Half-Life

Radioactive decay is a probabilistic process where unstable atomic nuclei spontaneously transform into more stable configurations by emitting radiation. This process is characterized by a half-life, which is the time required for half of the original quantity of a radioactive substance to decay.

What is a Half-Life?

The half-life of a radioactive isotope is a constant value unique to each element or isotope. It provides a measure of the element's stability:

- Short half-life: indicates rapid decay (e.g., seconds or minutes).
- Long half-life: indicates a relatively stable isotope (e.g., millions of years).

In our case, Element X has a half-life of 10 minutes, meaning that every 10 minutes, half of the remaining Element X will have decayed.

The Decay Law: Mathematical Foundations

The decay of a radioactive isotope follows an exponential law, described mathematically as:

$N(t) = N_0 \times e^{(-\lambda t)}$

Where:
- $N(t)$: the quantity of substance remaining at time t.
- N_0: the initial quantity at time $t=0$.
- λ (lambda): decay constant, specific to each isotope.
- t: elapsed time.

Connecting Half-Life and Decay Constant

The decay constant λ relates to the half-life ($T_{1/2}$) via:

$\lambda = \ln(2) / T_{1/2}$

Since $\ln(2) \approx 0.693$, for Element X:

$\lambda = 0.693 / 10 \text{ minutes} = 0.0693 \text{ min}^{-1}$

This decay constant tells us the probability per unit time that a single nucleus will decay.

Applying the Concepts to Our Scenario

Given Data:
- Half-life ($T_1/_2$): 10 minutes
- Initial mass (N_0): 270 grams

Goal:
- Determine how much Element X remains after a certain period.
- Calculate the decay process or remaining mass after specific time intervals.

Step-by-Step Calculation Approach

Step 1: Calculate the Decay Constant

As previously established:

$\lambda = 0.693 / T_1/_2 = 0.693 / 10$ minutes $= 0.0693$ min^{-1}

Step 2: Determine Remaining Quantity After a Given Time

Using the decay law:

$N(t) = N_0 \times e^\wedge(-\lambda t)$

Where:
- $N(t)$: remaining mass after time t
- N_0: initial mass (270 grams)
- λ: decay constant
- t: elapsed time in minutes

Step 3: Example Calculations

Suppose we want to find out how much Element X remains after:

- 10 minutes (one half-life)
- 30 minutes (three half-lives)
- 60 minutes (six half-lives)

Practical Examples: Calculating Remaining Mass

After 10 Minutes (One Half-Life)

$t = 10$ minutes

$N(10) = 270 \times e^{(-0.0693 \times 10)}$
$= 270 \times e^{(-0.693)}$
Recall that $e^{(-0.693)} \approx 0.5$

Therefore:

$N(10) \approx 270 \times 0.5 = 135$ grams

This confirms the concept of half-life—after one half-life, half of the initial mass remains.

After 30 Minutes (Three Half-Lives)

$t = 30$ minutes

$N(30) = 270 \times e^{(-0.0693 \times 30)}$
$= 270 \times e^{(-2.079)}$
Calculate $e^{(-2.079)}$:

$e^{(-2.079)} \approx 0.125$

Thus:

$N(30) \approx 270 \times 0.125 = 33.75$ grams

Alternatively, since each half-life halves the remaining amount, after 3 half-lives:

Remaining mass $= N_0 / 2^3 = 270 / 8 = 33.75$ grams

This matches our exponential calculation, demonstrating the consistency of the decay law.

After 60 Minutes (Six Half-Lives)

$t = 60$ minutes

$N(60) = 270 \times e^{(-0.0693 \times 60)}$
$= 270 \times e^{(-4.158)}$
$e^{(-4.158)} \approx 0.0157$

Remaining mass:

$N(60) \approx 270 \times 0.0157 \approx 4.24$ grams

Similarly, after 6 half-lives:

Remaining mass = $270 / 2^6 = 270 / 64 \approx 4.21875$ grams

Close agreement again.

Visualizing the Decay: Graphs and Predictions

Plotting $N(t)$ over time shows an exponential decay curve. Key points include:

- The mass halves every 10 minutes.
- The decay progresses rapidly initially and slows down over time.
- After about 7 half-lives (~70 minutes), less than 1% remains.

Additional Considerations

Cumulative Decay and Activity

Radioactive decay isn't just about remaining mass; it's also about activity (decays per second). The activity (A) at time t relates to the initial activity (A_0):

$A(t) = A_0 \times e^{(-\lambda t)}$

If you know the initial activity, you can determine how many decays occur over a certain period, useful in applications like radiometric dating or medical treatments.

Mass vs. Number of Nuclei

Calculations are often easier when considering the number of nuclei:

- Use molar masses and Avogadro's number.

- Convert grams to moles: grams / molar mass.
- Convert moles to nuclei: moles × Avogadro's number.

This allows precise modeling of decay on a microscopic level, especially in scientific research.

Real-World Applications and Implications

Understanding decay calculations has practical significance:

- Radiometric Dating: Determining the age of rocks and fossils.
- Nuclear Medicine: Planning doses of radioactive tracers.
- Nuclear Power: Managing fuel decay and waste.
- Safety Protocols: Estimating radiation levels over time.

In all these cases, knowing how much of an element remains after a given time informs safety, efficiency, and scientific accuracy.

Summary of Key Steps for Decay Calculations

1. Identify the half-life ($T_1/_2$).
2. Calculate the decay constant (λ): $\lambda = \ln(2) / T_1/_2$.
3. Use the decay law ($N(t) = N_0 \times e^{(-\lambda t)}$) to find remaining quantities.
4. Apply exponential calculations or half-life shortcuts for practical time intervals.
5. Interpret results in context—remaining mass, activity, or number of nuclei.

Final Thoughts

The process of analyzing radioactive decay, especially for elements like Element X with a known half-life, exemplifies the beautiful intersection of mathematics and physics. The exponential decay law provides a powerful tool for predicting how substances change over time, essential for fields ranging from geology to medicine. Through understanding and applying these principles, scientists and engineers can make informed decisions, ensuring safety and advancing knowledge.

Remember, every decay calculation is rooted in fundamental constants and laws, revealing the underlying order in what might seem like random atomic transformations. Whether you're tracking the decay of radioactive isotopes or simply exploring the science of half-lives, mastering these calculations is an essential step toward scientific literacy.

Interested in further reading? Dive into topics like nuclear decay series, radiometric dating techniques, or the applications of decay law in medical physics to deepen your understanding of this fascinating subject.

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