Evaluate SSS 4xy Dv Where E Is The Region Bounded By Z = 2x2 + 2y2 - 7 And Z = 1. O A. O O B. -32 3 Oc

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Introduction to Surface Integrals and the Region E

Surface integrals are fundamental concepts in multivariable calculus, often used to compute flux, surface area, and other quantities associated with a surface in three-dimensional space. In this article, we will explore how to evaluate a specific surface integral involving the scalar function $4xy$, over a region E bounded by two surfaces: $Z = 2x^2 + 2y^2 - 7$ and $Z = 1$.

The integral in question is:

$$
\iiint_{E} 4xy \, dV
$$

where E is the volume enclosed between the surfaces $Z = 2x^2 + 2y^2 - 7$ (a paraboloid opening upward) and $Z = 1$ (a horizontal plane).

Understanding this problem involves several key steps:
- Identifying the region E in 3D space
- Converting the volume integral into an easier form using symmetry or coordinate transformations
- Applying the appropriate limits of integration
- Calculating the integral accurately

This comprehensive guide aims to demystify each step, provide insights into the mathematical techniques involved, and optimize the solution process for better understanding and application in related problems.

Understanding the Geometric Region E

The Surfaces Defining E

The region E is bounded by:
- The paraboloid $Z = 2x^2 + 2y^2 - 7$
- The plane $Z = 1$

Visually, the paraboloid opens upward, with its vertex at the point $(0,0,-7)$. The plane $Z = 1$ intersects the paraboloid along a closed curve, forming the boundary of the region E.

Finding the Intersection Curve

To determine the boundary of the region E in the xy-plane, set the two surface equations equal:

$$
2x^2 + 2y^2 - 7 = 1
$$

Simplify:

$$
2x^2 + 2y^2 = 8
$$
$$
x^2 + y^2 = 4
$$

This is a circle of radius 2 centered at the origin in the xy-plane.

Visualizing the Region

The volume E lies between:
- The paraboloid $Z = 2x^2 + 2y^2 - 7$, which dips down to $Z = -7$ at the origin
- The plane $Z = 1$

Within the circle $x^2 + y^2 \leq 4$, the paraboloid's height varies from $Z = -7$ at the center to $Z = 1$ on the boundary, where it intersects the plane.

Setting Up the Triple Integral

Choosing the Coordinate System

Given the symmetry of the region about the z-axis, cylindrical coordinates are optimal:

$$
x = r \cos \theta, \quad y = r \sin \theta, \quad z = z
$$

Where:
- $r \geq 0$
- $0 \leq \theta \leq 2\pi$
- z varies between the paraboloid and the plane

Expressing the Surfaces in Cylindrical Coordinates

- Paraboloid:

$$

$$
z = 2r^2 - 7
$$

- Plane:

$$
z = 1
$$

Determining the Limits of Integration

- Radial coordinate r:

$$
x^2 + y^2 \leq 4 \Rightarrow 0 \leq r \leq 2
$$

- Angular coordinate θ:

$$
0 \leq \theta \leq 2\pi
$$

- Vertical coordinate z:

$$
\text{From paraboloid to plane:} \quad z = 2r^2 - 7 \quad \text{to} \quad z = 1
$$

Writing the Volume Element

In cylindrical coordinates, the volume element:

$$
dV = r \, dz \, dr \, d\theta
$$

Final Integral Setup

$$
\iiint_{E} 4xy \, dV = \int_{0}^{2\pi} \int_{0}^{2} \int_{2r^2 - 7}^{1} 4 (r \cos \theta)(r \sin \theta) \, r \, dz \, dr \, d\theta
$$

Simplify the integrand:

$$
4xy = 4 (r \cos \theta)(r \sin \theta) = 4 r^2 \cos \theta \sin \theta
$$

Including $r \, dz \, dr \, d\theta$:

$$
\text{Integrand} \times r = 4 r^2 \cos \theta \sin \theta \times r = 4 r^3 \cos \theta \sin \theta
$$

Thus, the integral becomes:

$$
\boxed{
\iiint_{E} 4xy \, dV = \int_{0}^{2\pi} \int_{0}^{2} \int_{2r^2 - 7}^{1} 4 r^3 \cos \theta \sin \theta \, dz \, dr \, d\theta
}
$$

Computing the Integral Step-by-Step

Integrating with Respect to z

Since the integrand does not depend on z:

$$
\int_{2r^2 - 7}^{1} dz = (1) - (2r^2 - 7) = 8 - 2 r^2
$$

Substituting Back

The integral reduces to:

$$
\int_{0}^{2\pi} \int_{0}^{2} 4 r^3 \cos \theta \sin \theta \times (8 - 2 r^2) \, dr \, d\theta
$$

or,

$$
\int_{0}^{2\pi} \int_{0}^{2} 4 r^3 \cos \theta \sin \theta (8 - 2 r^2) \, dr \, d\theta
$$

Simplify the Expression

Factor constants:

$$
4 r^3 (8 - 2 r^2) \cos \theta \sin \theta
$$

which simplifies to:

$$
4 r^3 \times (8 - 2 r^2) \times \cos \theta \sin \theta
$$

\]

or,

$$
4 r^3 (8 - 2 r^2) \cos \theta \sin \theta
$$

Note that:

$$
4 r^3 (8 - 2 r^2) = 4 r^3 \times 8 - 4 r^3 \times 2 r^2 = 32 r^3 - 8 r^5
$$

The integral becomes:

$$
\int_{0}^{2\pi} \int_{0}^{2} (32 r^3 - 8 r^5) \cos \theta \sin \theta \, dr \, d\theta
$$

Handling the Angular Integral

Recognizing Symmetry

The angular part:

$$
\int_{0}^{2\pi} \cos \theta \sin \theta \, d\theta
$$

can be simplified using the double-angle identity:

$$
\sin 2\theta = 2 \sin \theta \cos \theta
$$

So:

$$
\cos \theta \sin \theta = \frac{1}{2} \sin 2\theta
$$

Thus, the angular integral becomes:

$$
\frac{1}{2} \int_{0}^{2\pi} \sin 2\theta \, d\theta
$$

Evaluate:

$$
\int_{0}^{2\pi} \sin 2\theta \, d\theta = -\frac{1}{2} \cos 2\theta \Big|_0^{2\pi} = -\frac{1}{2} (\cos 4\pi - \cos 0) = -\frac{1}{2} (1 - 1) = 0
$$

Therefore,

$$
\int_{0}^{2\pi} \cos \theta \sin \theta \, d\theta = 0
$$

Implication for the Volume Integral

Since the angular integral evaluates to zero, the entire volume integral evaluates to zero:

$$
\boxed{
\iiint_{E} 4xy \, dV = 0
}
$$

This result aligns with the symmetry of the problem: the integrand $4xy$ is an odd function with respect to certain axes, and the symmetric limits cancel out positive and negative contributions.

Final Result

Frequently Asked Questions

What is the region D bounded by in the problem involving $Z = 2x^2 + 2y^2 - 7$ and $Z = 1$?

The region D is bounded by the surfaces $Z = 2x^2 + 2y^2 - 7$ and $Z = 1$.

How do you find the limits of integration for the region D in the given problem?

First, set the two equations equal to find the boundary: $2x^2 + 2y^2 - 7 = 1$, which simplifies to $2x^2 + 2y^2 = 8$, or $x^2 + y^2 = 4$. The limits are then based on the circle $x^2 + y^2 \leq 4$.

What is the significance of the surfaces $Z = 2x^2 + 2y^2 - 7$ and $Z = 1$ in the context of volume calculation?

They define the lower and upper bounds of the region in the Z-direction, allowing us to compute the volume between these surfaces over the bounded region D in the xy-plane.

How is the volume V of the region D computed in this problem?

The volume V is computed using a double integral over the region D of the difference between the upper and lower surfaces: $V = \iint_D (Z_{upper} - Z_{lower})\, dA$, which simplifies to $\iint_D (1 - (2x^2 + 2y^2 - 7))\, dA$.

What is the value of the volume V for the region D, given the equations?

Calculating the integral yields $V = 32\pi$, but based on the options, the closest answer is -32, which suggests a sign consideration; the correct magnitude is 32π.

Which option correctly represents the evaluated integral for the volume D?

Option B: -32 (assuming the problem expects a numerical value, the correct magnitude is 32π, but among the options, B indicates -32).

Additional Resources

Surface Area Calculation of the Region Bounded by $Z = 2x^2 + 2y^2 - 7$ and $Z = 1$

Introduction

When tackling the evaluation of surface areas in multivariable calculus, especially over regions bounded by specific surfaces, it becomes essential to understand the underlying principles, the geometric interpretation, and the methodical steps involved. Today, we focus on a classic yet insightful problem: evaluating the surface integral of the form SSS 4xy Dv where E is the region bounded by the surfaces $Z = 2x^2 + 2y^2 - 7$ and $Z = 1$.

This problem not only exemplifies the elegance of calculus in translating geometric regions into computational formulas but also demonstrates the practical application of surface integrals in fields like physics, engineering, and computer graphics.

Understanding the Problem: The Surfaces and the Region

The Surfaces

- Surface 1: \(Z = 2x^2 + 2y^2 - 7 \)
- Surface 2: \(Z = 1 \)

These two surfaces intersect and create a bounded region in space, which we'll call E.

The Region E

- Bounded between the paraboloid $Z = 2x^2 + 2y^2 - 7$ and the plane $Z = 1$.
- Objective: Evaluate the surface integral $\iint_E 4xy \, dv$, where the differential volume element dv refers to the volume between these surfaces.

Geometric Interpretation and Significance

Visualizing the region:

- Paraboloid: Opens upward with its vertex at $Z = -7$ when $x = y = 0$. The shape is symmetric about the z-axis.
- Plane: Horizontal plane at $Z = 1$.

The region E is essentially the "shell" between the paraboloid and the plane, extending over the area where the paraboloid dips below or reaches up to the plane.

Understanding this geometric setup is crucial because it guides the choice of coordinate systems and the subsequent algebraic simplifications needed for the integral evaluation.

Step-by-Step Approach to the Solution

1. Find the Boundary in the xy-Plane

To evaluate the surface integral over E, it’s practical to relate the surfaces via their intersection and then project onto the xy-plane.

- Set Z values equal to find the intersection:

$$
2x^2 + 2y^2 - 7 = 1
$$

- Solve for x and y:

$$
2x^2 + 2y^2 = 8
$$

$$
x^2 + y^2 = 4
$$

This is a circle of radius 2 centered at the origin in the xy-plane.

- Region in the xy-plane: $x^2 + y^2 \leq 4$.

2. Express the Surface Integral

Since the surfaces are functions of x and y, the volume element dv between the surfaces can be expressed as:

$$
dv = (z_{\text{top}} - z_{\text{bottom}}) \, dx \, dy
$$

where:

- $z_{\text{top}} = 1$
- $z_{\text{bottom}} = 2x^2 + 2y^2 - 7$

Thus,

$$
dv = [1 - (2x^2 + 2y^2 - 7)] \, dx \, dy = [8 - 2x^2 - 2y^2] \, dx \, dy
$$

The integral becomes:

$$
\iint_{D} 4xy \times [8 - 2x^2 - 2y^2] \, dx \, dy
$$

where D is the disk $x^2 + y^2 \leq 4$.

Coordinate Transformation: Converting to Polar Coordinates

Given the circular symmetry of the boundary, polar coordinates are an ideal choice.

- Transformations:

$$
x = r \cos \theta, \quad y = r \sin \theta
$$

- Limits:

$$
r: 0 \to 2 \quad (\text{radius of the circle})
$$
$$
\theta: 0 \to 2\pi
$$

- Jacobian:

$$
dx\,dy = r\, dr\, d\theta
$$

Express the integrand in terms of r and θ:

$$
4xy = 4 \times r \cos \theta \times r \sin \theta = 4 r^2 \cos \theta \sin \theta
$$

and

$$
8 - 2x^2 - 2y^2 = 8 - 2r^2 (\cos^2 \theta + \sin^2 \theta) = 8 - 2r^2
$$

because $\cos^2 \theta + \sin^2 \theta = 1$.

---

4. Formulate the Integral in Polar Coordinates

Putting everything together:

$$
\text{Integral} = \int_{0}^{2\pi} \int_{0}^{2} \left(4 r^2 \cos \theta \sin \theta \right) \left(8 - 2r^2 \right) r\, dr\, d\theta
$$

Simplify:

$$
= \int_{0}^{2\pi} \int_{0}^{2} 4 r^2 \cos \theta \sin \theta (8 - 2r^2) r\, dr\, d\theta
$$

$$
= \int_{0}^{2\pi} \int_{0}^{2} 4 r^3 \cos \theta \sin \theta (8 - 2r^2) \, dr\, d\theta
$$

---

5. Simplifying the Integral

Notice that the integrand can be separated into a product involving θ and r:

$$
I = \int_{0}^{2\pi} \left(\cos \theta \sin \theta \right) d\theta \times \int_{0}^{2} 4 r^3 (8 - 2r^2) dr
$$

But before splitting, observe the symmetry:

- The angular part:

$$
\int_{0}^{2\pi} \cos \theta \sin \theta \, d\theta
$$

- The radial part:

$$
\int_{0}^{2} 4 r^3 (8 - 2r^2) dr
$$

---

6. Evaluating the Angular Integral

Recall that:

$$
\sin 2\theta = 2 \sin \theta \cos \theta
$$

Thus,

$$
\cos \theta \sin \theta = \frac{1}{2} \sin 2\theta
$$

The integral becomes:

$$
\int_{0}^{2\pi} \cos \theta \sin \theta \, d\theta = \frac{1}{2} \int_{0}^{2\pi} \sin 2\theta \, d\theta
$$

Integrate:

$$
= \frac{1}{2} \left[-\frac{1}{2} \cos 2\theta \right]_0^{2\pi} = -\frac{1}{4} [\cos 4\pi - \cos 0] = -\frac{1}{4} (1 - 1) = 0
$$

Result: The angular integral evaluates to zero.

Implication:

Since the angular integral is zero, the entire surface integral evaluates to zero.

Final Conclusion:

\[
\boxed{
\iint_E 4xy \, dv = 0
}
\]

The symmetry of the integrand over the symmetric region causes the positive and negative contributions to cancel out.

Analysis of the Multiple Choice Options

Given the original problem's options:

- A. (No options provided)
- B. \(-32\)
- C. (No options provided)

Based on the detailed calculations, the integral evaluates to 0, which may suggest that the correct choice (if listed) corresponds to zero. If the options are only the numerical values, then none of the provided options match the zero result, indicating perhaps a need to re-express or re-examine the options.

Broader Implications and Insights

This problem elegantly demonstrates the power of symmetry in multivariable calculus:

- Symmetry in the region and integrand can lead to integrals evaluating to zero without detailed computation.
- Coordinate transformations such as polar coordinates simplify the evaluation of integrals over circular regions.
- Understanding the geometric boundaries guides the choice of limits and the coordinate system.

Final Remarks

In conclusion, evaluating the surface integral \(\iint_E 4xy \, dv \) over the bounded region between the paraboloid \(Z = 2x^2 + 2y^2 - 7 \) and the plane \(Z

Evaluate Sss 4xy Dv Where E Is The Region Bounded By Z 2x2

2y2 7 And Z 1 O A O O B 32 3 Oc

Evaluate Sss 4xy Dv Where E Is The Region Bounded By Z 2x2 2y2 7 And Z 1 O A O O B 32 3 Oc

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