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When faced with evaluating line integrals, particularly those involving curves such as circles, understanding the underlying principles of vector calculus becomes essential. The problem statement, **Evaluate The Following Line Integral Along The Curve C.Cx2 + y2 ds; C Is The Circle Of Radius 11 centered At**, presents a classic scenario where the curve C is a circle of radius 11 centered at a specific point, and the integral involves the function \(Cx^2 + y^2 \). This article aims to guide you through the process of calculating this integral systematically, leveraging techniques such as parameterization, symmetry, and potential vector calculus theorems.

Understanding the Curve C and the Integral

Defining the Curve C

The curve C in this context is a circle of radius 11. Typically, such a circle is represented mathematically as:

- Centered at a point, say \((h, k)\)
- With radius \(r = 11\)

For simplicity, unless specified otherwise, we often consider the circle centered at the origin \((0, 0)\). If the problem states a different center, the same approach applies with appropriate adjustments.

The Mathematical Representation

Assuming the circle is centered at the origin, the parametric equations are:

```
x = 11 cos t
y = 11 sin t
```

where \(t \) varies from \(0 \) to \(2\pi \). The differential arc length \(ds \) can be computed as:

```
ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt
```

The Integrand Function

The integrand, as provided, is \(Cx^2 + y^2 \). Note that the letter \(C \) here is a coefficient, not the curve itself. To avoid confusion, let's clarify:

- Suppose the integrand is \(a x^2 + y^2 \), where \(a \) is a constant.

If the original problem considers \(C \) as a constant, then the integrand becomes \(C x^2 + y^2 \). We will proceed with this understanding.

Step-by-Step Solution to the Line Integral

1. Parameterize the Curve

Using the standard parametrization of a circle centered at the origin, radius 11:

```
x(t) = 11 cos t
y(t) = 11 sin t
```

where \(t \in [0, 2\pi] \). The derivatives are:

```
dx/dt = -11 sin t
dy/dt = 11 cos t
```

2. Compute \(ds \) (Differential Arc Length)

The differential arc length \(ds \) is:

```
ds = \sqrt{\left(-11 \sin t\right)^2 + \left(11 \cos t\right)^2} dt =
\sqrt{121 \sin^2 t + 121 \cos^2 t} dt = \sqrt{121 (\sin^2 t + \cos^2 t)} dt =
11 dt
```

Since \(\sin^2 t + \cos^2 t = 1 \), the expression simplifies neatly to:

```
ds = 11 dt
```

3. Express the Integrand Along the Curve

The integrand \(C x^2 + y^2 \) becomes:

$$C x^2 + y^2 = C (11 \cos t)^2 + (11 \sin t)^2 = C \times 121 \cos^2 t + 121 \sin^2 t$$

Factor out 121:

$$= 121 (C \cos^2 t + \sin^2 t)$$

4. Set Up the Line Integral

The line integral over the curve C is:

$$\int_C (C x^2 + y^2) ds$$

Substituting the parameterizations and ds, the integral becomes:

$$\int_{0}^{2\pi} [121 (C \cos^2 t + \sin^2 t)] \times 11 dt = 121 \times 11 \int_{0}^{2\pi} (C \cos^2 t + \sin^2 t) dt$$

Simplify the constant coefficient:

$$= 1331 \int_{0}^{2\pi} (C \cos^2 t + \sin^2 t) dt$$

Evaluating the Integral

1. Break Down the Integral

The integral separates into two parts:

$$\int_{0}^{2\pi} (C \cos^2 t + \sin^2 t) dt = C \int_{0}^{2\pi} \cos^2 t dt + \int_{0}^{2\pi} \sin^2 t dt$$

2. Use Standard Integrals for $\cos^2 t$ and $\sin^2 t$

Recall the identities:

- $\int_{0}^{2\pi} \cos^2 t dt = \pi$
- $\int_{0}^{2\pi} \sin^2 t dt = \pi$

This is because over one full period, the average value of $\cos^2 t$ and $\sin^2 t$ is $\frac{1}{2}$, and integrating over 2π yields:

$$\int_{0}^{2\pi} \cos^2 t \, dt = \int_{0}^{2\pi} \sin^2 t \, dt = \frac{1}{2} \times 2\pi = \pi$$

3. Compute the Combined Integral

$$C \int_{0}^{2\pi} \cos^2 t \, dt + \int_{0}^{2\pi} \sin^2 t \, dt = C \times \pi + \pi = \pi (C + 1)$$

Final Calculation of the Line Integral

1. Multiply by the Constant Coefficient

$$1331 \times \pi (C + 1)$$

2. Final Expression

The value of the line integral is:

$$\boxed{
\text{Line Integral} = 1331 \pi (C + 1)
}$$

This expression provides the total value of the line integral along the circle of radius 11 centered at the origin, considering the coefficient C.

Special Cases and Additional Considerations

When the Center Is Not at the Origin

If the circle is centered at a point (h, k), the parameterization adjusts accordingly:

$$x(t) = h + 11 \cos t$$
$$y(t) = k + 11 \sin t$$

In this case, the integrand would become more complex, but if the integrand depends only on $x^2 + y^2$, you can use the shift to evaluate the integral accordingly, possibly involving the distance from the origin to the center.

Role of Vector Calculus Theorems

- **Green’s Theorem:** Can be used to convert certain line integrals into double integrals over the region enclosed by the curve, simplifying calculations in some cases.

- **Applications:** Particularly useful if the integrand is a component of a vector field, or if the line integral represents work or circulation.

Conclusion

Evaluating line integrals along curves such as circles involves a combination of parameterization, understanding the integrand's behavior, and leveraging symmetry and standard integral results. In the specific case of the circle of radius 11 centered at the origin, the integral simplifies elegantly due to the symmetry of $\sin^2 t$ and $\cos^2 t$. The final answer, expressed as $1331 \pi (C + 1)$, encapsulates all these considerations.

Mastering these techniques enhances your ability to evaluate complex integrals in various applications across physics, engineering, and mathematics, providing a solid foundation in multivariable calculus and vector analysis.