Find A General Solution To The Differential Equation. 8 3 Y″ - By′ +9y=-8 The General Solution Is Y(t)

Find A General Solution To The Differential Equation. 8 3 Y″ - By′ + 9y = -8 The General Solution Is Y(t)

Differential equations are fundamental in mathematical modeling, describing phenomena in physics, engineering, biology, and many other scientific disciplines. The specific differential equation $8 \times 3 Y'' - B y' + 9 y = -8$ presents an interesting case that involves second-order linear differential equations with constant coefficients. Finding its general solution not only enhances understanding of differential equations but also equips you with methods applicable to a wide range of real-world problems. In this comprehensive guide, we will explore the step-by-step process of solving this differential equation, including the derivation of the complementary (homogeneous) solution and particular solution, along with insights into common techniques and key points to remember.

Understanding the Differential Equation

Before delving into solution methods, it's essential to interpret the given equation clearly:

$$
8 \times 3 Y'' - B y' + 9 y = -8
$$

Key points:

- The equation is linear, second-order, and nonhomogeneous.
- Coefficients are constants: 8×3 = 24, and the coefficient of y' is represented by $-B$, a parameter.
- The right-hand side is a constant, -8.

For clarity, rewriting the equation:

$$
24 y'' - B y' + 9 y = -8
$$

where $y(t)$ is the unknown function of t, and B is a constant parameter.

Step 1: Write the Standard Form

The first step in solving this differential equation is to express it in standard linear form:

$$
a y'' + b y' + c y = f(t)
$$

Matching terms:

- $a = 24$
- $b = -B$

- $c = 9$
- $f(t) = -8$

The standard form becomes:

$$
24 y'' - B y' + 9 y = -8
$$

Step 2: Find the Complementary Solution

The complementary (or homogeneous) solution, $y_h(t)$, is obtained by solving the associated homogeneous equation:

$$
24 y'' - B y' + 9 y = 0
$$

Characteristic Equation:

Assuming solutions of the form $y(t) = e^{rt}$, substitute into the homogeneous equation:

$$
24 r^2 e^{rt} - B r e^{rt} + 9 e^{rt} = 0
$$

Dividing through by $e^{rt} \neq 0$:

$$
24 r^2 - B r + 9 = 0
$$

This quadratic equation in r is:

$$
24 r^2 - B r + 9 = 0
$$

Solution for r:

$$
r = \frac{B \pm \sqrt{B^2 - 4 \times 24 \times 9}}{2 \times 24}
$$

$$
r = \frac{B \pm \sqrt{B^2 - 864}}{48}
$$

Analysis:

- The roots depend on the discriminant $D = B^2 - 864$.
- Depending on whether $D > 0$, $D = 0$, or $D < 0$, the nature of the roots (real and distinct, repeated, or complex conjugates) changes.

Case 1: Real and Distinct Roots ($D > 0$)

$$
r_{1,2} = \frac{B \pm \sqrt{B^2 - 864}}{48}
$$

Complementary solution:

$$
y_h(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t}
$$

where C_1, C_2 are arbitrary constants.

Case 2: Repeated Roots ($D = 0$)

$$
r = \frac{B}{48}
$$

Complementary solution:

$$
y_h(t) = (C_1 + C_2 t) e^{\frac{B}{48} t}
$$

Case 3: Complex Roots ($D < 0$)

$$
r = \frac{B}{48} \pm i \frac{\sqrt{864 - B^2}}{48}
$$

Complementary solution:

$$
y_h(t) = e^{\frac{B}{48} t} \left[C_1 \cos \left(\frac{\sqrt{864 - B^2}}{48} t \right) + C_2 \sin \left(\frac{\sqrt{864 - B^2}}{48} t \right) \right]
$$

Step 3: Find the Particular Solution $y_p(t)$

Since the nonhomogeneous term $f(t) = -8$ is a constant, we seek a particular solution $y_p(t)$ that is also a constant, say $y_p = A$.

Substitute $y_p = A$ into the differential equation:

$$
24 \times 0 - B \times 0 + 9 A = -8
$$

$$
9 A = -8
$$

$$
A = -\frac{8}{9}
$$

Thus, the particular solution:

$$
y_p(t) = -\frac{8}{9}
$$

Step 4: Write the General Solution

The general solution $y(t)$ combines the complementary solution and particular solution:

$$
\boxed{
y(t) = y_h(t) + y_p(t)
}
$$

Final form:

$$
y(t) = \begin{cases}
C_1 e^{r_1 t} + C_2 e^{r_2 t} - \frac{8}{9}, & \text{if roots are real and distinct} \\
(C_1 + C_2 t) e^{\frac{B}{48} t} - \frac{8}{9}, & \text{if roots are repeated} \\
e^{\frac{B}{48} t} \left[C_1 \cos \left(\frac{\sqrt{864 - B^2}}{48} t \right) + C_2 \sin \left(\frac{\sqrt{864 - B^2}}{48} t \right) \right] - \frac{8}{9}, & \text{if roots are complex}
\end{cases}
$$

Additional Insights and Techniques

Understanding the solution process involves several key points:

1. Discriminant Analysis

- The nature of roots determines the form of the complementary solution.
- Always compute the discriminant $D = B^2 - 864$.

2. Parameter B and Its Effect

- The value of B influences the roots, and consequently, the behavior of the solution.
- For example, if $B^2 = 864$, roots are repeated.
- If $B^2 > 864$, roots are real and distinct.
- If $B^2 < 864$, roots are complex conjugates.

3. Using the Method of Undetermined Coefficients

- Since the right-hand side is a constant, a constant particular solution is appropriate.
- For more complex $f(t)$, other methods like variation of parameters may be necessary.

4. Application of Initial Conditions

- To determine the arbitrary constants C_1 and C_2, initial or boundary conditions are needed.
- These conditions specify the value of $y(t)$ and $y'(t)$ at specific points.

5. Significance of the General Solution

- The general solution encompasses all possible solutions to the differential equation.
- Particular solutions are used to fit specific initial or boundary conditions.

Practical Applications of Solving Such Differential Equations

Differential equations similar to $24 y'' - B y' + 9 y = -8$ appear in various contexts:

- Mechanical systems: Modeling damped harmonic oscillators with constant forcing.
- Electrical circuits: Analyzing RLC circuits with constant voltage sources.
- Population dynamics: Describing systems with constant growth or decay rates.
- Thermal processes: Heat transfer models with steady internal heat sources.

Understanding how to find the general solution enables engineers and scientists to predict system behavior under different parameters and initial conditions.

Summary and Key Takeaways

- The given differential equation is a second-order linear differential equation with constant coefficients.
- The homogeneous solution depends on the roots of the characteristic equation, which are influenced by the parameter B.
- The particular solution for a constant nonhomogeneous term is a constant.
- The general solution is the sum of the homogeneous and particular solutions.
- Analyzing the discriminant helps determine the precise form of

Frequently Asked Questions

What is the general form of the differential equation $8 \cdot 3y'' - By' + 9y = -8$?

The differential equation can be rewritten as $24y'' - By' + 9y = -8$, where B is a parameter or coefficient, and the goal is to find its general solution.

How do you find the complementary (homogeneous) solution for 24y'' - By' + 9y = 0?

First, write the characteristic equation: $24r^2 - Br + 9 =$. Then, solve for r using the quadratic formula to find the roots, which determine the complementary solution.

What role does the particular solution play in solving the differential equation?

The particular solution accounts for the nonhomogeneous term (-8). It is a specific solution that, combined with the complementary solution, gives the general solution.

How can we determine the form of the particular solution for this differential equation?

Since the nonhomogeneous term is a constant (-8), we try a constant particular solution, say y_p = A, and substitute back into the differential equation to solve for A.

What is the general solution y(t) to the differential equation 24y'' - By' + 9y = -8?

The general solution is y(t) = y_h(t) + y_p, where y_h(t) is the homogeneous solution determined from the roots of the characteristic equation, and y_p is the particular solution found for the nonhomogeneous term.

How does the value of B influence the roots of the characteristic equation and the form of the solution?

The coefficient B affects the discriminant of the quadratic characteristic equation, which determines whether the roots are real and distinct, real and repeated, or complex conjugates, thus influencing the form of the homogeneous solution.

Can you summarize the steps to find the general solution to the differential equation?

Yes. First, write the homogeneous equation 24y'' - By' + 9y = 0, solve the characteristic equation for roots, and find the homogeneous solution. Then, determine a particular solution for the nonhomogeneous term (-8), and combine both to obtain the general solution y(t) = y_h(t) + y_p.

Additional Resources

Find A General Solution To The Differential Equation. 8 3 Y" - By' +9y=-8 The General Solution Is Y(t)

In the realm of differential equations, understanding how to find the general solution to a given equation is a fundamental skill for mathematicians, engineers, and scientists alike. Today, we explore a specific second-order linear differential equation:

$8 \cdot 3 \cdot y'' - B \cdot y' + 9 \cdot y = -8$

At first glance, this may appear complex, but with systematic steps and a solid understanding of differential equations, we can uncover its general solution—an expression that encompasses all possible solutions to the equation. This article aims to demystify the process, providing a detailed, reader-friendly explanation suitable for students, educators, and enthusiasts eager to deepen their understanding.

Understanding the Differential Equation

Before jumping into the solution, it's essential to interpret the given differential equation correctly.

The original form appears to be:

$$8 \cdot 3 \cdot y'' - B \cdot y' + 9 \cdot y = -8$$

which simplifies to:

$$24 \cdot y'' - B \cdot y' + 9 \cdot y = -8$$

Note: The variable B appears as a coefficient; its exact value isn't specified. For the purposes of solving the differential equation, either B is a known constant, or it remains as a parameter. If B is known, substitute its value; if not, the solution will be expressed in terms of B.

Step 1: Write the Differential Equation in Standard Form

To analyze and solve this second-order linear differential equation, it's helpful to write it in the standard form:

$$y'' + p(t) \cdot y' + q(t) \cdot y = r(t)$$

In our case, the coefficients are constants, so:

$$y'' - (B/24) \cdot y' + (9/24) \cdot y = -8/24$$

which simplifies to:

$$y'' - (B/24) \cdot y' + (3/8) \cdot y = -1/3$$

This form allows us to identify the homogeneous part and the particular solution more straightforwardly.

Step 2: Solve the Homogeneous Equation

The homogeneous equation is:

$$y'' - (B/24) \cdot y' + (3/8) \cdot y = 0$$

This is a second-order linear differential equation with constant coefficients. The general approach involves:

- Finding the characteristic equation
- Solving for the roots
- Formulating the general homogeneous solution

Characteristic Equation:

Assuming a solution of the form $y(t) = e^{rt}$, substitute into the homogeneous equation:

$$r^2 - (B/24) \cdot r + (3/8) = 0$$

This quadratic in r:

$r^2 - (B/24)\cdot r + (3/8) = 0$

Discriminant (D):

$D = [(B/24)]^2 - 4\cdot 1\cdot (3/8)$
$D = (B^2 / 576) - (12/8)$
$D = (B^2 / 576) - (3/2)$

Step 3: Analyze Roots Based on Discriminant

The nature of the roots depends on D:

- $D > 0$: Two distinct real roots ⍰ general solution involves exponentials
- $D = 0$: Repeated real root ⍰ solution involves exponential and polynomial terms
- $D < 0$: Complex conjugate roots ⍰ solution involves exponential and sine/cosine functions

Case 1: $D > 0$

Roots:

$r_1, r_2 = [\ (B/24) \pm$ ⍰ $D\ ] / 2$

Case 2: $D = 0$

Root:

$r = (B/24) / 2 = B/48$

Solution:

$y_h(t) = (A + Bt)\cdot e^{r t}$

Case 3: $D < 0$

Let:

$a = (B/24)/2 = B/48$

$b =$ ⍰ $(-D)/2$

Roots:

$r = a \pm ib$

Solution:

$y_h(t) = e^{a t}\,[C_1\cdot\cos(b t) + C_2\cdot\sin(b t)]$

Step 4: Find the Particular Solution

Since the nonhomogeneous term is -8, a constant, we look for a particular solution $y_p(t)$ as a constant:

$y_p(t) = A$

Substitute into the original differential equation:

$24 \cdot 0 - B \cdot 0 + 9 \cdot A = -8$

which simplifies to:

$9A = -8$

$A = -8/9$

Therefore, the particular solution:

$y_p(t) = -8/9$

Step 5: Write the General Solution

The general solution $y(t)$ is the sum of the homogeneous solution $y_h(t)$ and the particular solution $y_p(t)$:

$y(t) = y_h(t) + y_p(t)$

Depending on the discriminant, this appears as:

- For $D > 0$:

$y(t) = c_1 \cdot e^{r_1 t} + c_2 \cdot e^{r_2 t} - 8/9$

- For $D = 0$:

$y(t) = (A + Bt) \cdot e^{(B/48) t} - 8/9$

- For $D < 0$:

$y(t) = e^{(B/48) t} [C_1 \cdot \cos(b t) + C_2 \cdot \sin(b t)] - 8/9$

where c_1, c_2, C_1, C_2 are arbitrary constants determined by initial conditions, and $b = \sqrt{(-D)}/2$.

Practical Example: Assigning a Value to B

Suppose $B = 12$; then:

Discriminant:

$D = (12)^2 / 576 - 3/2 = 144/576 - 1.5 = 0.25 - 1.5 = -1.25$

Since $D < 0$, roots are complex conjugates.

Calculate:

$a = (B/24)/2 = (12/24)/2 = (0.5)/2 = 0.25$

$b = \sqrt{1.25} / 2 \approx 1.118 / 2 \approx 0.559$

The homogeneous solution:

$y_h(t) = e^{0.25 t} [C_1 \cdot \cos(0.559 t) + C_2 \cdot \sin(0.559 t)]$

And the particular solution remains:

$y_p(t) = -8/9 \approx -0.8889$

Final general solution:

$y(t) = e^{0.25 t} [C_1 \cdot \cos(0.559 t) + C_2 \cdot \sin(0.559 t)] - 0.8889$

Additional Insights and Applications

Understanding the solutions to such differential equations is more than an academic exercise; they are crucial in modeling real-world systems such as mechanical vibrations, electrical circuits, and thermal processes. The approach outlined—identifying the characteristic equation, analyzing roots, and constructing the general solution—is a foundational method in differential equations.

Key takeaways:

- The nature of the roots of the characteristic equation determines the form of the homogeneous solution.
- The particular solution often involves guessing based on the nonhomogeneous term; here, a constant sufficed.
- Constants in the solution are determined by initial or boundary conditions, which tailor the general solution to specific scenarios.
- Parameters like b influence the behavior of the solution, especially regarding oscillatory or exponential decay/growth patterns.

Conclusion

In tackling the differential equation $8 \cdot 3 \cdot y'' - b \cdot y' + 9 \cdot y = -8$, we unraveled a systematic process to find its general solution. The key steps involved rewriting the equation in standard form, solving the homogeneous part by examining the roots of the characteristic equation, and then including a particular solution to account for the nonhomogeneous component.

This approach exemplifies the power of classical methods in differential equations—tools that continue to underpin advancements across science and engineering. Whether modeling the damping of a mechanical system or analyzing electrical circuits, mastering these techniques equips practitioners with the means to interpret, predict, and innovate within complex systems.

Note: For specific applications, initial conditions are necessary to determine the arbitrary constants. Always consider the context of the problem when applying these solutions.

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