Find A Particular Solution Yp Of Y" -y' 2y = 8 Sin 2x Solve The Initial Value Problem Y" 2y' + 5y = 2x

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When dealing with differential equations, especially those involving nonhomogeneous terms such as sinusoidal functions or polynomial expressions, finding particular solutions is a crucial step. This article guides you through the process of solving the differential equation Y" - 2y' + 2y = 8 sin 2x for a particular solution \(Y_p\), and then applying initial conditions to solve the initial value problem (IVP):

\[
\boxed{
Y'' + 2Y' + 2Y = 2x
}
\]

We will explore methods such as the method of undetermined coefficients and variation of parameters, analyze the homogeneous equation, and interpret the solutions within the context of initial conditions.

Understanding the Differential Equations

Before diving into the solution techniques, it is essential to understand the structure of the equations involved.

The Main Differential Equation:

\[
Y'' - 2Y' + 2Y = 8 \sin 2x
\]
This is a second-order linear nonhomogeneous differential equation with constant coefficients, featuring a sinusoidal forcing term.

The Initial Value Problem (IVP):

\[
Y'' + 2Y' + 2Y = 2x

\]
Here, the right side is a polynomial (linear in x), which influences the choice of particular solution method.

Solving the Homogeneous Equations

The first step in solving nonhomogeneous differential equations is to find the general solution of the associated homogeneous equation.

Homogeneous Equation:

$$
Y'' + 2Y' + 2Y = 0
$$

Characteristic Equation:

$$
r^2 + 2r + 2 = 0
$$

Solve for r:

$$
r = \frac{-2 \pm \sqrt{4 - 8}}{2} = \frac{-2 \pm \sqrt{-4}}{2} = -1 \pm i
$$

Thus, the roots are complex conjugates: $-1 \pm i$.

Homogeneous Solution:

$$
Y_h(x) = e^{-x} (C_1 \cos x + C_2 \sin x)
$$

where C_1, C_2 are arbitrary constants.

Finding a Particular Solution Y_p

The form of the particular solution depends on the nonhomogeneous term.

For the equation $Y'' + 2Y' + 2Y = 2x$:

Since the right side is a polynomial (degree 1), the method of undetermined coefficients suggests trying a polynomial of degree 1 or 2. However, because the homogeneous solution involves exponential and trigonometric functions, and the forcing term is polynomial, the particular solution can be assumed as a polynomial:

$$
Y_p = Ax + B
$$

But, because the differential operator involves derivatives, we substitute Y_p into the differential equation to determine A and B.

Calculating Y_p:

Compute derivatives:
$$
Y_p = Ax + B
$$
$$
Y_p' = A
$$
$$
Y_p'' = 0
$$

Plug into the differential equation:

$$
0 + 2A + 2(Ax + B) = 2x
$$

Simplify:

$$
2A + 2A x + 2 B = 2x
$$

Matching coefficients:

- For x:

$$
2A x = 2x \Rightarrow 2A = 2 \Rightarrow A = 1
$$

- For constants:

$$
2A + 2B = 0 \Rightarrow 2(1) + 2B = 0 \Rightarrow 2 + 2B = 0 \Rightarrow 2B = -2 \Rightarrow B = -1
$$

Therefore:

$$
Y_p = x - 1
$$

This is the particular solution for the IVP.

Constructing the General Solution

The general solution to the differential equation combines the homogeneous and particular solutions:

$$
Y(x) = Y_h + Y_p = e^{-x} (C_1 \cos x + C_2 \sin x) + x - 1
$$

Applying Initial Conditions to Solve the IVP

Suppose initial conditions are given, such as:

$$
Y(0) = y_0, \quad Y'(0) = y'_0
$$

To find C_1 and C_2, differentiate $Y(x)$:

$$
Y'(x) = \frac{d}{dx} \left[e^{-x} (C_1 \cos x + C_2 \sin x) + x - 1 \right]
$$

Compute derivatives:

$$
Y'(x) = e^{-x} \left[-C_1 \cos x - C_2 \sin x + C_1 \sin x - C_2 \cos x \right] + 1
$$

Simplify inside brackets:

$$
Y'(x) = e^{-x} \left[-C_1 \cos x - C_2 \sin x + C_1 \sin x - C_2 \cos x \right] + 1
$$

or:

$$
Y'(x) = e^{-x} \left[(-C_1 - C_2) \cos x + (C_1 - C_2) \sin x \right] + 1
$$

Now, plug in the initial conditions:

$$
Y(0) = e^{0} (C_1 \cos 0 + C_2 \sin 0) + 0 - 1 = C_1 + 0 - 1 = y_0
$$
$$
\Rightarrow C_1 = y_0 + 1
$$

Similarly,

$$
Y'(0) = e^{0} \left[(-C_1 - C_2) \cos 0 + (C_1 - C_2) \sin 0 \right] + 1 = (-C_1 - C_2) + 1 = y'_0
$$

Substitute $C_1 = y_0 + 1$:

$$
(- (y_0 + 1) - C_2) + 1 = y'_0
$$

$$
- y_0 - 1 - C_2 + 1 = y'_0 \Rightarrow - y_0 - C_2 = y'_0
$$

$$
C_2 = - y'_0 - y_0
$$

Final solution:

$$
Y(x) = e^{-x} \left[(y_0 + 1) \cos x + (- y'_0 - y_0) \sin x \right] + x - 1
$$

Summary of Methods and Results

- Homogeneous solution: $Y_h = e^{-x} (C_1 \cos x + C_2 \sin x)$
- Particular solution (polynomial method): $Y_p = x - 1$
- General solution: $Y(x) = Y_h + Y_p$

Additional Techniques for Complex Forcing Terms

While the polynomial approach works well here, for more complex nonhomogeneous terms such as $8 \sin 2x$, other methods are necessary.

Method of Undetermined Coefficients (for sinusoidal forcing):

- Assume particular solution of the form:

$$
Y_p = A \sin 2x + B \cos 2x

\]
- Calculate derivatives and substitute into the differential equation.
- Solve for \(A\) and \(B\) by equating coefficients.

Variation of Parameters:

- Use when the forcing term does not match the form suitable for undetermined coefficients.
- Construct a particular solution based on the homogeneous solutions and integrals involving the forcing function.

Conclusion and Practical Tips

- Always analyze the homogeneous equation first to understand the structure of the general solution.
- Choose the particular solution form based on the type of nonhomogeneous term:
- Polynomial → polynomial.
- Sinusoidal → sinusoidal.
- Exponential → exponential.
- Use initial conditions to solve for arbitrary constants.
- For complex forcing functions, variation of parameters provides a systematic approach.
- Practice with different equations to develop intuition for selecting the right method.

Keywords for SEO Optimization

- Differential equations
- Particular solution of differential equations
- Solving nonhomogeneous ODEs
- Method of undetermined coefficients
- Variation of parameters
- Homogeneous differential equations
- Initial value problem
- Sinusoidal forcing term
- Polynomial forcing term
- Second-order linear ODE

By understanding these principles and techniques, you

Frequently Asked Questions

What is the differential equation given for finding the particular solution Yp?

The differential equation is Y" - y' - 2y = 8 sin 2x.

How do you approach finding a particular solution Yp for Y" - y' - 2y = 8 sin 2x?

Use the method of undetermined coefficients, assuming a particular solution form based on the right-hand side (8 sin 2x), typically involving sine and cosine terms.

What is the general method to solve the initial value problem Y" + 2Y' + 5Y = 2x'?

First, find the complementary solution by solving the homogeneous equation, then find a particular solution for the nonhomogeneous part, and finally apply initial conditions if provided.

What is the characteristic equation for the homogeneous differential equation Y" + 2Y' + 5Y = 0?

The characteristic equation is r^2 + 2r + 5 = 0.

How do you find the particular solution Yp for Y" - y' - 2y = 8 sin 2x?

Assume a particular solution of the form Yp = A sin 2x + B cos 2x and determine A and B by substituting into the differential equation.

What is the significance of the initial conditions in solving the initial value problem?

Initial conditions allow you to determine the specific constants in the general solution, leading to a particular solution that satisfies those conditions.

Why do we use the method of undetermined coefficients in this context?

Because the nonhomogeneous term involves sinusoidal functions, and this method provides a systematic way to find particular solutions for such types of forcing functions.

What steps are involved in solving the differential equation Y" + 2Y' + 5Y = 2x'?

Solve the homogeneous equation for the complementary solution, find a particular solution for the right-hand side, then combine and apply initial conditions if applicable.

How do you verify that the particular solution Yp is correct?

Substitute Yp into the original differential equation; if the left side equals the right side, then Yp is a valid particular solution.

What are common forms of particular solutions for equations with sinusoidal forcing functions?

Typically, particular solutions are assumed to be of the form $A \sin nx + B \cos nx$, where A and B are constants to be determined.

Additional Resources

Differential Equations

Introduction

When delving into the realm of differential equations, one encounters a fascinating interplay between theory and application. These equations serve as mathematical models for a multitude of phenomena—ranging from physics and engineering to biology and economics. Today, we explore a specific type of problem involving second-order linear differential equations: how to find a particular solution Y_p to the differential equation

$$
Y'' - y' - 2y = 8 \sin 2x
$$

and subsequently solve the initial value problem (IVP):

$$
Y'' + 2Y' + 5Y = 2x
$$

This comprehensive examination aims to clarify each step, explain the underlying concepts, and demonstrate the methods used by experts to tackle such problems efficiently.

Understanding the Differential Equations

Before proceeding to solutions, let's clarify what each equation entails.

Equation 1: $Y'' - y' - 2y = 8 \sin 2x$

- Type: Nonhomogeneous second-order linear differential equation with constant coefficients.
- Known as the forced equation because of the non-zero right side $8 \sin 2x$.

Equation 2 (initial value problem): $Y'' + 2Y' + 5Y = 2x$

- Also a nonhomogeneous second-order linear differential equation, but with different coefficients.
- The goal is not only to find the general solution but also to determine the particular solution satisfying initial conditions (which are assumed to be given, e.g., $Y(0)$ and $Y'(0)$), though specifics are not provided here).

Part 1: Finding a Particular Solution Y_p for $Y'' - y' - 2y = 8 \sin 2x$

Step 1: Homogeneous Equation and Its Complement

The first step involves solving the associated homogeneous equation:

$$
Y'' - y' - 2y = 0
$$

which simplifies to:

$$
Y'' - Y' - 2Y = 0
$$

Characteristic Equation:

$$
r^2 - r - 2 = 0
$$

Solving for r:

$$
r = \frac{1 \pm \sqrt{1 + 8}}{2} = \frac{1 \pm 3}{2}
$$

yielding:

$$
r = 2 \quad \text{and} \quad r = -1
$$

Homogeneous Solution:

$$
Y_h = C_1 e^{2x} + C_2 e^{-x}
$$

where C_1 and C_2 are arbitrary constants.

Step 2: Choosing the Method for Particular Solution — Undetermined Coefficients or Variation of Parameters?

Given that the forcing term $8 \sin 2x$ is a sinusoid, the method of undetermined coefficients is suitable because sinusoidal functions are standard particular solutions for such right-hand sides.

However, because the homogeneous solutions involve exponentials, and the nonhomogeneous term is sinusoidal, the trial particular solution takes the form:

$$
Y_p = A \sin 2x + B \cos 2x
$$

where A and B are coefficients to be determined.

Step 3: Plugging Y_p into the Differential Equation

Calculate derivatives:

$$
Y_p' = 2A \cos 2x - 2B \sin 2x
$$
$$
Y_p'' = -4A \sin 2x - 4B \cos 2x
$$

Substitute into the original differential equation:

$$
Y'' - Y' - 2Y = 8 \sin 2x
$$

becomes:

$$
(-4A \sin 2x - 4B \cos 2x) - (2A \cos 2x - 2B \sin 2x) - 2(A \sin 2x + B \cos 2x) = 8 \sin 2x
$$

Simplify term-by-term:

$$
-4A \sin 2x - 4B \cos 2x - 2A \cos 2x + 2B \sin 2x - 2A \sin 2x - 2B \cos 2x = 8 \sin 2x
$$

Group similar terms:

- $\sin 2x$: $-4A + 2B - 2A = (-4A - 2A) + 2B = -6A + 2B$
- $\cos 2x$: $-4B - 2A - 2B = (-4B - 2B) - 2A = -6B - 2A$

So the equation reduces to:

$$
(-6A + 2B) \sin 2x + (-6B - 2A) \cos 2x = 8 \sin 2x
$$

Matching coefficients of $\sin 2x$ and $\cos 2x$:

\[
-6A + 2B = 8 \quad (1)
\]
\[
-6B - 2A = 0 \quad (2)
\]

Step 4: Solving for \(A \) and \(B \)

From equation (2):

\[
-6B - 2A = 0 \Rightarrow 6B = -2A \Rightarrow B = -\frac{A}{3}
\]

Substitute into equation (1):

\[
-6A + 2(-\frac{A}{3}) = 8
\]
\[
-6A - \frac{2A}{3} = 8
\]

Express with common denominator:

\[
-6A - \frac{2A}{3} = -\frac{18A}{3} - \frac{2A}{3} = -\frac{20A}{3}
\]

Set equal to 8:

\[
-\frac{20A}{3} = 8 \Rightarrow -20A = 24 \Rightarrow A = -\frac{24}{20} = -\frac{6}{5}
\]

Calculate \(B \):

\[
B = -\frac{A}{3} = -\left(-\frac{6}{5}\right) \times \frac{1}{3} = \frac{6}{5} \times \frac{1}{3} = \frac{6}{15} = \frac{2}{5}

\]

Part 1: Final Particular Solution

\[
\boxed{
Y_p = -\frac{6}{5} \sin 2x + \frac{2}{5} \cos 2x
}
\]

This solution captures the particular response of the system to the sinusoidal forcing term.

Part 2: Solving the Initial Value Problem \(Y'' + 2Y' + 5Y = 2x \)

Now, let's examine the second differential equation, which involves a linear polynomial forcing term \(2x \).

Step 1: Homogeneous Solution

Homogeneous equation:

\[
Y'' + 2Y' + 5Y = 0
\]

Characteristic Equation:

\[
r^2 + 2r + 5 = 0
\]

Discriminant:

\[
\Delta = 2^2 - 4 \times 1 \times 5 = 4 - 20 = -16
\]

Since \(\Delta < 0\), roots are complex conjugates:

$$
r = \frac{-2 \pm \sqrt{-16}}{2} = -1 \pm 2i
$$

Homogeneous Solution:

$$
Y_h = e^{-x} (C_1 \cos 2x + C_2 \sin 2x)
$$

Step 2: Finding the Particular Solution Y_p

Because the forcing term $2x$ is a polynomial, the method of undetermined coefficients is suitable again.

Trial particular solution:

$$
Y_p = A x + B
$$

Calculate derivatives:

$$
Y_p' = A
$$

$$
Y_p'' = 0
$$

Substitute into the differential equation:

$$
0 + 2A + 5(Ax + B) = 2x
$$

Simplify:

$$
2A + 5A x + 5B = 2x
$$

Matching coefficients:

- For \(x \):

\[
5A = 2 \Rightarrow A = \frac{2}{5}
\]

- For constants:

\[
2A + 5B = 0 \Rightarrow 2 \times \frac{2}{5} + 5B = 0
\]
\[
\frac{4}{5} + 5B = 0 \Rightarrow 5B = -\frac{4}{5} \Rightarrow B = -\frac{4}{25}
\]

Part 2: Final General Solution

\[
Y(x) = e^{-x

Find A Particular Solution Yp Of Y Y 2y 8 Sin 2x Solve The Initial Value Problem Y 2y 5y 2x

Find A Particular Solution Yp Of Y Y 2y 8 Sin 2x Solve The Initial Value Problem Y 2y 5y 2x

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