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Introduction to Orthogonality in R4

Understanding the concept of orthogonality in vector spaces is crucial in fields such as linear algebra, engineering, computer science, and mathematics. When vectors are orthogonal, they are perpendicular in a geometric sense, meaning their dot product equals zero. This property is fundamental in applications like orthogonal projections, signal processing, and data analysis.

In this article, we focus on a specific problem: finding all possible values of (a, B, C, D) in R4 such that a given set of vectors is orthogonal. The set includes vectors with some unknown components, and our goal is to determine all these components that satisfy the orthogonality conditions.

Understanding the Problem Statement

The problem involves a set of vectors in four-dimensional space, R4:

- Vector 1: (1, 2, 1, 0)
- Vector 2: (1, 1, 1, 3)
- Vector 3: (2, 1, a, B)
- Vector 4: (C, D, ..., ...)

(Note: The original problem seems to be incomplete, but we proceed with the assumption that the third vector involves parameters a and B, and the fourth involves C and D.)

Our goal is to find all possible values of a, B, C, and D such that these vectors are mutually orthogonal.

Key concepts:

- Orthogonality: Two vectors u and v are orthogonal if their dot product $u \cdot v = 0$.
- Mutual Orthogonality: All pairs of vectors in the set are orthogonal to each other.

Given that, the conditions for orthogonality among the vectors are:

- $u_1 \cdot u_2 = 0$
- $u_1 \cdot u_3 = 0$
- $u_1 \cdot u_4 = 0$

- $u_2 \cdot u_3 = 0$
- $u_2 \cdot u_4 = 0$
- $u_3 \cdot u_4 = 0$

By solving these equations, we can find all the possible values of (a, B, C, D).

Step-by-Step Solution Approach

To solve for the unknown components, follow these steps:

1. Write down the dot product equations for each pair of vectors.
2. Substitute the known vectors and parameters.
3. Solve the resulting system of equations for the unknowns.
4. Determine all solutions that satisfy the orthogonality conditions.

Let's proceed with the detailed calculations.

Step 1: Dot Product of Known Vectors

Vectors:

- $u_1 = (1, 2, 1, 0)$
- $u_2 = (1, 1, 1, 3)$
- $u_3 = (2, 1, a, B)$
- $u_4 = (C, D, ..., ...)$

(Assuming u_4 is a vector with unknown components C and D.)

Note: Since the problem statement is incomplete, for the sake of completeness, we'll assume the fourth vector is (C, D, E, F), where E and F are additional unknowns. But as per the initial problem, only a, B, C, D are variables, so perhaps u_4 is (C, D, 0, 0). For simplicity, we assume $u_4 = (C, D, 0, 0)$.

Dot product equations:

1. $u_1 \cdot u_2 = (1)(1) + (2)(1) + (1)(1) + (0)(3) = 1 + 2 + 1 + 0 = 4$

Since this is not zero, the vectors u_1 and u_2 are not orthogonal unless we interpret the problem differently. But the goal is to find the unknowns such that the entire set is orthogonal, which involves the other pairs.

Assumption: The initial set may be intended to be mutually orthogonal, and perhaps the first two vectors are fixed, while the third and fourth vectors are parameterized.

Step 2: Define the Conditions for Orthogonality

Assuming the vectors are:

- $u_1 = (1, 2, 1, 0)$
- $u_2 = (1, 1, 1, 3)$
- $u_3 = (2, 1, a, B)$
- $u_4 = (C, D, 0, 0)$

The orthogonality conditions are:

- $u_1 \cdot u_2 = 0$
- $u_1 \cdot u_3 = 0$
- $u_1 \cdot u_4 = 0$
- $u_2 \cdot u_3 = 0$
- $u_2 \cdot u_4 = 0$
- $u_3 \cdot u_4 = 0$

Step 3: Write Out Each Dot Product Equation

1. $u_1 \cdot u_2 = 0$

$(1)(1) + (2)(1) + (1)(1) + (0)(3) = 1 + 2 + 1 + 0 = 4$

Result: $4 \neq 0$, so vectors u_1 and u_2 are not orthogonal unless the problem expects the set to be orthogonal excluding the pair (u_1, u_2), or perhaps the initial vectors are different.

Clarification:

Given the initial confusion, perhaps the problem is to find (a, B, C, D) such that the vectors:

- $v_1 = (1, 2, 1, 0)$
- $v_2 = (1, 1, 1, 3)$
- $v_3 = (2, 1, a, B)$
- $v_4 = (C, D, 0, 0)$

are mutually orthogonal except for the pair (v_1, v_2). Alternatively, perhaps the problem wants the set to be orthogonal with respect to these vectors, and the initial set is just a starting point.

Step 4: Focus on the core problem — solving for (a, B, C, D)

Assuming the goal is to find (a, B, C, D) such that the following conditions hold:

- $u_1 \cdot u_3 = 0$
- $u_1 \cdot u_4 = 0$
- $u_2 \cdot u_3 = 0$
- $u_2 \cdot u_4 = 0$
- $u_3 \cdot u_4 = 0$

Step 5: Set Up Equations and Solve

Equation 1: $u_1 \cdot u_3 = 0$

$(1)(2) + (2)(1) + (1)(a) + (0)(B) = 2 + 2 + a + 0 = 4 + a = 0$

$\Rightarrow a = -4$

Equation 2: $u_1 \cdot u_4 = 0$

$(1)(C) + (2)(D) + (1)(0) + (0)(0) = C + 2D + 0 + 0 = C + 2D = 0$

$\Rightarrow C = -2D$

Equation 3: $u_2 \cdot u_3 = 0$

$(1)(2) + (1)(1) + (1)(a) + (3)(B) = 2 + 1 + a + 3B$

Substitute $a = -4$:

$2 + 1 - 4 + 3B = (-1) + 3B$

Set equal to zero:

$-1 + 3B = 0$

$\Rightarrow 3B = 1$

$\Rightarrow B = 1/3$

Equation 4: $u_2 \cdot u_4 = 0$

$(1)(C) + (1)(D) + (1)(0) + (3)(0) = C + D + 0 + 0 = C + D$

Recall $C = -2D$ from earlier:

$C + D = -2D + D = -D$

Set equal to zero:

$-D = 0$

$\Rightarrow D = 0$

Then C = -2D = 0

Equation 5: $u_3 \cdot u_4 = 0$

(2)(C) + (1)(D) + (a)(0) + (B)(0) = 2C + D + 0 + 0 = 2C + D

Recall C = 0 and D = 0:

20 + 0 = 0

Equation satisfied.

Final Solution:

Variable	Value
a	-4
B	1/3
C	0
D	0

Conclusion and Summary

In this detailed analysis, we've demonstrated how to determine the unknown components (a, B, C, D) in R4 such that the set of vectors is mutually

Frequently Asked Questions

How do you determine if a set of vectors in R4 is orthogonal?

To determine if a set of vectors in R4 is orthogonal, you calculate the dot product between every pair of distinct vectors. If all pairwise dot products are zero, then the set is orthogonal.

What is the significance of finding vectors (a, B, C, D) such that the set is orthogonal?

Finding such vectors allows us to identify additional vectors that are orthogonal to the given set, which can help in constructing orthogonal complements or extending the set to an orthogonal basis.

Given the set {(1, 2, 1, 0), (1, 1, 1, 3), (2, 1, ..., how do I find all (a, B, C, D) in R4 to make the set orthogonal?

You find all vectors (a, B, C, D) such that their dot product with each of the given vectors is zero. This involves solving the system of equations formed by setting the dot products equal to zero.

How do I set up the equations to find (a, B, C, D) orthogonal to the given vectors?

You set up equations by computing the dot product of (a, B, C, D) with each given vector: (a, B, C, D) · (1, 2, 1, 0) = 0, (a, B, C, D) · (1, 1, 1, 3) = 0, and so on. Solving these equations yields the set of all such vectors.

Can you provide the step-by-step process to find all vectors orthogonal to the given set in R4?

Yes. First, write the dot product equations for each vector: $a + 2B + C + 0 \cdot D = 0$, $a + B + C + 3D = 0$, and $2a + B + C + \ldots$. Then, solve this system of linear equations to express (a, B, C, D) in terms of free parameters, representing the entire set of orthogonal vectors.

What is the geometric interpretation of the set of all vectors orthogonal to a given set in R4?

The set of all vectors orthogonal to a given set forms the orthogonal complement of the subspace spanned by that set. Geometrically, it consists of all vectors perpendicular to every vector in the original set.

Are the vectors in the set {(1, 2, 1, 0), (1, 1, 1, 3), (2, 1, ...} orthogonal to each other?

Not necessarily. To determine if the vectors are mutually orthogonal, you must compute the dot product between each pair. If all pairwise dot products are zero, then the vectors are orthogonal to each other; otherwise, they are not.

Additional Resources

Find All (a, B, C, D) in R^4 Such That the Given Set Is Orthogonal — this problem lies at the heart of linear algebra's exploration of vector spaces and orthogonality. When dealing with sets of vectors in R^4, understanding how to determine the specific parameters that make these vectors orthogonal is crucial, especially in applications spanning computer graphics, data analysis, and engineering. In this guide, we'll dissect the process step-by-step, providing a comprehensive approach to finding all solutions for (a, B, C, D) such that the vectors meet the orthogonality criteria.

Introduction: Understanding Orthogonality in R^4

In the realm of vector spaces, orthogonality refers to the perpendicularity of vectors. Two vectors are orthogonal if their dot product equals zero. Extending this to a set of vectors, the entire set is orthogonal if every pair of vectors in the set is orthogonal.

Given the set:

$$
\{ \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \} = \{ (1, 2, 1, 0), \quad (1, 1, 1, 3), \quad (2, 1, a, B) \}
$$

Our goal is to find all real values of a, B, C, D such that these vectors are pairwise orthogonal. While the problem statement mentions parameters a, B, C, D, the given vectors only involve a and B, suggesting that C and D might be related to other vectors or parameters in an extended context. For clarity, we'll focus on the vectors explicitly provided and their pairwise dot products, aiming to find all parameters that satisfy the orthogonality conditions.

Step 1: Recall the Dot Product and Orthogonality Conditions

The dot product of two vectors $\mathbf{u} = (u_1, u_2, u_3, u_4)$ and $\mathbf{v} = (v_1, v_2, v_3, v_4)$ in R^4 is:

$$
\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + u_3 v_3 + u_4 v_4
$$

For the vectors to be orthogonal:

$$
\mathbf{u} \cdot \mathbf{v} = 0
$$

Applying this to our set, we need to compute:

- $\mathbf{v}_1 \cdot \mathbf{v}_2 = 0$
- $\mathbf{v}_1 \cdot \mathbf{v}_3 = 0$
- $\mathbf{v}_2 \cdot \mathbf{v}_3 = 0$

Step 2: Calculate the Pairwise Dot Products

2.1 Dot product of $\mathbf{v}_1$ and $\mathbf{v}_2$:

$$
(1, 2, 1, 0) \cdot (1, 1, 1, 3) = (1)(1) + (2)(1) + (1)(1) + (0)(3) = 1 + 2 + 1 + 0 = 4
$$

Observation: Since this dot product equals 4, unless the vectors are scaled or modified, they are not orthogonal. To make them orthogonal, additional conditions or modifications are needed, or perhaps the problem expects us to find parameters within the third vector so that the set becomes orthogonal.

2.2 Dot product of $\mathbf{v}_1$ and $\mathbf{v}_3$:

$$
(1, 2, 1, 0) \cdot (2, 1, a, B) = (1)(2) + (2)(1) + (1)(a) + (0)(B) = 2 + 2 + a + 0 = 4 + a
$$

Set equal to zero for orthogonality:

$$
4 + a = 0 \Rightarrow a = -4
$$

Result: To make $\mathbf{v}_1$ and $\mathbf{v}_3$ orthogonal, a must be -4.

2.3 Dot product of $\mathbf{v}_2$ and $\mathbf{v}_3$:

$$
(1, 1, 1, 3) \cdot (2, 1, a, B) = (1)(2) + (1)(1) + (1)(a) + (3)(B) = 2 + 1 + a + 3B
$$

Set equal to zero:

$$
2 + 1 + a + 3B = 0
$$
$$
3 + a + 3B = 0
$$

Recall that $a = -4$ from the previous step, so substitute:

$$
3 - 4 + 3B = 0 \Rightarrow -1 + 3B = 0 \Rightarrow 3B = 1 \Rightarrow B = \frac{1}{3}
$$

Step 3: Summarize the Conditions

From the calculations above, the parameters must satisfy:

- $a = -4$
- $B = \frac{1}{3}$

Now, revisit the initial vectors to verify whether these choices satisfy the orthogonality conditions.

3.1 Check $\mathbf{v}_1 \cdot \mathbf{v}_2$:

\[

$$
(1, 2, 1, 0) \cdot (1, 1, 1, 3) = 4
$$

Since this equals 4, the vectors are not orthogonal unless scaled appropriately.

Important clarification: The problem statement mentions "Find all (a, B, C, D) such that the set is orthogonal." Given the initial vectors, the set is not orthogonal with the current vectors unless we modify or consider scalar multiples.

Step 4: Extending the Problem — Making the Set Orthogonal

To make the entire set orthogonal, we need all pairwise dot products to be zero. Given that $\mathbf{v}_1$ and $\mathbf{v}_2$ have a dot product of 4, the set is not orthogonal as is.

4.1 Adjusting $\mathbf{v}_2$ or scaling vectors

One approach is to see if scaling vectors can produce orthogonality. Alternatively, perhaps the problem expects the set to be orthogonal after choosing certain parameters or considering specific linear combinations.

Step 5: Reinterpreting the Problem — Orthogonality of the Set with Parameters a, B, C, D

Suppose the third vector involves parameters C and D:

$$
\mathbf{v}_3 = (2, 1, a, B)
$$

and perhaps the problem involves constructing vectors with parameters C and D in similar fashion, like:

$$
\mathbf{v}_3 = (2, 1, C, D)
$$

In that context, the goal would be to find all (a, B, C, D) such that the set:

$$
\left\{ (1, 2, 1, 0), \quad (1, 1, 1, 3), \quad (2, 1, C, D) \right\}
$$

is orthogonal, i.e., pairwise dot products are zero.

Step 6: Solving for the Extended Set

6.1 Dot product of \(\mathbf{v}_1 \) and \(\mathbf{v}_3 \):

\[
(1, 2, 1, 0) \cdot (2, 1, C, D) = 1 \times 2 + 2 \times 1 + 1 \times C + 0 \times D = 2 + 2 + C + 0 = 4 + C
\]

Set to zero:

\[
4 + C = 0 \Rightarrow C = -4
\]

6.2 Dot product of \(\mathbf{v}_2 \) and \(\mathbf{v}_3 \):

\[
(1, 1, 1, 3) \cdot (2, 1, C, D) = 1 \times 2 + 1 \times 1 + 1 \times C + 3 \times D = 2 + 1 + C + 3D
\]

Set to zero:

\[
2 + 1 + C + 3D = 0 \Rightarrow 3 + C + 3D = 0
\]

Substitute \(C = -4 \):

\[
3 - 4 + 3D = 0 \Rightarrow -1 + 3D = 0 \Rightarrow 3D = 1 \Rightarrow D = \frac{1}{3}
\]

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Find All A B C D In R4 Such That The Given Set Is Orthogonal 1 2 1 0 1 1 1 3 2 1

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