Find All Second-order Partial Derivatives Of The Given Function. Z = 6x In (3x^5 Y^3) Zxx = ____ (Type

Find All Second-order Partial Derivatives Of The Given Function. Z = 6x In (3x^5 Y^3) Zxx = ____ (Type

In multivariable calculus, understanding the behavior of functions involving multiple variables often requires finding their derivatives with respect to each variable. Among these derivatives, second-order partial derivatives provide insights into the curvature and concavity of the function in different directions. In this article, we will explore the process of computing all second-order partial derivatives of the given function:

Z = 6x $\ln (3x^5 y^3)$

Specifically, we will focus on calculating the second-order partial derivatives, including Z_{xx}, Z_{xy}, Z_{yx}, and Z_{yy}. To do this systematically, we'll start by finding the first-order derivatives and then differentiate again to obtain the second-order derivatives. Let's dive into the detailed step-by-step process.

Understanding the Function and Its Components

Function Breakdown

- The function is: $Z = 6x \ln(3x^5 y^3)$
- Variables involved: x and y
- Goal: Find all second-order partial derivatives, especially Z_{xx}

Key Concepts

- **Partial derivatives**: derivatives with respect to one variable, treating others as constants.
- **Second-order derivatives**: derivatives of the first derivatives, providing curvature information.

- Chain rule and product rule are essential tools for differentiation here.

Step 1: Simplify the Function

Rewrite the Function

Express the function in a more manageable form by expanding the logarithm:

$$
Z = 6x \ln(3x^5 y^3)
$$

$$
\begin{align}
\ln(3x^5 y^3) &= \ln 3 + \ln x^5 + \ln y^3 \\
&= \ln 3 + 5 \ln x + 3 \ln y
\end{align}
$$

Thus,

$$
Z = 6x (\ln 3 + 5 \ln x + 3 \ln y)
$$

Now, distribute $6x$:

$$
Z = 6x \ln 3 + 30x \ln x + 18x \ln y
$$

This form simplifies differentiation significantly.

Step 2: First-order Partial Derivatives

Calculate $Z_x = \frac{\partial Z}{\partial x}$

$$
Z = 6x \ln 3 + 30x \ln x + 18x \ln y
$$

Differentiate term-by-term:

1. $\frac{\partial}{\partial x} (6x \ln 3) = 6 \ln 3$ (since $\ln 3$ is constant)
2. $\frac{\partial}{\partial x} (30x \ln x)$

Apply the product rule:

$$
\frac{\partial}{\partial x} (x \ln x) = \ln x + 1
$$

Therefore:

$$
\frac{\partial}{\partial x} (30x \ln x) = 30 (\ln x + 1) = 30 \ln x + 30
$$

3. $\frac{\partial}{\partial x} (18x \ln y)$

Since $\ln y$ is constant with respect to x:

$$
= 18 \ln y
$$

Putting it together:

$$
Z_x = 6 \ln 3 + 30 \ln x + 30 + 18 \ln y
$$

Calculate $Z_y = \frac{\partial Z}{\partial y}$

From the simplified form:

$$
Z = 6x \ln 3 + 30x \ln x + 18x \ln y
$$

Differentiate term-by-term:

1. $\frac{\partial}{\partial y} (6x \ln 3) = 0$
2. $\frac{\partial}{\partial y} (30x \ln x) = 0$
3. $\frac{\partial}{\partial y} (18x \ln y)$

Since x is treated as constant:

$$
= 18x \frac{1}{y} = \frac{18x}{y}
$$

Thus,

\[

$$
Z_y = \frac{18x}{y}
$$

Step 3: Computing Second-order Partial Derivatives

Calculate $Z_{xx} = \frac{\partial^2 Z}{\partial x^2}$

Recall the first derivative:

$$
Z_x = 6 \ln 3 + 30 \ln x + 30 + 18 \ln y
$$

Differentiate Z_x with respect to x:

- $6 \ln 3$ and 30 are constants, derivatives are zero.
- $30 \ln x$:

$$
\frac{\partial}{\partial x} (30 \ln x) = 30 \times \frac{1}{x} = \frac{30}{x}
$$
- $18 \ln y$ is constant with respect to x:

$$
\frac{\partial}{\partial x} (18 \ln y) = 0
$$

Therefore,

$$
Z_{xx} = \frac{30}{x}
$$

Calculate $Z_{yy} = \frac{\partial^2 Z}{\partial y^2}$

Recall:

$$
Z_y = \frac{18x}{y}
$$

Differentiate with respect to y:

$$
Z_{yy} = \frac{\partial}{\partial y} \left(\frac{18x}{y} \right) = 18x \times \frac{\partial}{\partial y} \left(y^{-1} \right) = 18x \times (- y^{-2}) = - \frac{18x}{y^2}
$$

Calculate $Z_{xy} = \frac{\partial^2 Z}{\partial y \partial x}$

Differentiate Z_x with respect to y:

Recall:

$$
Z_x = 6 \ln 3 + 30 \ln x + 30 + 18 \ln y
$$

Only the last term depends on y:

$$
\frac{\partial}{\partial y} (18 \ln y) = \frac{18}{y}
$$

Thus:

$$
Z_{xy} = \frac{18}{y}
$$

Similarly, Z_{yx} should be equal to Z_{xy} due to the symmetry of mixed partial derivatives (assuming continuous second derivatives), which confirms:

$$
Z_{yx} = \frac{18}{y}
$$

Summary of Second-Order Partial Derivatives

- $Z_{xx} = \frac{30}{x}$
- $Z_{yy} = - \frac{18x}{y^2}$
- $Z_{xy} = Z_{yx} = \frac{18}{y}$

Conclusion and Final Remarks

Understanding how to compute all second-order partial derivatives of functions involving multiple variables is crucial for analyzing their curvature and stability. In this example, we've demonstrated a systematic approach starting from simplifying the function, computing first derivatives, and then differentiating again to obtain second derivatives.

Knowing these derivatives allows you to explore the Hessian matrix of the function, which is essential in optimization problems, analyzing critical points, and studying the function's behavior in different directions.

Key Takeaways

1. Always simplify the function as much as possible before differentiating.
2. Use the product rule and chain rule carefully, especially with composed functions like logarithms.
3. Verify the symmetry of mixed partial derivatives when applicable.

With practice, calculating second-order partial derivatives becomes more intuitive, empowering you to analyze complex multivariable functions effectively.

Frequently Asked Questions

How do you find the second-order partial derivative Zxx of the function Z = 6x ln(3x^5 y^3)?

First, find the first partial derivative Zx by differentiating Z with respect to x, then differentiate Zx with respect to x again to obtain Zxx.

What is the first step in computing Zxx for Z = 6x ln(3x^5 y^3)?

Apply the product rule to differentiate 6x and ln(3x^5 y^3) with respect to x, simplifying each term accordingly.

How do you differentiate ln(3x^5 y^3) with respect to x?

Use the chain rule: d/dx [ln(3x^5 y^3)] = (1 / (3x^5 y^3)) d/dx (3x^5 y^3), which simplifies to (1 / (3x^5 y^3)) (15x^4 y^3).

What is the simplified form of the first derivative Zx for Z = 6x ln(3x^5 y^3)?

Zx = 6 ln(3x^5 y^3) + 6x (15x^4 y^3 / 3x^5 y^3) = 6 ln(3x^5 y^3) + 6x (15x^4 / 3x^5) = 6 ln(3x^5 y^3) + 6x (5 / x) = 6 ln(3x^5 y^3) + 30.

How do you differentiate Zx to find Zxx?

Differentiate Zx with respect to x, applying the chain rule to the logarithmic term and the power rule to the polynomial terms.

What is the second derivative Zxx of Z = 6x ln(3x^5 y^3)?

Zxx = d/dx [6 ln(3x^5 y^3) + 30] = 6 d/dx [ln(3x^5 y^3)] + 0 = 6 (15x^4 / x) = 6 15x^3 = 90x^3.

Are there any terms involving y when computing Zxx of the given function?

No, since the second derivative with respect to x involves only x-dependent terms, y is treated as a constant during differentiation.

What is the importance of understanding second-order partial derivatives like Zxx?

Second-order partial derivatives help analyze the curvature, concavity, and local extrema of multivariable functions, which are essential in optimization and differential equations.

Can you provide the final expression for Zxx for Z = 6x ln(3x^5 y^3)?

Yes, the second-order partial derivative Zxx = 90x^3.

Additional Resources

Find All Second-Order Partial Derivatives Of The Given Function. Z = 6x ln (3x^5 Y^3) Zxx = ____

When working with multivariable calculus, understanding how to find all second-order partial derivatives of a function is a fundamental skill. In particular, the task of finding all second-order partial derivatives of the given function Z = 6x ln (3x^5 Y^3) involves applying partial differentiation rules meticulously and systematically. Mastering this process not only deepens comprehension of partial derivatives but also enhances problem-solving skills crucial for advanced calculus, engineering, physics, and related fields.

In this comprehensive guide, we will walk through the process step-by-step, providing clear explanations, formulas, and tips to ensure you can confidently compute all second-order partial derivatives for this specific function.

Understanding the Function

Before diving into derivatives, let's analyze the given function:

$$ Z = 6x \ln (3x^5 Y^3) $$

At first glance, we notice that Z is a product of $6x$ and a natural logarithm function involving both x and Y. This suggests that the function is multivariable, depending on x and Y, and that it involves products and compositions, which require careful application of differentiation rules.

Step 1: Simplify the Function

To make differentiation easier, it helps to simplify the logarithmic expression:

$$
\ln (3x^5 Y^3) = \ln 3 + \ln x^5 + \ln Y^3
$$

Using logarithm properties:

- $\ln a b = \ln a + \ln b$
- $\ln a^k = k \ln a$

we get:

$$
\ln (3x^5 Y^3) = \ln 3 + 5 \ln x + 3 \ln Y
$$

Thus, the function simplifies to:

$$
Z = 6x \left(\ln 3 + 5 \ln x + 3 \ln Y \right)
$$

Distribute $6x$:

$$
Z = 6x \ln 3 + 30x \ln x + 18x \ln Y
$$

This form is more manageable for differentiation.

Step 2: Find the First-Order Partial Derivatives

2.1 Partial derivative with respect to x (Z_x)

We differentiate Z with respect to x, treating Y as a constant:

$$
Z = 6x \ln 3 + 30x \ln x + 18x \ln Y
$$

Differentiate each term:

- $\frac{\partial}{\partial x} (6x \ln 3) = 6 \ln 3$ (since $\ln 3$ is constant)
- $\frac{\partial}{\partial x} (30x \ln x)$:

Apply the product rule:

$$
\frac{d}{dx} (x \ln x) = \ln x + 1
$$

Thus,

$$
\frac{\partial}{\partial x} (30x \ln x) = 30 (\ln x + 1)
$$

- $\frac{\partial}{\partial x} (18x \ln Y)$:

Since $\ln Y$ is constant with respect to x,

$$
= 18 \ln Y
$$

Putting it all together:

$$
Z_x = 6 \ln 3 + 30 (\ln x + 1) + 18 \ln Y
$$
$$
Z_x = 6 \ln 3 + 30 \ln x + 30 + 18 \ln Y
$$

2.2 Partial derivative with respect to Y (Z_Y)

Now, differentiate Z with respect to Y, treating x as a constant:

$$
Z = 6x \ln 3 + 30x \ln x + 18x \ln Y
$$

All terms except $18x \ln Y$ are constants with respect to Y:

$$
\frac{\partial}{\partial Y} (18x \ln Y) = 18x \times \frac{1}{Y} = \frac{18x}{Y}
$$

Hence,

$$
Z_Y = \frac{18x}{Y}
$$

Step 3: Find All Second-Order Partial Derivatives

Second derivatives involve differentiating the first derivatives again, considering mixed derivatives where necessary.

3.1 Second derivative with respect to x (Z_{xx})

Differentiate Z_x with respect to x:

$$
Z_x = 6 \ln 3 + 30 \ln x + 30 + 18 \ln Y
$$

Constants:

- $6 \ln 3$: derivative is 0
- 30: derivative is 0
- $18 \ln Y$: derivative with respect to x is 0 (since Y is treated as constant)

Remaining term:

$$
\frac{d}{dx} (30 \ln x) = 30 \times \frac{1}{x} = \frac{30}{x}
$$

Therefore,

$$
Z_{xx} = \frac{30}{x}
$$

3.2 Second derivative with respect to Y (Z_{yy})

Differentiate Z_Y with respect to Y:

$$
Z_Y = \frac{18x}{Y}
$$

Treating x as a constant:

$$
Z_{yy} = \frac{d}{dY} \left(\frac{18x}{Y} \right) = 18x \times \left(- \frac{1}{Y^2}\right) = -\frac{18x}{Y^2}
$$

3.3 Mixed second derivative Z_{xy}

Differentiate Z_x with respect to Y:

Recall:

$$
Z_x = 6 \ln 3 + 30 \ln x + 30 + 18 \ln Y
$$

All terms except $18 \ln Y$ are independent of Y:

$$
\frac{\partial}{\partial Y} (18 \ln Y) = 18 \times \frac{1}{Y} = \frac{18}{Y}
$$

Thus,

$$
Z_{xy} = \frac{18}{Y}
$$

3.4 Mixed second derivative Z_{yx}

Differentiate Z_Y with respect to x:

$$
Z_Y = \frac{18x}{Y}
$$

Treating Y as constant:

$$
Z_{yx} = \frac{\partial}{\partial x} \left(\frac{18x}{Y} \right) = \frac{18}{Y}
$$

Note that:

$$
Z_{xy} = Z_{yx} = \frac{18}{Y}
$$

which confirms the symmetry of mixed partial derivatives for sufficiently smooth functions.

Summary of All Second-Order Partial Derivatives

Derivative	Expression
Z_{xx}	$\frac{30}{x}$
Z_{yy}	$- \frac{18x}{Y^2}$
Z_{xy} / Z_{yx}	$\frac{18}{Y}$

Final Remarks and Tips

- Logarithmic properties simplify complex functions, making derivatives easier.
- Always distribute and simplify before differentiating.
- When differentiating products, use the product rule.
- Remember that partial derivatives with respect to one variable treat other variables as constants.
- Mixed derivatives should be checked for symmetry ($Z_{xy} = Z_{yx}$) in well-behaved functions.
- Be cautious with division by variables; ensure variables are within their domain to avoid undefined expressions.

Conclusion

Mastering the process of finding all second-order partial derivatives involves careful application of differentiation rules, algebraic simplification, and understanding the structure of multivariable functions. For the function $Z = 6x \ln (3x^5 Y^3)$, we demonstrated how to simplify, differentiate, and organize the derivatives systematically. Whether you're working on theoretical problems or applied scenarios, these skills form the backbone of advanced calculus and mathematical analysis. Keep practicing with different functions to build confidence and proficiency in multivariable differentiation!

Find All Second Order Partial Derivatives Of The Given Function Z 6x In 3x 5 Y 3 Zxx Type

Find All Second Order Partial Derivatives Of The Given Function Z 6x In 3x 5 Y 3 Zxx Type

Related Articles

- [In A Yoga Class, 75% Of The Students Are Women. There Are 24 Women In The Class. How Many Students Are]
- [How Does The Organization Of The US Government Reflect The Ideas Of Enlightenment Philosopher Baron De]
- [If Your Core Temperature Becomes Colder, It Is More Difficult For Oxygen To Dissociate From Hemoglobin]

Back to Home