Find An Equation For The Parabola That Has Its Vertex At The Origin And Satisfies The Given Condition.

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Understanding the equations of parabolas is fundamental in algebra and geometry, especially when analyzing their shapes, properties, and applications in real-world scenarios. When tasked with finding the equation of a parabola that has its vertex at the origin (0,0) and satisfies certain conditions, it becomes essential to understand the basic forms of parabola equations, how to manipulate them, and how to apply the given conditions to determine the specific equation.

In this comprehensive guide, we will explore the process of deriving the equation of a parabola with its vertex at the origin that meets various conditions. We will cover the standard forms, vertex form, focus-directrix property, and how to utilize given points or slopes to find the specific parabola equation. Additionally, we will incorporate numerous examples and step-by-step solutions to clarify the concepts and methods involved.

Understanding the Basic Forms of Parabola Equations

To find the equation of a parabola with its vertex at the origin, it is crucial to understand the different forms the parabola's equation can take. The most common forms are:

1. Standard Form

- The standard form of a parabola that opens upward or downward is:

$$
y = ax^2 + bx + c
$$

- When the vertex is at the origin (0,0), the equation simplifies to:

$$
y = ax^2
$$

since there are no linear or constant terms (b = 0, c = 0).

2. Vertex Form

- The vertex form explicitly shows the vertex's coordinates:

$$
y = a(x - h)^2 + k
$$

- When the vertex is at (0,0), this reduces to:

$$
y = ax^2
$$

- The parameter 'a' determines the parabola's opening direction and "width" (or "narrowness").

3. Focus-Directrix Form

- A parabola can also be defined by its focus and directrix:

$$
\text{Distance from any point } (x, y) \text{ to focus} = \text{Distance from } (x, y) \text{ to directrix}
$$

- For a parabola with vertex at the origin, focus at (0, p) (for a parabola opening upward), the equation becomes:

$$
y = \frac{x^2}{4p}
$$

- Similarly, for a parabola opening downward, the focus is at (0, -p), leading to:

$$
y = -\frac{x^2}{4p}
$$

Deriving the Equation Based on Given Conditions

When solving for the specific parabola, the key is to understand what conditions are provided and how they influence the equation. Typical conditions include:

- A point lying on the parabola.
- A slope (derivative) at a specific point.
- The focus or directrix location.
- The parabola passing through multiple points.

Let's examine how to utilize these conditions.

Condition 1: The Parabola Passes Through a Given Point

Suppose the parabola has its vertex at the origin, and it passes through a point (x_1, y_1). The steps are:

1. Write the general form of the parabola: $y = ax^2$.
2. Substitute the point into the equation:

$$
y_1 = a x_1^2
$$

3. Solve for a:

$$
a = \frac{y_1}{x_1^2}
$$

4. Write the specific equation:

$$
y = \frac{y_1}{x_1^2} x^2
$$

Example:

Find the equation of the parabola with vertex at (0,0) passing through (2, 8).

Solution:

$$
a = \frac{8}{(2)^2} = \frac{8}{4} = 2
$$

Equation:

\[

\boxed{ y = 2x^2 }
\]

Condition 2: The Slope at a Point

If the condition involves the slope (derivative) at a particular point, follow these steps:

1. Write the general form \(y = ax^2 \).
2. Find the derivative \(y' = 2ax \).
3. Substitute the \(x\) coordinate of the point to find the slope:

\[
\text{slope} = 2a x
\]

4. Set the derivative equal to the given slope and solve for \(a\).

Example:

Find the parabola with vertex at (0,0), passing through (1,1), and having a slope of 2 at that point.

Solution:

- From the point:

\[
1 = a \times 1^2 \Rightarrow a = 1
\]

- Derivative:

\[
y' = 2a x = 2 \times 1 \times 1 = 2
\]

which matches the given slope. So, the parabola is:

\[
\boxed{ y = x^2 }
\]

Condition 3: Focus or Directrix Constraints

When the focus or directrix is specified, use the focus-directrix form:

$$
y = \frac{x^2}{4p}
$$

where p is the distance from the vertex to the focus.

Example:

Find the parabola with vertex at (0,0) and focus at (0, 3).

Solution:

- The focus is at (0, p), so $p = 3$.
- Equation:

$$
y = \frac{x^2}{4p} = \frac{x^2}{4 \times 3} = \frac{x^2}{12}
$$

Thus, the parabola's equation:

$$
\boxed{ y = \frac{x^2}{12} }
$$

Summary of Methods to Find the Parabola Equation

Here is a quick reference:

Condition	Approach	Equation Form	Example
Passes through a point (x_1, y_1)	Substitute into $y = ax^2$	$a = y_1 / x_1^2$	$y=2x^2$
Slope at a point	Derivative approach	$y' = 2ax$	$y = x^2$
Focus at $(0,p)$	Use focus-directrix form	$y = x^2 / (4p)$	$y = x^2/12$

Additional Examples and Practice Problems

To solidify understanding, let's work through additional examples of deriving parabola equations based on mixed conditions.

Example 1: Parabola with Vertex at (0,0), passing through (3, 9), and having a focus at (0, 4)

Step 1: Use focus-directrix form:

- Focus at (0, 4), so $p = 4$.
- Equation:

$$
y = \frac{x^2}{4p} = \frac{x^2}{16}
$$

Step 2: Verify if the point (3, 9) lies on this parabola:

$$
\text{LHS} = 9, \quad \text{RHS} = \frac{(3)^2}{16} = \frac{9}{16} \approx 0.5625
$$

No, it doesn't. So, the focus at (0,4) doesn't match the point.

Step 3: Alternatively, find a from the point:

$$
9 = a \times 3^2 \Rightarrow a = \frac{9}{9} = 1
$$

Equation:

$$
y = x^2
$$

Step 4: Check if focus at (0, p):

$$
p = y_{\text{vertex}} + \frac{1}{4a} = 0 + \frac{1}{4 \times 1} = \frac{1}{4}
$$

which is not 4, so the focus is at (0, 0.25). The initial focus at (0, 4) doesn't fit this point.

Conclusion: The parabola passing through (3,9) with vertex at (0,0) and focus at (0, 4) cannot be the same, so they define different parabola equations.

Conclusion and Final Tips

- When given a condition, identify whether it relates to a point, slope, focus, or directrix.
- Use the appropriate form of the parabola equation based on the condition:
- For points, use \(y = ax^

Frequently Asked Questions

How do I find the equation of a parabola with its vertex at the origin and a given focus?

For a parabola with vertex at the origin and focus at (0, p), the equation is $y^2 = 4px$ if the parabola opens vertically, or $x^2 = 4py$ if it opens horizontally. Use the focus coordinates to determine p and then write the corresponding equation.

What is the standard form of a parabola with vertex at the origin that opens upwards?

The standard form is $y^2 = 4ax$, where a is a real number indicating the distance from the vertex to the focus. For a parabola opening upwards or downwards, the form is $x^2 = 4ay$.

How can I derive the equation of a parabola given its directrix and vertex at the origin?

The parabola is the set of points equidistant from the focus and directrix. If the directrix is $y = -d$, and the focus is at (0, d), the equation becomes $y = (1/4d) x^2$. Simplify accordingly to find the specific equation.

What condition must be satisfied for a parabola with vertex at the origin?

The focus must be located along the axis of symmetry, and the parabola's equation reflects that symmetry. If the focus is at (0, p), the parabola's equation depends on whether it opens vertically or horizontally, with $y^2 = 4px$ or $x^2 = 4py$ respectively.

Can you give an example of finding the parabola equation with vertex at the origin and focus at (0, 3)?

Yes. Since the focus is at (0, 3), the parabola opens upward. The equation is $y^2 = 4px$, with $p = 3$. So, the equation is $y^2 = 12x$.

How do I determine the equation of a parabola if I know it passes through a specific point and has vertex at the origin?

Use the general form of the parabola with vertex at the origin, substitute the coordinates of the known point into the equation, and solve for the parameter (like a or p) to find the specific parabola.

What is the process to find an equation for a parabola with a given focus and vertex at the origin?

Identify the focus's position relative to the vertex to determine p, then write the standard form $y^2 = 4px$ (if vertical) or $x^2 = 4py$ (if horizontal). Substitute p to get the specific equation.

How does the orientation of the parabola affect its equation when the vertex is at the origin?

If the parabola opens vertically (up or down), the equation is $y^2 = 4ax$. If it opens horizontally (left or right), the equation is $x^2 = 4ay$. The sign of a determines the direction of opening.

What are common mistakes to avoid when deriving the equation of a parabola with vertex at the origin?

Common mistakes include mixing up the orientation (vertical vs horizontal), incorrectly identifying the focus or directrix, and forgetting the sign conventions for the parameters. Always verify the direction of opening and the position of the focus.

How can I verify that the equation I found correctly represents the parabola with the specified vertex and condition?

Check that the vertex (0,0) satisfies the equation, confirm the focus or directrix matches the given condition, and verify additional points on the parabola satisfy the equation. Visualizing the graph can also help confirm correctness.

Additional Resources

Parabola Equation Derivation: A Comprehensive Guide to Finding the Equation of a Parabola with Its Vertex at the Origin and Satisfying Given Conditions

Understanding the fundamental equations of conic sections, especially parabolas, is essential not only in mathematics but also in various applied fields such as physics, engineering, and computer graphics. When tasked with finding the equation of a parabola that has its vertex at the origin and satisfies specific conditions, a methodical approach rooted in algebra and geometry is necessary. This article provides an in-depth exploration of how to derive such equations, examining key concepts, standard forms, and the impact of different conditions on the parabola's equation.

Foundations of Parabola Equations

Before diving into the derivation process, it is crucial to understand the basic properties and standard forms of a parabola.

What Is a Parabola?

A parabola is a symmetric, U-shaped conic section that can be described as the set of all points equidistant from a fixed point called the focus and a fixed line called the directrix. Its unique geometric property makes it significant in optics, projectile motion, and satellite dish design.

Standard Forms of a Parabola Equation

The equation of a parabola depends on its orientation and position:

- Vertex at the Origin, Opening Up or Down:

$$
y = ax^{2}
$$

- Vertex at the Origin, Opening Left or Right:

$$
x = ay^{2}
$$

\]

- General Form for a Vertical Parabola (Vertex at (h, k)):

$$
(x - h)^2 = 4p(y - k)
$$

- General Form for a Horizontal Parabola (Vertex at (h, k)):

$$
(y - k)^2 = 4p(x - h)
$$

In these forms, p indicates the distance from the vertex to the focus, and the sign of p indicates the direction of opening.

Key Assumptions and Conditions

When the problem specifies that the parabola has its vertex at the origin, the focus shifts to the standard forms with $h = 0$ and $k = 0$. The specific conditions provided further tailor the equation.

Common conditions include:

- The parabola passes through a specific point (x_0, y_0).
- The parabola has a specific focus point (x_f, y_f).
- The parabola has a known directrix.
- The parabola's width or "spread" is specified via the parameter p.

In practice, to derive the equation, you need to:

1. Identify the orientation (vertical or horizontal).
2. Use the given point(s), focus, or directrix to determine parameters.
3. Plug known values into standard forms to solve for unknowns.

Deriving the Equation Step-by-Step

Let's now explore the systematic process of deriving the parabola's equation given various conditions, assuming the vertex at the origin.

Case 1: Parabola Opening Up or Down, Passing Through a Given Point

Scenario: The parabola opens upward or downward with vertex at (0, 0), and passes through a point (x_0, y_0).

Step 1: Choose the appropriate standard form. For vertical orientation:

$$
y = ax^2
$$

Step 2: Use the point (x_0, y_0) to find a:

$$
y_0 = a x_0^2
\Rightarrow a = \frac{y_0}{x_0^2}
$$

Step 3: Write the equation:

$$
\boxed{
y = \frac{y_0}{x_0^2} x^2
}
$$

Note: If $x_0 = 0$, the point lies at the vertex; otherwise, the equation is fully determined.

Case 2: Using Focus and Directrix

Scenario: The parabola has vertex at (0, 0), focus at $(0, p)$, and directrix at $y = -p$.

Step 1: Recall the geometric definition: each point (x, y) on the parabola is equidistant from the focus and directrix.

Step 2: Write the distances:

- Distance to focus:

$$
\sqrt{(x - 0)^2 + (y - p)^2}
$$

- Distance to directrix $y = -p$:

$$
| y + p |
$$

Step 3: Set equal:

$$
\sqrt{x^2 + (y - p)^2} = | y + p |
$$

Since $y + p \geq 0$ when $y \geq -p$, square both sides:

$$
x^2 + (y - p)^2 = (y + p)^2
$$

Expand:

$$
x^2 + y^2 - 2 p y + p^2 = y^2 + 2 p y + p^2
$$

Subtract $y^2 + p^2$:

$$
x^2 - 2 p y = 2 p y
$$

Combine like terms:

$$
x^2 = 4 p y
$$

Result:

$$
\boxed{
y = \frac{x^2}{4 p}
}
$$

This is the standard form of a parabola opening upward with vertex at the origin and focus at $(0, p)$.

Case 3: Parabola Opening Left or Right with Focus and Directrix

Scenario: The focus at $(p, 0)$, directrix at $x = -p$.

Step 1: Using the same geometric definition:

$$
\sqrt{(x - p)^2 + y^2} = | x + p |
$$

Step 2: Square both sides:

$$
(x - p)^2 + y^2 = (x + p)^2
$$

Expand:

$$
x^2 - 2 p x + p^2 + y^2 = x^2 + 2 p x + p^2
$$

Subtract $x^2 + p^2$:

$$
- 2 p x + y^2 = 2 p x
$$

Rearranged:

$$
y^2 = 4 p x
$$

Equation:

$$
\boxed{
x = \frac{y^2}{4 p}
}
$$

This describes a parabola opening to the right with vertex at the origin.

Incorporating Additional Conditions

The above cases demonstrate standard derivations. However, real-world problems often specify more nuanced conditions, such as:

- The parabola passing through multiple points.
- The parabola having a specific focal length.
- The parabola satisfying certain symmetry or geometric properties.

Approach for these scenarios:

1. Set up the standard form based on orientation.
2. Use the given points or parameters to substitute into the equation.
3. Solve for unknowns such as a, p, or other parameters.
4. Verify the resulting equation satisfies all conditions.

Practical Examples

Example 1: Find the equation of a parabola with vertex at the origin, passing through $(2, 8)$, opening upward.

Solution:

- Use $y = a x^2$.
- Plug in $(2, 8)$:

$$
8 = a \times 2^2 \Rightarrow 8 = 4a \Rightarrow a = 2
$$

- Equation:

$$
\boxed{
y = 2 x^2
}
$$

Example 2: Find the equation of a parabola with focus at $(0, 3)$ and vertex at the origin.

Solution:

- Since focus is at $(0, p)$, $p = 3$.

- Equation:

$$
y = \frac{x^2}{4 p} = \frac{x^2}{12}
$$

- Final equation:

$$
\boxed{
y = \frac{x^2}{12}
}
$$

Conclusion and Practical Tips

Deriving the equation of a parabola with its vertex at the origin involves understanding the standard forms and the geometric or algebraic conditions that define its shape. Whether using the focus-directrix definition or point-based substitution, the core process revolves around:

- Choosing the appropriate standard form.
- Applying known conditions to solve for parameters.
- Verifying that the resulting equation satisfies all given constraints.

Expert Tips:

- Always clarify the parabola's orientation before selecting the standard form.
- Use the geometric definition involving focus and directrix when possible, as it provides clear insight into the parabola's parameters.
- For multiple conditions, set up systems of equations to determine unknowns.
- Remember that the vertex at the origin simplifies many equations, but ensure all assumptions align with the problem's conditions.

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