Find An Equation Of The Line: A Parallel To The Line Y = -2x 5, Passing Through (-1/2; 3/2) B Parallel

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Understanding how to find the equation of a line parallel to a given line and passing through a specific point is a fundamental skill in coordinate geometry. Whether you're a student preparing for exams, a math enthusiast, or a professional solving real-world problems, mastering this concept is essential. In this comprehensive guide, we will explore step-by-step methods to derive the equation of such a line, focusing on the given line $y = -2x + 5$ and the point $\left(-\frac{1}{2}, \frac{3}{2} \right)$. We will delve into the concepts of parallel lines, slope calculation, point-slope form, and the slope-intercept form, providing detailed explanations, examples, and tips along the way.

Understanding the Basics of Line Equations and Parallel Lines

Before we dive into solving the specific problem, it's important to review some foundational concepts related to line equations and the properties of parallel lines.

Equation of a Line in Coordinate Geometry

A straight line on the Cartesian plane can be represented by various forms of equations, with the most common being:

- Slope-Intercept Form: $y = mx + b$, where:
- m is the slope of the line.
- b is the y-intercept, the point where the line crosses the y-axis.

- Point-Slope Form: $y - y_1 = m(x - x_1)$, where:
- (x_1, y_1) is a point on the line.
- m is the slope of the line.

- Standard Form: $Ax + By + C = 0$.

While each form has its uses, for our purposes, the slope-intercept and point-slope forms are most intuitive.

What Does It Mean for Lines to Be Parallel?

Two lines are parallel if they lie in the same plane and never intersect, no matter how far extended. Mathematically, parallel lines share:

- The same slope m.
- Different y-intercepts b, unless they are the same line (which would mean they coincide).

Therefore, to find a line parallel to a given line, the key is to determine the slope of the original line and then use that same slope for the new line.

Analyzing the Given Line: Y = -2x + 5

The line provided in the problem is:

$$y = -2x + 5$$

This is in slope-intercept form, where:

- Slope m = -2
- Y-intercept b = 5

This line has a slope of -2, which indicates it descends at a rate of 2 units vertically for every 1 unit horizontally to the right.

Identifying the Slope for the Parallel Line

Since the new line must be parallel to the given line, it must:

- Have the same slope, $m = -2$.
- Pass through the point $\left(-\frac{1}{2}, \frac{3}{2} \right)$.

Deriving the Equation of the Parallel Line

With the slope and a point provided, we can now find the equation of the required line.

Using the Point-Slope Form

The point-slope form of a line's equation is:

$$ y - y_1 = m(x - x_1) $$

Where:

- $ (x_1, y_1) = \left(-\frac{1}{2}, \frac{3}{2} \right) $
- $ m = -2 $

Plugging in the values:

$$ y - \frac{3}{2} = -2 \left(x - \left(-\frac{1}{2} \right) \right) $$

Simplify inside the parentheses:

$$ y - \frac{3}{2} = -2 \left(x + \frac{1}{2} \right) $$

Distribute $ -2 $:

$$ y - \frac{3}{2} = -2x - 1 $$

Now, solve for $ y $:

$$ y = -2x - 1 + \frac{3}{2} $$

Expressing $ -1 $ as $ -\frac{2}{2} $ to combine with $ \frac{3}{2} $:

$$ y = -2x + \left(-\frac{2}{2} + \frac{3}{2} \right) $$

$$ y = -2x + \frac{1}{2} $$

Final Equation in Slope-Intercept Form:

$$ y = -2x + \frac{1}{2} $$

Summary of the Process

To find the equation of a line parallel to a given line passing through a specific point, follow these steps:

1. Identify the slope of the given line. For $y = -2x + 5$, the slope is -2.

2. Use the same slope for the new line since parallel lines share identical slopes.

3. Apply the point-slope form with the known point $\left(-\frac{1}{2}, \frac{3}{2} \right)$:

$$y - y_1 = m(x - x_1)$$

4. Substitute the known values and simplify to find the equation in the slope-intercept form.

Additional Tips for Solving Similar Problems

- Always verify the slope of the original line before proceeding.
- Convert the point into decimal or fractional form consistently.
- Use the point-slope form for easier calculation when passing through a specific point.
- Simplify carefully to avoid algebraic errors, especially with fractions.
- Express the final answer in your preferred form (slope-intercept, standard, or point-slope).

Real-World Applications of Parallel Line Equations

Understanding how to find equations of parallel lines isn't just a theoretical exercise; it has practical applications across various fields:

- Engineering: Designing parallel beams or components.
- Architecture: Ensuring walls or features are parallel.
- Navigation: Plotting routes that run parallel to existing paths.
- Data Analysis: Drawing parallel trend lines for comparative purposes.

Practice Problems to Reinforce Learning

To solidify your understanding, try solving these problems:

1. Find the equation of a line parallel to $y = 3x - 4$ passing through $(2, 5)$.

2. Determine the equation of a line parallel to $y = \frac{1}{2}x + 1$ passing through $\left(0, -3 \right)$.

3. Given the line $y = -5x + 2$, find the equation of the line parallel passing through $\left(4, 7 \right)$.

Conclusion

Mastering the process of finding the equation of a line parallel to a given line and passing through a specific point is crucial in coordinate geometry. By understanding the properties of parallel lines, accurately calculating slopes, and applying the point-slope form, you can confidently derive the equations needed for various mathematical and real-world applications. Remember to verify your steps, handle fractions carefully, and express your final answer clearly. With practice, this process becomes intuitive, empowering you to tackle more complex geometric problems with ease.

Keywords: line equation, parallel lines, slope, point-slope form, coordinate geometry, algebra, math problems, slope-intercept form, find line equation, passing through point

Frequently Asked Questions

How do you find the equation of a line parallel to y = -2x + 5 passing through the point (-1/2, 3/2)?

Since the lines are parallel, they have the same slope of -2. Using point-slope form with point (-1/2, 3/2): y - 3/2 = -2(x + 1/2). Simplify to get the equation in slope-intercept form.

What is the slope of a line parallel to y = -2x + 5?

The slope of the parallel line is also -2 because parallel lines have identical slopes.

How do you write the equation of a line passing through a point with a given slope?

Use the point-slope form: $y - y_1 = m(x - x_1)$, where m is the slope and (x_1, y_1) is the point the line passes through.

What is the equation of the line parallel to y = -2x + 5 passing through (-1/2, 3/2)?

Using point-slope form: y - 3/2 = -2(x + 1/2). Simplify to get y = -2x + 1.

How do you convert a point-slope form to slope-intercept form?

Distribute and simplify to solve for y, resulting in the form y = mx + b.

Why do parallel lines have the same slope?

Parallel lines never intersect, which only occurs if their slopes are equal, ensuring they run in the same direction.

Is the point (-1/2, 3/2) in fraction form or decimal, and does it affect the equation?

The point is given in fractional form, but converting to decimal doesn't change the calculation; use either form as preferred.

Can the equation of the parallel line be written in standard form?

Yes, after finding the slope-intercept form, you can rearrange to standard form: Ax + By = C.

What are the steps to find the equation of a parallel line passing through a specific point?

Identify the slope from the given line, use the point-slope formula with the given point, then simplify to the desired form.

How does knowing the original line's equation help in finding the parallel line?

Because parallel lines share the same slope, the original line's equation directly provides the slope needed for the new line.

Additional Resources

Find An Equation Of The Line: A Parallel To The Line Y = -2x + 5, Passing Through (-1/2, 3/2) B Parallel

Understanding how to find the equation of a line parallel to a given line and passing through a specific point is a fundamental skill in coordinate geometry. This process involves grasping the concepts of slope, parallel lines, and point-slope form, which are essential in various mathematical and real-world applications. In this comprehensive guide, we'll explore the step-by-step method to determine the equation of a line parallel to \(y = -2x + 5 \), passing through the point \(-\frac{1}{2}, \frac{3}{2}\). We'll also discuss the underlying principles, common pitfalls, and practical tips to master this topic.

Understanding the Basic Concepts

Before diving into the problem, it's crucial to understand some foundational ideas in coordinate geometry.

1. Slope of a Line

The slope, denoted by m, measures the steepness of a line. For a line given in the form \(y = mx + b \), the coefficient m is the slope.

- In the line \(y = -2x + 5 \), the slope is -2.
- The slope indicates how much y changes for a unit change in x.

2. Parallel Lines

Parallel lines are lines in the same plane that never intersect and have identical slopes.

- If two lines are parallel, their slopes are equal.
- Thus, if the original line has slope m, any line parallel to it will also have slope m.

3. Point-Slope Form

The point-slope form of a line's equation is:
\[
y - y_1 = m (x - x_1)
\]
where \((x_1, y_1)\) is a point on the line, and m is the slope.

- This form is particularly useful when you know a point through which the line passes and its slope.

Step-by-Step Solution Approach

The goal is to find the equation of a line parallel to \(y = -2x + 5 \), passing through the point \((- \frac{1}{2}, \frac{3}{2})\). Let's break down the process:

1. Identify the Slope of the Given Line

Since the original line is \(y = -2x + 5 \), its slope m is -2.

2. Determine the Slope of the Parallel Line

Lines parallel to each other share the same slope:

\[
m_{parallel} = -2
\]

3. Use the Point-Slope Form with the Given Point

Plug the point \((- \frac{1}{2}, \frac{3}{2})\) and the slope \(-2\) into the point-slope form:

\[
y - y_1 = m (x - x_1)
\]
\[
y - \frac{3}{2} = -2 \left(x - \left(-\frac{1}{2}\right) \right)
\]
\[
y - \frac{3}{2} = -2 \left(x + \frac{1}{2} \right)
\]

4. Simplify the Equation

Distribute the slope:

$$
y - \frac{3}{2} = -2x - 1
$$

Add $\frac{3}{2}$ to both sides to solve for y:

$$
y = -2x - 1 + \frac{3}{2}
$$

Express -1 as $-\frac{2}{2}$:

$$
y = -2x - \frac{2}{2} + \frac{3}{2}
$$
$$
y = -2x + \frac{1}{2}
$$

Final Equation:

$$
\boxed{ y = -2x + \frac{1}{2} }
$$

This is the equation of the line parallel to $y = -2x + 5$, passing through the point $(-\frac{1}{2}, \frac{3}{2})$.

Additional Methods and Variations

While the point-slope form is straightforward, other methods can be used to derive the same line equation.

Using Slope-Intercept Form

Once the slope is known, you can directly substitute the point into the slope-intercept form $y = mx + b$ to find b:

$$
\frac{3}{2} = -2 \left(-\frac{1}{2} \right) + b
$$

$$
\frac{3}{2} = 1 + b
$$

$$
b = \frac{3}{2} - 1 = \frac{3}{2} - \frac{2}{2} = \frac{1}{2}
$$

Hence, the line's equation is:

$$
y = -2x + \frac{1}{2}
$$

Features and Practical Insights

Understanding this process offers several advantages:

- Clarity in geometric relationships: Recognizing that parallel lines share the same slope simplifies the problem.
- Versatility: The method applies to any line and point, making it a universal skill.
- Foundation for advanced topics: Concepts like perpendicular lines, distance between points and lines, and transformations build upon these principles.

Common Mistakes and How to Avoid Them

Despite the straightforward nature of the problem, students often encounter pitfalls:

- Mixing up the slope: Remember, the slope of the parallel line must be identical to the original line's slope.
- Incorrectly substituting points: Be cautious with the signs of the coordinates, especially with fractions.
- Forgetting to simplify: Always simplify the equation to the slope-intercept form for clarity.
- Confusing the point-slope and slope-intercept forms: They serve different purposes; choose the one that simplifies your calculations.

Additional Tips and Practice Problems

- Practice with various points and lines to enhance understanding.
- When given two points, find the slope first, then use point-slope form.
- Verify your answer by checking if the point satisfies the final equation.

Practice Problem: Find the equation of a line parallel to $y = 3x - 4$, passing through the point $(2, -1)$.

Conclusion

Finding an equation of a line parallel to a given line and passing through a specific point is an essential skill that combines understanding of slopes, equations of lines, and coordinate geometry principles. By identifying the slope of the original line and leveraging the point-slope form, you can efficiently derive the desired equation. Mastering this process not only enhances your mathematical proficiency but also prepares you for more advanced topics in geometry, algebra, and their applications in real-world scenarios.

Remember, the key steps are:

1. Determine the slope of the original line.
2. Use the same slope for the parallel line.
3. Substitute the point into the point-slope form.
4. Simplify to the slope-intercept form if needed.

With consistent practice, you'll find these problems become routine, empowering you to approach a wide range of geometric challenges with confidence.

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