Find The Area Of The Surface Generated When The Given Curve Is Revolved About The Given Ax Y=4sqrt(x),

Find The Area Of The Surface Generated When The Given Curve Is Revolved About The Given Axis: Y = 4√x

Understanding the surface area generated by the revolution of a curve around an axis is a fundamental concept in calculus. It combines the principles of integration with geometric intuition to determine the surface area of a three-dimensional shape formed by rotating a two-dimensional curve. In this article, we focus on the specific problem: finding the surface area generated when the curve $y = 4\sqrt{x}$ is revolved about a given axis, typically the x-axis or y-axis.

This problem is not only mathematically interesting but also has practical applications in engineering, physics, and manufacturing, where surface area calculations are essential for designing objects, calculating material usage, and analyzing physical phenomena.

Understanding the Problem: The Curve y = 4√x

Before delving into the mathematics, it's important to understand the characteristics of the given curve:

- The equation $y = 4\sqrt{x}$ describes a square root function scaled vertically by a factor of 4.
- The domain of the function is $x \geq 0$, as the square root function is defined for non-negative x.
- The range of the function is $y \geq 0$, since the square root yields non-negative results.
- The curve starts at the origin (0, 0) and increases gradually as x increases.

Visualizing this curve helps in understanding how the surface area will be generated when it is revolved about an axis.

Revolution About the X-Axis and Y-Axis: Key Concepts

The surface area generated by revolving a curve around an axis can vary significantly depending on the axis chosen. The two common axes of revolution are:

1. Revolution About the X-Axis

- The curve $y = 4\sqrt{x}$ is rotated around the x-axis.
- The resulting surface resembles a "bowl" or "shell" extending along the x-axis.
- The surface area depends on the length of the curve and the distance from each point to the axis.

2. Revolution About the Y-Axis

- The curve is revolved around the y-axis.
- The surface resembles a "bottle" or "vase" shape.
- The calculation involves different limits and variables.

For this article, we'll primarily focus on the revolution about the x-axis, which is the most common scenario in calculus problems involving $y = 4\sqrt{x}$.

Mathematical Foundation: Surface Area Formula

The general formula for the surface area (SA) generated when a curve $y = f(x)$ is revolved about the x-axis, over the interval [a, b], is:

$$SA = 2\pi \int_a^b y \sqrt{1 + (dy/dx)^2}\, dx$$

Where:
- $y = f(x)$ is the curve being revolved.
- dy/dx is the derivative of y with respect to x.
- The limits a and b define the interval over which the surface area is computed.

Similarly, if the rotation is about the y-axis, an analogous formula applies, involving integration with respect to y, but for the current focus, the x-axis is our primary axis.

Step-by-Step Solution: Calculating Surface Area for $y = 4\sqrt{x}$

To find the surface area of the surface generated when the curve $y = 4\sqrt{x}$ is revolved about the x-axis, follow these steps:

Step 1: Define the interval of interest

- Suppose we are interested in the surface area between $x = a$ and $x = b$.
- For example, from $x = 0$ to $x = c$, where $c > 0$.

Step 2: Find the derivative dy/dx

Given $y = 4\sqrt{x} = 4x^{(1/2)}$:

$dy/dx = 4 (1/2) x^{-1/2} = 2 x^{-1/2} = 2 / \sqrt{x}$

Step 3: Calculate $(dy/dx)^2$

$(dy/dx)^2 = (2 / \sqrt{x})^2 = 4 / x$

Step 4: Set up the surface area integral

Using the formula:

$SA = 2\pi \int_a^b y \sqrt{1 + (dy/dx)^2}\, dx$

Substitute $y = 4\sqrt{x}$ and $(dy/dx)^2 = 4 / x$:

$SA = 2\pi \int_a^b 4\sqrt{x} \sqrt{1 + 4 / x}\, dx$

Simplify inside the square root:

$1 + 4 / x = (x + 4) / x$

Thus,

$\sqrt{1 + 4 / x} = \sqrt{(x + 4)/ x} = \sqrt{x + 4} / \sqrt{x}$

Therefore, the integrand becomes:

$4\sqrt{x} (\sqrt{x + 4} / \sqrt{x}) = 4 \sqrt{x + 4}$

Step 5: Rewrite the integral

$SA = 2\pi \int_a^b 4 \sqrt{x + 4}\, dx = 8\pi \int_a^b \sqrt{x + 4}\, dx$

Now, the problem reduces to evaluating:

$SA = 8\pi \int_a^b \sqrt{x + 4}\, dx$

Step 6: Integrate $\sqrt{x + 4}\, dx$

Let $u = x + 4$, so that $du = dx$.

When $x = a$, $u = a + 4$.

When $x = b$, $u = b + 4$.

The integral becomes:

$\int \sqrt{u}\, du = \int u^{1/2}\, du$

The integral of $u^{1/2}$ is:

$(2/3) u^{3/2}$

Thus,

$SA = 8\pi\ (2/3)\ [u^{3/2}]$ evaluated from $u = a + 4$ to $u = b + 4$

$SA = (16\pi/3)\ [(b + 4)^{3/2} - (a + 4)^{3/2}]$

Final Expression for Surface Area

The surface area generated when the curve $y = 4\sqrt{x}$ is revolved about the x-axis over the interval [a, b] is:

$$\mathbf{SA = (16\pi/3)\ [(b + 4)^{3/2} - (a + 4)^{3/2}]}$$

This formula allows you to compute the surface area for any chosen interval.

Specific Example: From x = 0 to x = 4

Let’s compute the surface area when x ranges from 0 to 4:

- $a = 0$, $b = 4$

Plug into the formula:

$SA = (16\pi/3)\ [(4 + 4)^{3/2} - (0 + 4)^{3/2}]$

Calculate each term:

$(8)^{3/2} = (8)^{1}\ (8)^{1/2} = 8\ \sqrt{8} = 8\ 2\sqrt{2} = 16\sqrt{2}$

$(4)^{3/2} = 4\ \sqrt{4} = 4\ 2 = 8$

Therefore,

$SA = (16\pi/3)\ (16\sqrt{2} - 8) = (16\pi/3)\ 8\ (2\sqrt{2} - 1)$

Simplify:

$SA = (16\pi/3)\ 8\ (2\sqrt{2} - 1) = (16\ 8\ \pi/3)\ (2\sqrt{2} - 1) = (128\pi/3)\ (2\sqrt{2} - 1)$

Hence, the surface area is:

$SA = (128\pi/3)\ (2\sqrt{2} - 1)$

This numerical value can be approximated as:

- $2\sqrt{2} \approx 2\ 1.4142 \approx 2.8284$

- $2.8284 - 1 = 1.8284$

- $SA \approx (128\pi/3)\ 1.8284 \approx (128\ 3.1416\ /\ 3)\ 1.8284 \approx (128\ 1.0472)\ 1.8284 \approx 134.07\ 1.8284 \approx 245.31$ square units

This comprehensive calculation illustrates how the surface area can be precisely determined.

Extensions and Variations

While the example above considers revolution about the x-axis, similar methods apply if the curve is revolved around the y-axis, or if the interval changes.

Revolution About the Y-Axis

- The formula involves integrating with respect to y.
- Express x in terms of y: $x = (y/4)^2$
- Derive dx/dy, and set up the integral accordingly.

Other Intervals and Curves

- The approach remains similar: find the derivative, simplify the integrand, and evaluate the integral over the specified limits.

Frequently Asked Questions

What is the general formula for finding the surface area of a curve revolved about the x-axis?

The surface area S when a curve $y = f(x)$ from $x = a$ to $x = b$ is revolved about the x-axis is given by $S = 2\pi \int_a^b y \sqrt{1 + (dy/dx)^2}\, dx$.

Given the curve y = 4√x, how do we find dy/dx?

For $y = 4\sqrt{x} = 4x^{1/2}$, $dy/dx = 4 (1/2) x^{-1/2} = 2 / \sqrt{x}$.

What are the limits of integration when calculating the surface area for the curve y = 4√x?

The limits depend on the interval over which the surface is generated. For example, from $x = a$ to $x = b$, where a and b are specific bounds chosen for the problem.

How do we set up the integral for the surface area when revolving y = 4√x about the x-axis from x = 1 to x = 4?

Using the formula, $S = 2\pi \int_1^4 y \sqrt{1 + (dy/dx)^2}\, dx$. Substituting $y = 4\sqrt{x}$ and $dy/dx = 2/\sqrt{x}$ gives the integral to evaluate.

How do you evaluate the integral for the surface area of y = 4√x revolved about the x-axis?

Substitute y and dy/dx into the formula, simplify the integrand, and perform the integration with respect to x over the given limits.

What is the significance of the derivative dy/dx in calculating the surface area?

The derivative dy/dx accounts for the slope of the curve, ensuring the surface area calculation incorporates the curve's shape accurately.

Can the surface area of the curve $y = 4\sqrt{x}$ be calculated for any interval? How does the interval affect the calculation?

Yes, the surface area can be computed over any interval; the limits of integration determine the portion of the curve revolved, affecting the total surface area.

What tools or methods are commonly used to evaluate the integral for the surface area in such problems?

Standard calculus techniques, substitution methods, and sometimes numerical integration or computer algebra systems are used to evaluate the integral accurately.

Why is understanding the derivative dy/dx important in the context of solid of revolution problems?

Because it appears in the integrand as part of the arc length element, helping to accurately model the surface generated by revolving the curve.

Additional Resources

Find The Area Of The Surface Generated When The Given Curve Is Revolved About The Given Axis: $Y = 4\sqrt{x}$

Calculating the surface area generated by revolving a curve around an axis is a fundamental concept in calculus, particularly in the study of solids of revolution. In this article, we delve into the problem of finding the surface area when the curve $y = 4\sqrt{x}$ is revolved about a specified axis. This exploration will guide you through understanding the theoretical foundation, setting up the integral, performing calculations, and interpreting the results. Whether you're a student preparing for exams or a mathematics enthusiast, this comprehensive review aims to clarify the process and provide insights into the application of surface area formulas in calculus.

Understanding the Problem

Before embarking on the calculation, it’s essential to comprehend what the problem entails and the key concepts involved.

Curve Description:

The given curve is $y = 4\sqrt{x}$, which can also be written as $y = 4x^{1/2}$. This is a standard square root function scaled vertically by a factor of 4.

Axis of Revolution:

While the problem states "revolved about the given axis," the specific axis is crucial for setting up the surface area integral. Common axes include:

- The x-axis ($y=0$)
- The y-axis ($x=0$)
- A vertical or horizontal line, such as $x = a$ or $y = b$

Assuming the axis is the x-axis (a typical scenario), the surface area generated by revolving the curve around this axis is calculated accordingly. If the axis differs, the method adapts, but the core principles remain similar.

Surface Area of a Surface of Revolution:

The general formula for the surface area S when revolving a curve $y = f(x)$ from $x=a$ to $x=b$ about the x-axis is:

$$
S = 2\pi \int_{a}^{b} y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx
$$

This formula accounts for the infinitesimal surface element dS generated by revolving a small segment of the curve.

Setting Up the Surface Area Integral

To proceed, define the specific parameters and limits for the problem.

Choosing the Limits of Integration

- If the problem specifies a particular interval, say x from $x=a$ to $x=b$, use that.
- If not specified, a common approach is to consider the interval from $x=0$ up to some $x = c$.

For demonstration, suppose we want to find the surface area generated when revolving the curve from $x=0$ to $x=X$.

Deriving the Formula

Given:

$$
y = 4\sqrt{x} = 4x^{1/2}
$$

Calculate:

\[

$$
\frac{dy}{dx} = \frac{d}{dx} (4x^{1/2}) = 4 \times \frac{1}{2} x^{-1/2} = 2 x^{-1/2} = \frac{2}{\sqrt{x}}
$$

Then,

$$
1 + \left(\frac{dy}{dx}\right)^2 = 1 + \left(\frac{2}{\sqrt{x}}\right)^2 = 1 + \frac{4}{x} = \frac{x + 4}{x}
$$

The integrand becomes:

$$
y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} = 4\sqrt{x} \times \sqrt{\frac{x + 4}{x}} = 4\sqrt{x} \times \frac{\sqrt{x + 4}}{\sqrt{x}} = 4 \times \sqrt{x + 4}
$$

Thus, the surface area differential:

$$
dS = 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = 2\pi \times 4 \sqrt{x + 4} \, dx = 8\pi \sqrt{x + 4} \, dx
$$

Final integral:

$$
S = \int_{a}^{b} dS = 8\pi \int_{a}^{b} \sqrt{x + 4} \, dx
$$

Calculating the Surface Area

Example: Surface Area from $x=0$ to $x=R$

Suppose we want to compute the surface area when x varies from 0 to R. The integral becomes:

$$
S = 8\pi \int_{0}^{R} \sqrt{x + 4} \, dx
$$

Performing the Integration

Let:

$$
u = x + 4 \Rightarrow du = dx
$$

When $x=0$, $u=4$

When $x=R$, $u= R + 4$

Therefore,

$$
S = 8\pi \int_{u=4}^{u=R+4} \sqrt{u} \, du
$$

Recall:

$$
\int \sqrt{u} \, du = \int u^{1/2} \, du = \frac{2}{3} u^{3/2} + C
$$

Thus,

$$
S = 8\pi \times \frac{2}{3} \left[u^{3/2} \right]_{4}^{R+4} = \frac{16\pi}{3} \left[(R+4)^{3/2} - 4^{3/2} \right]
$$

Note:

$$
4^{3/2} = (4)^{1} \times (4)^{1/2} = 4 \times 2 = 8
$$

Therefore, the surface area is:

$$
S(R) = \frac{16\pi}{3} \left[(R+4)^{3/2} - 8 \right]
$$

Interpreting the Results

This formula allows us to compute the surface area for any upper limit R. For specific values, plug in the number and simplify.

Example:

Calculate the surface area generated when $R=4$:

$$
S(4) = \frac{16\pi}{3} \left[(4+4)^{3/2} - 8 \right] = \frac{16\pi}{3} \left[8^{3/2} - 8 \right]
$$

Calculate $8^{3/2}$:

$$
8^{3/2} = (8)^{1} \times (8)^{1/2} = 8 \times 2.8284 \approx 22.627
$$

Thus,

$$
S(4) \approx \frac{16\pi}{3} (22.627 - 8) = \frac{16\pi}{3} \times 14.627
$$

$$
\approx \frac{16 \times 3.1416}{3} \times 14.627
$$

Calculate numerator:

$$
16 \times 3.1416 \approx 50.265
$$

Divide by 3:

$$
\frac{50.265}{3} \approx 16.755
$$

Multiply by 14.627:

$$
16.755 \times 14.627 \approx 244.9
$$

So, the approximate surface area:

$$
S(4) \approx 244.9 \text{ square units}
$$

This method demonstrates how the integral approach provides a precise and workable solution.

Extensions and Variations

The methodology outlined here can be adapted for different axes of revolution or different intervals.

Revolving Around the y-Axis

- The formula for surface area changes to:

$$
S = 2\pi \int_{a}^{b} x \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx
$$

- The derivation process then involves expressing x in terms of y or adjusting the integral accordingly.

Revolving About Other Lines

- For axes such as $x = k$ or $y = c$, the radius of revolution adjusts accordingly, and the integral setup modifies to account for the distance from the axis.

Features, Pros, and Cons

Features:

- Provides a systematic approach to calculating surface areas of solids of revolution.
- Utilizes substitution to simplify integrals involving square roots.
- Applicable to various functions and axes with appropriate modifications.

Pros:

- Offers precise analytical solutions.
- Enhances understanding of geometric interpretations of calculus.
- Can be extended to complex functions and multiple intervals.

Cons:

- Requires familiarity with integration techniques, including substitution.
- Can become algebraically intensive for complicated functions.
- Assumes the function is continuous and differentiable over the interval.

Conclusion

Calculating the surface area generated when revolving a curve like $y = 4\sqrt{x}$ about an axis involves

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