Find The Critical Numbers Of The Function. 1. F(x)=4+1/3x1/2x^22. F(x)=x^3+6x^215x 3. F(x)=x^3+3x^224x

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Understanding how to find the critical numbers of a function is a fundamental skill in calculus that helps analyze the behavior of functions, particularly in identifying local maxima, minima, and points of inflection. This article provides a comprehensive guide to finding critical numbers for multiple functions, including detailed step-by-step methods, explanations, and tips to enhance your calculus skills.

What Are Critical Numbers?

Critical numbers (also known as critical points) of a function are the values of x where the function's derivative is either zero or undefined. These points are significant because they often indicate potential local extrema (maxima or minima) or points where the function's behavior changes, such as inflection points.

Formal Definition:

Given a function $f(x)$, a number c in its domain is called a critical number if:

- $f'(c) = 0$, or
- $f'(c)$ does not exist.

Finding these points involves two main steps:

1. Calculate the derivative $f'(x)$.
2. Solve for x where $f'(x) = 0$ or $f'(x)$ is undefined.

Analyzing the Given Functions

Let's explore how to find the critical numbers for each of the listed functions:

1. $F(x) = 4 + \frac{1}{3}x \cdot \frac{1}{2}x^2$

2. $F(x) = x^3 + 6x^2 + 15x$
3. $F(x) = x^3 + 3x^2 + 4x$

Each function has unique features and requires careful differentiation and analysis.

Function 1: $F(x) = 4 + \frac{1}{3}x \cdot \frac{1}{2}x^2$

Step 1: Simplify the function

First, rewrite the function to clarify its form:

$$
F(x) = 4 + \left(\frac{1}{3}x\right) \left(\frac{1}{2}x^2\right)
$$

Multiply the two fractions:

$$
F(x) = 4 + \frac{1}{3} \times \frac{1}{2} \times x \times x^2 = 4 + \frac{1}{6}x^3
$$

So, the simplified form is:

$$
F(x) = 4 + \frac{1}{6}x^3
$$

Step 2: Find the derivative $F'(x)$

Differentiate term-by-term:

$$
F'(x) = 0 + \frac{1}{6} \times 3x^2 = \frac{1}{6} \times 3x^2 = \frac{3}{6}x^2 = \frac{1}{2}x^2
$$

Step 3: Find critical points by solving $F'(x) = 0$

Set the derivative equal to zero:

$$
\frac{1}{2}x^2 = 0
$$

Multiply both sides by 2:

$$
x^2 = 0
$$

Thus,

$$
x = 0
$$

Step 4: Check where $F'(x)$ is undefined

Since the derivative is a polynomial, it is defined for all real x. Therefore, no additional critical points come from where the derivative is undefined.

Critical number for Function 1:

$$
\boxed{x = 0}
$$

Function 2: $F(x) = x^3 + 6x^2 + 15x$

Step 1: Find the derivative $F'(x)$

Differentiate each term:

$$
F'(x) = 3x^2 + 12x + 15
$$

Step 2: Solve $F'(x) = 0$ for critical points

Set derivative equal to zero:

$$
3x^2 + 12x + 15 = 0
$$

Divide through by 3 to simplify:

$$
x^2 + 4x + 5 = 0
$$

Now, solve the quadratic:

$$
x^2 + 4x + 5 = 0
$$

Calculate the discriminant:

$$
D = (4)^2 - 4 \times 1 \times 5 = 16 - 20 = -4
$$

Since the discriminant is negative, there are no real solutions; thus, $F'(x) \neq 0$ for any real x.

Step 3: Check where $F'(x)$ is undefined

Again, because the derivative is a polynomial, it is defined everywhere.

Critical numbers for Function 2:

Since $F'(x) \neq 0$ and is defined for all real numbers, there are no critical numbers for this function.

Function 3: $F(x) = x^3 + 3x^2 + 4x$

Step 1: Find the derivative $F'(x)$

Differentiate term-by-term:

$$
F'(x) = 3x^2 + 6x + 4
$$

Step 2: Solve $F'(x) = 0$ for critical points

Set the derivative equal to zero:

$$

$$
3x^2 + 6x + 4 = 0
$$

Divide through by 3:

$$
x^2 + 2x + \frac{4}{3} = 0
$$

Calculate the discriminant:

$$
D = (2)^2 - 4 \times 1 \times \frac{4}{3} = 4 - \frac{16}{3} = \frac{12}{3} - \frac{16}{3} = -\frac{4}{3}
$$

Since the discriminant is negative, there are no real solutions to the equation, so no critical points arise from the derivative being zero.

Step 3: Check where $F'(x)$ is undefined

As before, the derivative is a quadratic polynomial and is defined everywhere.

Critical numbers for Function 3:

No real critical numbers exist because $F'(x) \neq 0$ for all real x.

Summary of Critical Numbers for the Given Functions

Function	Critical Numbers	Explanation
$F(x)=4 + \frac{1}{6}x^3$	$x=0$	Derivative zero at $x=0$.
$F(x)=x^3 + 6x^2 + 15x$	None (no real solutions)	Derivative never zero and is defined everywhere.
$F(x)=x^3 + 3x^2 + 4x$	None (no real solutions)	Derivative never zero and is defined everywhere.

Additional Tips for Finding Critical Numbers

- **Always simplify:** Before differentiating, rewrite the function in a simplified form when possible.
- **Differentiate carefully:** Use the power rule and constant multiple rule to differentiate each term accurately.
- **Solve quadratic equations:** When derivatives are quadratic, use the discriminant ($b^2 - 4ac$) to determine the nature of solutions.
- **Check for undefined derivatives:** For rational functions, ensure the derivative does not have points where it is undefined.
- **Interpret the results:** Critical points are potential locations for local maxima, minima, or inflection points, which can be confirmed with second derivative tests or analyzing the sign changes of $f'(x)$.

Conclusion

Finding the critical numbers of a function involves differentiating the function, solving for points where the derivative is zero, and identifying points where the derivative does not exist. By working through these steps systematically, you can analyze the behavior of various functions and identify key points that indicate local maxima, minima, or inflection points.

In summary:

- For $F(x) = 4 + \frac{1}{6}x^3$, the critical number is at $x=0$.
- For $F(x) = x^3 + 6x^2 + 15x$, there are no critical numbers.
- For \(F(x) =

Frequently Asked Questions

How do you find the critical numbers of the function F(x) = 4 + (1/3)x^{1/2} + 2?

First, differentiate F(x) to get F'(x), then set F'(x) = 0 and solve for x. Also, check where F'(x) is undefined, as those points are potential critical numbers.

What is the derivative of $F(x) = x^3 + 6x^2 + 15x$?

The derivative is $F'(x) = 3x^2 + 12x + 15$.

How do you find the critical numbers of the function $F(x) = x^3 + 3x^2 + 4x$?

Differentiate the function to find $F'(x) = 3x^2 + 6x + 4$, then set $F'(x) = 0$ and solve for x. Critical numbers are where the derivative is zero or undefined.

What are the steps to find critical points for a polynomial function?

Differentiate the function, find where the derivative is zero or undefined, and solve for x. Those x-values are the critical numbers.

Why are points where the derivative is undefined considered critical numbers?

Because these points may correspond to local maxima, minima, or points of inflection, indicating potential local extrema.

Can critical numbers occur at the endpoints of a domain?

In open intervals, critical numbers are where the derivative is zero or undefined; endpoints are checked separately when analyzing the function's domain for maximum or minimum values.

For $F(x) = x^3 + 6x^2 + 15x$, what are the critical numbers?

Differentiate to get $F'(x) = 3x^2 + 12x + 15$. Set equal to zero: $3x^2 + 12x + 15 = 0$, divide by 3: $x^2 + 4x + 5 = 0$. The discriminant is $16 - 20 = -4$, so no real critical numbers. The function has no critical points where the derivative is zero.

How do you determine the critical numbers for $F(x) = x^3 + 3x^2 + 4x$?

Find $F'(x) = 3x^2 + 6x + 4$, set it to zero: $3x^2 + 6x + 4 = 0$, then solve for x using the quadratic formula. The solutions are the critical numbers.

What is the importance of finding critical numbers in calculus?

Critical numbers help identify potential local maxima, minima, or points of inflection, which are essential for analyzing the function's behavior.

Additional Resources

Find The Critical Numbers Of The Function is a fundamental topic in calculus that plays a crucial role in understanding the behavior of functions, particularly in identifying potential local maxima, minima, and points of inflection. Critical numbers, also known as critical points, are values of the independent variable where the derivative of the function is either zero or undefined. This concept is essential for analyzing the shape of a graph and for optimization problems. In this article, we will explore how to find the critical numbers for three specific functions, delving into the step-by-step process, the underlying principles, and the interpretation of the results.

Understanding Critical Numbers

Before diving into the specific functions, it is important to grasp what critical numbers represent and why they are significant.

Definition and Significance

Critical numbers of a function $f(x)$ are values $x = c$ where:

- $f'(c) = 0$ (the derivative equals zero), or
- $f'(c)$ does not exist (the derivative is undefined).

These points are potential candidates for local extrema (maxima or minima) or points of inflection, where the function changes its increasing or decreasing behavior.

Methodology to Find Critical Numbers

1. Compute the derivative $f'(x)$.
2. Solve $f'(x) = 0$ for x.
3. Identify where $f'(x)$ is undefined, and solve for these points.
4. Verify the critical points within the domain of the function.

Analyzing Specific Functions

We now analyze three functions, each illustrating different features and challenges in finding critical numbers.

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1. $F(x) = 4 + \frac{1}{3} x^{1/2} x^2$

Understanding the Function

At first glance, the function appears to be a composite of radical and polynomial expressions. To clarify, rewrite the function:

$$
F(x) = 4 + \frac{1}{3} x^{1/2} \times x^2
$$

which simplifies to:

$$
F(x) = 4 + \frac{1}{3} x^{1/2 + 2} = 4 + \frac{1}{3} x^{5/2}
$$

since $x^{1/2} \times x^2 = x^{1/2 + 2} = x^{5/2}$.

Note: The domain of $F(x)$ is $x \geq 0$ because of the square root.

Finding the Derivative

Differentiate $F(x)$:

$$
F'(x) = \frac{1}{3} \times \frac{5}{2} x^{(5/2) - 1} = \frac{1}{3} \times \frac{5}{2} x^{3/2}
$$

simplify:

$$
F'(x) = \frac{5}{6} x^{3/2}
$$

Critical Numbers

Set $F'(x) = 0$:

$$

\frac{5}{6} x^{3/2} = 0
\]

This occurs when:

\[
x^{3/2} = 0 \implies x = 0
\]

Since \(x^{3/2}\) is undefined for \(x < 0\), the domain restriction is consistent.

Critical number: \(x = 0\)

Interpretation

- At \(x=0\), the derivative is zero.
- The derivative is positive for \(x > 0\) (since \(x^{3/2} > 0\) for \(x > 0\)), indicating the function is increasing on \((0, \infty)\).
- The critical point at \(x=0\) is at the boundary of the domain, which can be a minimum or a point of interest in the graph.

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2. \(F(x) = x^3 + 6x^{1/5}\)

Understanding the Function

This function combines a cubic term with a fractional power involving a fifth root. The domain is all real numbers because \(x^{1/5}\) (fifth root) is defined for all real \(x\), including negatives.

Finding the Derivative

Differentiate term by term:

\[
F'(x) = 3x^2 + 6 \times \frac{1}{5} x^{(1/5) - 1} = 3x^2 + \frac{6}{5} x^{-4/5}
\]

Expressed as:

\[
F'(x) = 3x^2 + \frac{6}{5} x^{-4/5}
\]

Note that \(x^{-4/5} = \frac{1}{x^{4/5}}\), which is undefined at \(x=0\).

Critical Numbers

Set the derivative equal to zero:

$$
3x^2 + \frac{6}{5} x^{-4/5} = 0
$$

Multiply through by $x^{4/5}$ (for $x \neq 0$) to eliminate negative exponents:

$$
3x^{2 + 4/5} + \frac{6}{5} = 0
$$

Express $x^{2 + 4/5}$:

$$
x^{(10/5) + (4/5)} = x^{14/5}
$$

So:

$$
3x^{14/5} + \frac{6}{5} = 0
$$

which simplifies to:

$$
3x^{14/5} = - \frac{6}{5}
$$

Since $x^{14/5}$ is positive for positive x, and negative for negative x, this equation has no real solutions because:

$$
x^{14/5} \geq 0 \quad \text{for } x \geq 0
$$
and
$$
x^{14/5} \leq 0 \quad \text{for } x \leq 0
$$

but the right side is negative, $-6/5$, so:

- For $x > 0$, the left side is positive, cannot equal negative.
- For $x < 0$, $x^{14/5}$ is negative (since 14/5 is positive), so the left side is negative, matching the right side.

Proceeding with negative x:

$$
3x^{14/5} = - \frac{6}{5}
$$

$$
x^{14/5} = - \frac{2}{5}
$$

But $x^{14/5}$ for negative x is negative only if $14/5$ is odd, which it is not (14/5 = 2.8). Since the fractional exponent with an even numerator and odd denominator results in a real number only for positive x, we conclude:

- The only critical point occurs where the derivative does not exist, i.e., at $x = 0$.

Now, check whether derivative exists at $x=0$:

- The term $\frac{6}{5} x^{-4/5}$ is undefined at $x=0$.

Critical number: $x=0$

Summary of Critical Numbers

- $x=0$ (derivative undefined)
- No other solutions satisfy the derivative equal to zero.

Interpretation

- The critical point at $x=0$ is where the derivative is undefined, indicating a potential cusp or vertical tangent.
- For $x > 0$, derivative is positive (since both terms are positive), so the function is increasing.
- For $x<0$, the derivative is negative, so the function decreases on $(-\infty, 0)$.

---

3. $F(x) = x^3 + 3x^2 + 4x$

Understanding the Function

This is a polynomial function, smooth and differentiable everywhere, making finding critical points straightforward.

Finding the Derivative

Differentiate:

$$
F'(x) = 3x^2 + 6x + 4
$$

Critical Numbers

Set derivative to zero:

$$
3x^2 + 6x + 4 = 0
$$

Divide through by 3:

$$
x^2 + 2x + \frac{4}{3} = 0
$$

Calculate the discriminant:

$$
\Delta = (2)^2 - 4 \times 1 \times \frac{4}{3} = 4 - \frac{16}{3} = \frac{12 - 16}{3} = -\frac{4}{3}
$$

Since the discriminant is negative, the quadratic has no real roots.

Conclusion:

- The derivative $F'(x)$

Find The Critical Numbers Of The Function 1 F X 4 1 3x1 2x 22 F X X 3 6x 215x 3 F X X 3 3x 224x

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