Find The Critical Points Of The Function F(x)=x^2-9/x^2-4x+3 Use A Comma To Separate Multiple Critical

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Understanding how to find the critical points of a function is fundamental in calculus, as these points help identify local maxima, minima, and points of inflection. In this article, we will thoroughly analyze the function F(x) = (x^2 - 9) / (x^2 - 4x + 3), demonstrate the step-by-step process to find its critical points, and discuss their significance in the context of the function's behavior. Whether you're a student preparing for exams or a math enthusiast seeking a deeper understanding, this guide aims to clarify the process and provide practical insights.

Understanding the Function F(x)

Before diving into the process of finding critical points, it is crucial to understand the structure of the function:

$$ F(x) = \frac{x^2 - 9}{x^2 - 4x + 3} $$

This is a rational function, meaning it is the ratio of two polynomials. Rational functions often have interesting features such as vertical asymptotes, horizontal asymptotes, and critical points that can be identified through calculus techniques.

Simplification and Domain Considerations

Let's analyze the numerator and denominator:

- Numerator: $x^2 - 9 = (x - 3)(x + 3)$

- Denominator: $x^2 - 4x + 3 = (x - 1)(x - 3)$

The domain of F(x) excludes points where the denominator is zero, i.e., at $x = 1$ and $x = 3$. Therefore:

$$ \text{Domain} = \mathbb{R} \setminus \{1, 3\} $$

Understanding these restrictions is essential because the function is undefined at these points, and they are potential locations for vertical asymptotes.

Step-by-Step Method to Find Critical Points

Critical points occur where the derivative $F'(x)$ is zero or undefined, provided the point is within the domain of the function.

Step 1: Compute the Derivative $F'(x)$

Since F(x) is a quotient, we will apply the quotient rule:

$$
F'(x) = \frac{v(x) \cdot u'(x) - u(x) \cdot v'(x)}{[v(x)]^2}
$$

where:

- $u(x) = x^2 - 9$

- $v(x) = x^2 - 4x + 3$

Calculating derivatives:

- $u'(x) = 2x$

- $v'(x) = 2x - 4$

Applying the quotient rule:

$$
F'(x) = \frac{(x^2 - 4x + 3)(2x) - (x^2 - 9)(2x - 4)}{(x^2 - 4x + 3)^2}
$$

Step 2: Simplify the Derivative Expression

Let's expand numerator step-by-step:

Numerator:

$$
N(x) = (x^2 - 4x + 3)(2x) - (x^2 - 9)(2x - 4)
$$

First term:

$$
(x^2 - 4x + 3)(2x) = 2x^3 - 8x^2 + 6x
$$

Second term:

$$
(x^2 - 9)(2x - 4) = x^2(2x - 4) - 9(2x - 4) = 2x^3 - 4x^2 - 18x + 36
$$

Putting it together:

$$
N(x) = [2x^3 - 8x^2 + 6x] - [2x^3 - 4x^2 - 18x + 36]
$$

Distribute the minus:

$$
N(x) = 2x^3 - 8x^2 + 6x - 2x^3 + 4x^2 + 18x - 36
$$

Combine like terms:

- $2x^3 - 2x^3 = 0$

- $-8x^2 + 4x^2 = -4x^2$

- $6x + 18x = 24x$

- Constant term: -36

Thus:

$$
N(x) = -4x^2 + 24x - 36
$$

So the derivative simplifies to:

$$
F'(x) = \frac{-4x^2 + 24x - 36}{(x^2 - 4x + 3)^2}
$$

Step 3: Find Critical Points by Setting $F'(x) = 0$ or $F'(x)$ undefined

Since the denominator is always positive except where the function is undefined (at $x = 1$ and $x=3$), the critical points are determined by numerator zeros and points where the derivative is undefined.

Set numerator equal to zero:

$$
-4x^2 + 24x - 36 = 0
$$

Divide both sides by -4 to simplify:

$$
x^2 - 6x + 9 = 0
$$

This quadratic factors as:

$$
(x - 3)^2 = 0
$$

which has a double root at:

$$
x = 3
$$

Now, check points where $F'(x)$ is undefined, i.e., where the denominator is zero:

$$
x^2 - 4x + 3 = 0
$$

which factors as:

$$
(x - 1)(x - 3) = 0
$$

Thus, $x = 1$ and $x = 3$. Recall that $x=3$ is not in the domain because the original function is undefined there, so we do not consider it a critical point, but it is a point of discontinuity (vertical asymptote). For $x=1$, the function is undefined, so $x=1$ is also not a critical point but a vertical asymptote.

Summary of Critical Points

From the above analysis:

- The only candidate for a critical point from the derivative being zero is at $x=3$, but since the function is undefined there, it is not a critical point.

- The points where the derivative is undefined are at $x=1$ and $x=3$, both outside the domain; hence, neither are critical points.

- Therefore, there are no critical points within the domain of the function.

Interpreting the Results and Function Behavior

Although the derivative analysis indicates no critical points within the domain, it is essential to analyze the behavior of the function near the points where it is undefined and at infinity to understand its overall shape.

Behavior Near Vertical Asymptotes

- As $x \to 1^-$, analyze the limits:

$$
F(x) \to -\infty \quad \text{or} \quad +\infty
$$

- As $x \to 1^+$, similar analysis applies.

- As $x \to 3^-$, the function tends toward $+\infty$ or $-\infty$.

- As $x \to 3^+$, the behavior continues accordingly.

These asymptotic behaviors indicate the function has vertical asymptotes at $x=1$ and $x=3$, and the function's graph exhibits unbounded growth near these points.

Behavior at Infinity

- As $x \to \pm \infty$, the dominant terms are:

$$
F(x) \approx \frac{x^2}{x^2} = 1
$$

- Therefore, the horizontal asymptote is at $y=1$.

Conclusion and Summary

In this comprehensive analysis, we have determined that the function $F(x) = \frac{x^2 - 9}{x^2 - 4x + 3}$ has no critical points within its domain because the points where the derivative equals zero or is undefined coincide with the function's discontinuities at $x=1$ and $x=3$. The critical point candidate at $x=3$ lies outside the domain, and at $x=1$, the function is undefined, representing a vertical asymptote rather than a critical point.

Key takeaways:

- The derivative $F'(x)$ simplifies to $\frac{-4x^2 + 24x - 36}{(x^2 - 4x + 3)^2}$.

- The numerator factors to $(x - 3)^2$, indicating a repeated root at $x=3$.

- Critical points occur where $F'(x) = 0$ or is undefined within the domain, but here, only

Frequently Asked Questions

What are the critical points of the function F(x)=x^2-9/x^2-4x+3?

To find the critical points, first compute the derivative F'(x), set it equal to zero, and solve for x. The critical points occur where F'(x)=0 or where F'(x) is undefined.

How do I differentiate F(x)=x^2 - 9/(x^2 - 4x + 3)?

Use the quotient rule or rewrite the function as F(x)=x^2 - 9/(x^2 - 4x + 3) and differentiate term by term, applying the quotient rule to the second term.

What are the steps to find critical points for this rational function?

Step 1: Find the derivative F'(x). Step 2: Set F'(x)=0 and solve for x. Step 3: Find where F'(x) is undefined, which occurs where the denominator is zero. Step 4: Verify solutions are within the domain.

How do I determine the domain of F(x)=x^2 - 9/(x^2 - 4x + 3)?

The domain excludes values that make the denominator zero: solve x^2 - 4x + 3=0, which factors to (x-1)(x-3)=0. So, the domain is all real numbers except x=1 and x=3.

What are the critical points of F(x)=x^2-9/x^2-4x+3, considering the domain restrictions?

Critical points are at the solutions of F'(x)=0 and where F'(x) is undefined, excluding x=1 and x=3 since these are outside the domain. Solve for x accordingly.

How can I classify the critical points as maxima, minima, or saddle points?

Use the second derivative test or analyze the sign changes of F'(x) around the critical points to determine whether they are local maxima, minima, or saddle points.

Additional Resources

Find The Critical Points Of The Function $F(x)=x^2 - 9 / x^2 - 4x + 3$

Understanding the behavior of mathematical functions is fundamental in calculus, particularly when analyzing their critical points. Critical points are the values of x where the derivative of a function either equals zero or does not exist, often indicating potential maxima, minima, or points of inflection. In this article, we delve into the process of identifying the critical points of the function $F(x) = \frac{x^2 - 9}{x^2 - 4x + 3}$, exploring the necessary calculus techniques, the implications of domain restrictions, and the significance of the critical points in understanding the function's overall behavior.

Introduction to Critical Points and Their Significance

Critical points serve as pivotal markers in the landscape of a function's graph. They are the potential locations where the function attains local maximums or minimums, or where the function's slope changes direction. In real-world applications—ranging from physics to economics—identifying these points allows analysts and scientists to predict behavior, optimize outcomes, and understand underlying trends.

For a given function $F(x)$, the critical points are determined by solving the equation $F'(x) = 0$, where $F'(x)$ is the derivative of $F(x)$, or by finding where the derivative does not exist within the function's domain. The importance of these points stems from their relationship with the function's increasing/decreasing intervals and concavity, which can be analyzed through the first and second derivative tests.

Understanding the Function $F(x) = \frac{x^2 - 9}{x^2 - 4x + 3}$

Before proceeding to find the critical points, it is essential to understand the structure and domain of the function.

Function Structure and Simplification

The function is a rational function, expressed as a ratio of two polynomials:

$$
F(x) = \frac{x^2 - 9}{x^2 - 4x + 3}
$$

Noticing that the numerator can be factored as a difference of squares:

$$
x^2 - 9 = (x - 3)(x + 3)
$$

Similarly, the denominator can be factored:

$$
x^2 - 4x + 3 = (x - 1)(x - 3)
$$

Thus, the function simplifies to:

$$
F(x) = \frac{(x - 3)(x + 3)}{(x - 1)(x - 3)}
$$

However, note that $x = 3$ is a common factor in numerator and denominator, leading to a canceled term but also indicating a potential point of discontinuity. Simplifying, we get:

$$
F(x) = \frac{x + 3}{x - 1}, \quad \text{for } x \neq 3
$$

This simplification is crucial because it reveals the domain restrictions: the original function is undefined at the zeros of the denominator, i.e., at $x = 1$ and $x = 3$. The point $x = 3$ is a removable discontinuity (a hole), since the factor cancels out, but $x = 1$ is an vertical asymptote.

Domain of $F(x)$:

$$
\boxed{
x \in \mathbb{R} \setminus \{1, 3\}
}
$$

Calculating the Derivative $F'(x)$: Step-by-Step

The critical points are found where $F'(x) = 0$ or undefined within the domain. Since the function simplifies to $F(x) = \frac{x + 3}{x - 1}$ for $x \neq 3$, we proceed to differentiate this simplified form.

Applying the Quotient Rule

The quotient rule states:

$$
\frac{d}{dx} \left(\frac{u(x)}{v(x)} \right) = \frac{v(x) u'(x) - u(x) v'(x)}{[v(x)]^2}
$$

Let:

- $u(x) = x + 3$
- $v(x) = x - 1$

Calculate derivatives:

- $u'(x) = 1$
- $v'(x) = 1$

Applying the quotient rule:

$$
F'(x) = \frac{(x - 1)(1) - (x + 3)(1)}{(x - 1)^2} = \frac{x - 1 - x - 3}{(x - 1)^2} = \frac{-4}{(x - 1)^2}
$$

Result:

$$
F'(x) = \frac{-4}{(x - 1)^2}
$$

Identifying Critical Points

With the derivative calculated, the next step is to analyze where $F'(x) = 0$ or where the derivative is undefined within the domain.

Setting $F'(x) = 0$

$$
\frac{-4}{(x - 1)^2} = 0
$$

Since the numerator is a constant -4, which is never zero, and the denominator $(x - 1)^2$ is always positive for $x \neq 1$, it follows that:

$$
F'(x) \neq 0 \quad \text{for any } x \in \text{domain}
$$

Therefore, there are no points where the derivative equals zero. This indicates that the function has no local maxima or minima where the slope is zero.

Points Where $F'(x)$ is Undefined

The derivative $F'(x)$ is undefined where the denominator is zero:

$$
(x - 1)^2 = 0 \Rightarrow x = 1
$$

- Recall $x = 1$ is a vertical asymptote of the function, not within its domain.
- Since $x = 1$ is not in the domain, it cannot be a critical point.

However, the derivative is defined for all $x \neq 1$, and the only potential critical points would be where $F'(x) = 0$, which we've established does not occur.

Conclusion:

- The derivative is never zero within the domain.
- The derivative is undefined only at $x = 1$, which is outside the domain.

Analyzing the Behavior of the Function at Critical Points and Domain Boundaries

Given the derivative analysis, the absence of points where $F'(x) = 0$ suggests that the function does not have typical local maxima or minima. Yet, understanding the function's overall behavior requires examining its limits and the points of discontinuity.

Behavior at $x \to \pm \infty$

Since $F(x) = \frac{x + 3}{x - 1}$, as $x \to \pm \infty$:

$$
F(x) \sim \frac{x}{x} = 1
$$

More precisely,

$$
\lim_{x \to \pm \infty} F(x) = 1
$$

indicating a horizontal asymptote at $y = 1$.

Behavior near the vertical asymptote $x = 1$

- As $x \to 1^-$:

$$
F(x) = \frac{x + 3}{x - 1} \to -\infty
$$

- As $x \to 1^+$:

$$
F(x) \to +\infty
$$

This vertical asymptote at $x=1$ separates the domain into two intervals:

- $(-\infty, 1)$: $F(x)$ decreases from 1 (at negative infinity) to $-\infty$ approaching $x=1$.
- $(1, \infty)$: $F(x)$ increases from $+\infty$ at $x=1^+$ to 1 at infinity.

Implications of Critical Point Analysis in the Context of the Function's Behavior

The analysis reveals that the function has no critical points where the derivative is zero within its domain; however, it exhibits significant features at points of discontinuity and at the asymptotes.

Key observations:

- The function has a removable discontinuity at $x=3$, where the original function is undefined but simplifies to a continuous function elsewhere. Since $x=3$ cancels out, the function approaches a finite value there, and it does not qualify as a critical point.
- At $x=1$, a vertical asymptote exists, which is not within the domain and thus not a critical point.
- The function exhibits a vertical asymptote at $x=1$ and a horizontal asymptote at $y=1$.
- The derivative's sign analysis indicates the function is

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