Find The Directional Derivative Of F At The Given Point In The Direction Indicated By The Angle . F(x,

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Understanding how functions change as we move in specific directions is fundamental in multivariable calculus. The concept of the directional derivative provides a measure of the rate at which a function changes at a point when moving in a particular direction. When the direction is specified by an angle, the calculation involves translating that angle into a unit vector and then applying the gradient of the function. This article offers an in-depth exploration of how to find the directional derivative of a function $F(x, y)$ at a given point, in a direction specified by an angle.

Understanding the Directional Derivative

Definition of the Directional Derivative

The directional derivative of a function $F(x, y)$ at a point (x_0, y_0) in the direction of a unit vector $\mathbf{u} = (u_1, u_2)$ is defined as:

$$
D_{\mathbf{u}} F(x_0, y_0) = \lim_{h \to 0} \frac{F(x_0 + h u_1,\, y_0 + h u_2) - F(x_0, y_0)}{h}
$$

This derivative measures how rapidly F changes at (x_0, y_0) as we move a small step in the direction of $\mathbf{u}$.

Geometric Interpretation

- The directional derivative indicates the slope of the function in a specific direction.
- When the gradient ∇F exists and is continuous at (x_0, y_0), the directional derivative can be computed as the dot product:

$$
D_{\mathbf{u}} F(x_0, y_0) = \nabla F(x_0, y_0) \cdot \mathbf{u}
$$

- The gradient vector $\nabla F = \left(\frac{\partial F}{\partial x},\, \frac{\partial F}{\partial y} \right)$ points in the direction of the steepest ascent.

Converting an Angle into a Direction Vector

Angles and Direction Vectors

Suppose the given direction is indicated by an angle θ, measured from the positive x-axis:

- θ is typically in radians or degrees.
- The unit vector $\mathbf{u}$ in the direction of θ is:

$$
\mathbf{u} = (\cos \theta, \sin \theta)
$$

This vector points in the direction specified by the angle.

Example Conversion

- If $\theta = 0^\circ$ or 0 radians, then $\mathbf{u} = (1, 0)$.
- If $\theta = 90^\circ$ or $\frac{\pi}{2}$ radians, then $\mathbf{u} = (0, 1)$.
- For an arbitrary angle θ, the direction vector is:

$$
\mathbf{u} = (\cos \theta, \sin \theta)
$$

Calculating the Gradient of the Function $F(x, y)$

Partial Derivatives

To compute the gradient, you need the partial derivatives of F:

$$
\nabla F(x, y) = \left(\frac{\partial F}{\partial x},\, \frac{\partial F}{\partial y} \right)
$$

- These derivatives are evaluated at the point (x_0, y_0).

- The partial derivatives are obtained via differentiation rules applied to the specific form of F.

Example

Suppose $F(x, y) = x^2 y + 3 y$. Then:

$$
\frac{\partial F}{\partial x} = 2 x y, \quad \frac{\partial F}{\partial y} = x^2 + 3
$$

Evaluating at (x_0, y_0), for example at $(1, 2)$:

$$
\nabla F(1, 2) = (2 \times 1 \times 2,\, 1^2 + 3) = (4, 4)
$$

Computing the Directional Derivative

Step-by-step Procedure

1. Identify the point (x_0, y_0): The point at which the derivative is to be computed.
2. Determine the angle θ: The angle indicating the direction.
3. Convert θ to a unit vector $\mathbf{u}$: Use $\mathbf{u} = (\cos \theta, \sin \theta)$.
4. Compute the gradient ∇F at (x_0, y_0): Find the partial derivatives and evaluate at the point.
5. Calculate the dot product: Multiply the gradient vector by the unit vector:

$$
D_{\mathbf{u}} F(x_0, y_0) = \nabla F(x_0, y_0) \cdot \mathbf{u}
$$

which simplifies to:

$$
D_{\mathbf{u}} F(x_0, y_0) = \frac{\partial F}{\partial x}(x_0, y_0) \cos \theta + \frac{\partial F}{\partial y}(x_0, y_0) \sin \theta
$$

Worked Example

Suppose:

- $F(x, y) = x^3 y - 2 x y^2$,
- The point of interest is $(2, 1)$,
- The direction is given by $\theta = 45^\circ$.

Step 1: Convert angle to radians

$$
\theta = 45^\circ = \frac{\pi}{4} \text{ radians}
$$

Step 2: Find the unit vector $\mathbf{u}$

$$
\mathbf{u} = (\cos \frac{\pi}{4}, \sin \frac{\pi}{4}) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)
$$

Step 3: Compute partial derivatives of F

$$
\frac{\partial F}{\partial x} = 3 x^2 y - 2 y^2
$$
$$
\frac{\partial F}{\partial y} = x^3 - 4 x y
$$

Step 4: Evaluate at $(2, 1)$

$$
\frac{\partial F}{\partial x}(2, 1) = 3 \times 4 \times 1 - 2 \times 1 = 12 - 2 = 10
$$
$$
\frac{\partial F}{\partial y}(2, 1) = 8 - 8 = 0
$$

Step 5: Calculate the directional derivative

$$
D_{\mathbf{u}} F(2, 1) = 10 \times \frac{\sqrt{2}}{2} + 0 \times \frac{\sqrt{2}}{2} = 10 \times \frac{\sqrt{2}}{2} = 5 \sqrt{2}
$$

Result:

The directional derivative of F at $(2, 1)$ in the direction of 45° is $5 \sqrt{2}$.

Additional Considerations

Special Cases

- When the gradient is zero: The directional derivative in any direction is zero, indicating a critical point or flat region.
- When the angle is zero or 90°: The calculation simplifies to the partial derivatives along axes.

Limitations and Assumptions

- The function F must be differentiable at the point for the gradient to exist.
- The direction vector is assumed to be a unit vector; otherwise, normalization is required.

Summary

To find the directional derivative of a function $F(x, y)$ at a point (x_0, y_0) in a direction specified by an angle θ:

1. Convert the angle θ into a unit vector $\mathbf{u} = (\cos \theta, \sin \theta)$.
2. Calculate the gradient ∇F at the point.
3. Compute the dot product of the gradient with the direction vector.
4. The result gives the rate of change of F at \((

Frequently Asked Questions

What is the formula for finding the directional derivative of a function F at a point in a given direction?

The directional derivative of F at point P in the direction of a unit vector u is given by $D_u F(P) = \nabla F(P) \cdot u$, where $\nabla F(P)$ is the gradient of F at P.

How do I determine the unit vector corresponding to a given angle θ for the directional derivative?

The unit vector u in the direction of angle θ (measured from the positive x-axis) is $u = (\cos \theta, \sin \theta)$.

What is the role of the gradient vector in calculating the directional derivative?

The gradient vector ∇F at a point points in the direction of maximum increase of the function, and its dot product with the unit vector u gives the rate of change in that specific direction.

How do I compute the gradient of a multivariable function F(x, y)?

The gradient is computed as $\nabla F(x, y) = (\partial F/\partial x, \partial F/\partial y)$, where each partial derivative is evaluated at the given point.

If the angle θ is given in degrees, how do I convert it to radians for the unit vector?

To convert degrees to radians, use $\theta_{radians} = \theta_{degrees} \times (\pi/180)$. Then, the unit vector is $(\cos \theta_{radians}, \sin \theta_{radians})$.

Can the directional derivative be negative, and what does that indicate?

Yes, a negative directional derivative indicates that the function decreases in the specified direction at the point.

What is the significance of the magnitude of the gradient vector in relation to the directional derivative?

The magnitude of the gradient vector indicates the steepness of the maximum increase; the directional derivative in a particular direction is scaled by the cosine of the angle between the gradient and the direction vector.

How does changing the angle θ affect the value of the directional derivative?

Changing θ alters the direction u, thus affecting the dot product with ∇F, which can increase or decrease the directional derivative depending on the alignment with the gradient.

Are there special cases where the directional derivative is zero?

Yes, when the direction u is orthogonal to the gradient vector ∇F at the point, their dot product is zero, resulting in a zero directional derivative.

What steps should I follow to find the directional derivative of F at a point in the direction of an angle θ?

First, compute the gradient ∇F at the point, then convert θ to a unit vector $u = (\cos \theta, \sin \theta)$, and finally, calculate the dot product $\nabla F \cdot u$ to obtain the directional derivative.

Additional Resources

Find the Directional Derivative of F at the Given Point in the Direction Indicated by the Angle

Understanding how a multivariable function changes as we move in specific directions is fundamental in fields such as calculus, physics, engineering, and data science. The concept of the directional derivative extends the idea of a derivative from a single-variable setting to functions of multiple variables, providing a measure of the rate at which a function varies as we proceed from a point in a specified direction. This article aims to explore the process of calculating the directional derivative of a function $F(x, y)$ at a particular point, in the direction defined by an angle θ. We will systematically analyze the theoretical underpinnings, mathematical formulations, practical techniques, and applications, equipping readers with the tools to approach these problems confidently.

Understanding the Concept of the Directional Derivative

What Is the Directional Derivative?

In single-variable calculus, the derivative of a function at a point quantifies the instantaneous rate of change of the function as the variable varies. Extending this to multivariable functions, the directional derivative measures how the function changes as we move from a point (x_0, y_0) in a particular direction.

Mathematically, for a function $F(x, y)$, the directional derivative at point (x_0, y_0) in the direction of a unit vector $\mathbf{u} = (u_x, u_y)$ is defined as:

$$
D_{\mathbf{u}} F(x_0, y_0) = \lim_{h \to 0} \frac{F(x_0 + h u_x, y_0 + h u_y) - F(x_0, y_0)}{h}
$$

This limit, if it exists, indicates the instantaneous rate of change of F at (x_0, y_0) as we move in the direction $\mathbf{u}$.

Key Point: The vector $\mathbf{u}$ must be a unit vector to ensure the derivative measures the rate of change per unit length in the specified direction.

Significance in Multivariable Calculus

The directional derivative generalizes the concept of the slope of a tangent line in one dimension to the rate of change along an arbitrary direction in two or more dimensions. It provides insights into:

- Identifying the steepest ascent or descent at a point.
- Analyzing how functions behave locally.
- Optimizing functions by understanding their directional tendencies.
- Modeling physical phenomena such as heat flow, gradient fields, and velocity vectors.

The Mathematical Framework for Computing the Directional Derivative

Gradient Vector: The Core Concept

The gradient vector of F, denoted as ∇F, encapsulates the partial derivatives of the function:

$$
\nabla F(x, y) = \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y} \right)
$$

The gradient points in the direction of the maximum rate of increase of F, and its magnitude corresponds to that maximum rate.

Relationship to the Directional Derivative:

$$
D_{\mathbf{u}} F(x_0, y_0) = \nabla F(x_0, y_0) \cdot \mathbf{u} = \left(\frac{\partial F}{\partial x}(x_0, y_0), \frac{\partial F}{\partial y}(x_0, y_0) \right) \cdot (u_x, u_y)
$$

This dot product simplifies the calculation considerably once the gradient and the direction vector are known.

Expressing the Directional Vector via an Angle θ

When the direction is specified by an angle θ, measured from the positive x-axis, the unit vector $\mathbf{u}$ in that direction is given by:

$$
\mathbf{u} = (\cos \theta, \sin \theta)
$$

This parametrization allows for a straightforward substitution into the formula for the directional derivative.

Step-by-Step Procedure for Finding the Directional Derivative

The process involves several key steps, which we will detail below.

1. Identify the Point and the Function

Suppose you're given:
- A function $F(x, y)$.
- A specific point (x_0, y_0) at which to evaluate.
- An angle θ indicating the direction.

For example, $F(x, y) = x^2 y + 3xy^2$, at the point $(1, 2)$, in the direction of $\theta = 45^\circ$.

2. Compute the Gradient ∇F at the Point

Calculate the partial derivatives:

\[

\frac{\partial F}{\partial x} \quad \text{and} \quad \frac{\partial F}{\partial y}
\]

and evaluate them at \((x_0, y_0) \).

Continuing the example:

\[
\frac{\partial F}{\partial x} = 2xy + 3y^2
\]
\[
\frac{\partial F}{\partial y} = x^2 + 6xy
\]

At \((1, 2) \):

\[
\frac{\partial F}{\partial x} (1, 2) = 2 \times 1 \times 2 + 3 \times (2)^2 = 4 + 3 \times 4 = 4 + 12 = 16
\]
\[
\frac{\partial F}{\partial y} (1, 2) = 1^2 + 6 \times 1 \times 2 = 1 + 12 = 13
\]

So,

\[
\nabla F(1, 2) = (16, 13)
\]

3. Determine the Direction Vector \(\mathbf{u} \)

Convert the angle \(\theta \) to radians if necessary, then find:

\[
\mathbf{u} = (\cos \theta, \sin \theta)
\]

For \(\theta = 45^\circ \):

\[
\cos 45^\circ = \frac{\sqrt{2}}{2} \quad \text{and} \quad \sin 45^\circ = \frac{\sqrt{2}}{2}
\]

Thus,

\[
\mathbf{u} = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right)
\]

Since this is already a unit vector, no normalization is necessary.

4. Calculate the Directional Derivative

Use the dot product:

$$
D_{\mathbf{u}} F(x_0, y_0) = \nabla F(x_0, y_0) \cdot \mathbf{u}
$$

Substituting the values:

$$
D_{\mathbf{u}} F(1, 2) = (16, 13) \cdot \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right) = 16 \times \frac{\sqrt{2}}{2} + 13 \times \frac{\sqrt{2}}{2}
$$

Factor out $\frac{\sqrt{2}}{2}$:

$$
D_{\mathbf{u}} F(1, 2) = \left(16 + 13 \right) \times \frac{\sqrt{2}}{2} = 29 \times \frac{\sqrt{2}}{2}
$$

Simplify:

$$
D_{\mathbf{u}} F(1, 2) = \frac{29 \sqrt{2}}{2}
$$

This value indicates the instantaneous rate of change of F at the point $(1, 2)$ in the direction specified by $\theta = 45^\circ$.

Analytical Insights and Practical Considerations

Maximum and Minimum Rate of Change

The magnitude of the gradient vector $|\nabla F|$ at a point signifies the maximum rate of increase of the function at that point. Conversely, the negative of the gradient indicates the direction of the steepest descent.

Given the gradient $\nabla F = (a, b)$, the directional derivative in direction $\mathbf{u}$ varies between:

$$
-D_{\max} = -|\nabla F| \quad \text{(steepest descent)}
$$
$$
D_{\max} = |\nabla F| \quad \text{(steepest ascent)}
$$

where

$$
|\nabla F| = \sqrt{a^2 + b^2}
$$

In the example:

\[
|\nabla F(1, 2)| = \sqrt{16^2 + 13

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Find The Directional Derivative Of F At The Given Point In The Direction Indicated By The Angle F X

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