Find The Distance Between The Point And The Line Given By The Set Of Parametric Equations. (Round Your

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Understanding how to find the shortest distance between a point and a line is a fundamental concept in coordinate geometry, crucial for various applications in mathematics, engineering, computer graphics, and physics. When the line is expressed through parametric equations, the process involves a systematic approach that combines vector algebra and algebraic manipulation. This article offers an in-depth guide to calculating this distance accurately, including methods to handle parametric forms, rounding techniques, and practical examples.

Introduction to Distance From a Point to a Line

Determining the shortest distance between a point and a straight line is essential in many geometric problems. The shortest distance is always measured along the perpendicular from the point to the line. When the line is given in a standard form, such as $Ax + By + C = 0$, the formula is straightforward. However, in cases where the line is expressed parametrically, the process involves additional steps, including vector projections.

Parametric equations describe a line as a set of points generated by a parameter, typically denoted as t. This form is versatile, allowing for easy representation of lines in 3D and 2D spaces, especially when the line's direction vector and a point on the line are known.

Understanding Parametric Equations of a Line

A line in two-dimensional space can be represented parametrically as:

$$
\begin{cases}
x = x_0 + at \\
y = y_0 + bt
\end{cases}
$$

where:

- (x_0, y_0) is a specific point on the line.
- (a, b) is the direction vector of the line.
- t is the parameter, which varies over all real numbers.

In three-dimensional space, the parametric form extends to:

$$
\begin{cases}
x = x_0 + a t \\
y = y_0 + b t \\
z = z_0 + c t
\end{cases}
$$

This representation allows for a flexible description of a line's orientation and position.

Mathematical Approach to Finding the Distance

The core idea involves calculating the shortest distance from a point $P(x_p, y_p)$ to a line defined parametrically by a point $Q(x_0, y_0)$ and a direction vector $\vec{d} = (a, b)$.

Step 1: Identify the Known Elements

- Point P: The external point from which the distance is measured.
- Point Q: A specific point on the line, given by (x_0, y_0).
- Direction Vector $\vec{d}$: Indicates the line's orientation, given by (a, b).

Step 2: Formulate the Vector from Q to P

Construct the vector:

$$
\vec{QP} = (x_p - x_0, y_p - y_0)
$$

This vector points from Q on the line to the external point P.

Step 3: Calculate the Cross Product Magnitude

In 2D, the magnitude of the cross product between $\vec{QP}$ and $\vec{d}$ is:

$$
|\vec{QP} \times \vec{d}| = |(x_p - x_0)b - (y_p - y_0)a|
$$

This scalar value represents the area of the parallelogram formed by the two vectors.

Step 4: Apply the Distance Formula

The shortest distance D from the point to the line is given by:

$$
D = \frac{|\vec{QP} \times \vec{d}|}{|\vec{d}|}
$$

where $|\vec{d}| = \sqrt{a^2 + b^2}$ is the magnitude of the direction vector.

Step-by-Step Example

Suppose we are given:

- The point $P(4, 3)$
- The line parametrically defined by:
$$
x = 1 + 2t \\
y = -1 + 3t
$$

Find the shortest distance from P to this line, rounding your answer to two decimal places.

Step 1: Identify a point Q on the line and the direction vector $\vec{d}$

- Choose $t = 0$ for simplicity:

$$
Q(1 + 2\cdot 0, -1 + 3 \cdot 0) = (1, -1)
$$

- Direction vector:

$$
\vec{d} = (2, 3)
$$

Step 2: Calculate $\vec{QP}$

$$
\vec{QP} = (4 - 1, 3 - (-1)) = (3, 4)
$$

Step 3: Compute the cross product magnitude

$$
|\vec{QP} \times \vec{d}| = |(3)(3) - (4)(2)| = |9 - 8| = 1
$$

Step 4: Find the magnitude of $\vec{d}$

$$
|\vec{d}| = \sqrt{2^2 + 3^2} = \sqrt{4 + 9} = \sqrt{13} \approx 3.606
$$

Step 5: Calculate the distance

$$
D = \frac{1}{3.606} \approx 0.277
$$

Rounded to two decimal places, the shortest distance from $P(4, 3)$ to the line is approximately 0.28 units.

Handling Rounding and Precision

When rounding your answer:

- Use appropriate decimal places based on the problem's requirements.
- Maintain sufficient precision during intermediate calculations to avoid rounding errors.
- Typically, rounding to two decimal places is sufficient unless specified otherwise.

Extensions to 3D Space

The process is similar in three dimensions, with the cross product replaced by the vector cross product in 3D:

$$
|\vec{QP} \times \vec{d}| = \text{magnitude of the 3D cross product}
$$

and the formula becomes:

$$
D = \frac{|\vec{QP} \times \vec{d}|}{|\vec{d}|}

\]

where:

- $\vec{QP} = (x_p - x_0, y_p - y_0, z_p - z_0)$
- $\vec{d} = (a, b, c)$

This technique is particularly useful in applications involving spatial analysis and 3D modeling.

Practical Applications of Finding the Distance

Understanding how to find the distance between a point and a parametric line has numerous real-world applications:

- Engineering and Design: Ensuring components are within specified tolerances by measuring minimal distances.
- Computer Graphics: Calculating the shortest distance from a camera or object to a line or edge.
- Navigation and Robotics: Determining the proximity of a robot to obstacles represented by lines or curves.
- Physics: Analyzing the shortest distance between particles or objects moving along specified paths.
- Geospatial Analysis: Measuring the shortest path from a location to a linear feature like a road or river.

Summary of Key Steps

To summarize, the process of finding the distance from a point to a line given by parametric equations involves:

1. Identifying a specific point on the line Q and the direction vector $\vec{d}$.
2. Constructing the vector $\vec{QP}$ from Q to the external point P.
3. Calculating the cross product magnitude $|\vec{QP} \times \vec{d}|$.
4. Computing the magnitude of the direction vector $|\vec{d}|$.
5. Applying the formula $D = \frac{|\vec{QP} \times \vec{d}|}{|\vec{d}|}$.
6. Rounding the result to the desired decimal places.

By following these steps, you can effectively determine the shortest distance between any point and a line expressed parametrically.

Conclusion

Mastering the technique of calculating the distance from a point to a parametric line is indispensable in many fields of science and engineering. It combines fundamental concepts of vector algebra with geometric intuition, providing a robust method for spatial analysis. Whether working in two or three dimensions, understanding and applying this approach enhances problem-solving skills and supports complex analyses in practical scenarios. Remember to carefully select a point on the line, compute vectors accurately, and maintain precision throughout your calculations to arrive at reliable and meaningful results.

Frequently Asked Questions

How do you find the shortest distance from a point to a line given in parametric form?

To find the shortest distance, identify a point on the line and the direction vector from the parametric equations. Then, use the formula: distance = $|(P - P0) \times v| / |v|$, where P is the point outside the line, P0 is a point on the line, and v is the direction vector.

What is the general formula for calculating the distance between a point and a line given parametrically?

Given a point P and a line with a point P0 and direction vector v, the distance d is calculated as $d = |(P - P0) \times v| / |v|$. This formula involves the cross product of the vector from P0 to P and the line's direction vector.

How do you handle rounding the distance to a specific decimal place when solving this problem?

After computing the exact distance, use your calculator's rounding function to round the result to the specified number of decimal places, such as rounding to two decimal places if instructed.

Can the method for finding the distance change if the line is given in symmetric form instead of parametric form?

Yes, if the line is given in symmetric form, you can convert it to parametric form or directly apply the point-line distance formula by extracting a point and a direction vector from the symmetric equations. The core concept remains the same.

What are common mistakes to avoid when calculating the distance between a point and a parametric line?

Common mistakes include using the wrong point on the line, forgetting to take the magnitude of the cross product, not simplifying the vectors correctly, or failing to round the final answer accurately. Double-check the direction vector and ensure proper vector operations.

Additional Resources

Distance Calculation Between a Point and a Line Given by Parametric Equations

Understanding the geometric relationship between points and lines is fundamental in fields ranging from engineering and computer graphics to physics and mathematics. One of the most intriguing problems in analytic geometry involves determining the shortest distance from a specific point to a line defined through parametric equations. This task, though straightforward in principle, demands a nuanced approach, especially when dealing with parametric forms rather than standard equations.

In this comprehensive guide, we'll delve into the methods and formulas used to find the shortest distance from a point to a line given by parametric equations. We’ll cover the foundational concepts, step-by-step procedures, and practical tips, ensuring you gain a deep understanding of this essential topic.

Understanding the Core Concepts

Before diving into formulas and calculations, it is crucial to grasp the basic ideas underpinning the problem.

What Is a Point and a Line in Space?

- Point: A specific location in space, represented by coordinates such as (x_0, y_0, z_0) in three dimensions or (x_0, y_0) in two dimensions.

- Line: A one-dimensional object extending infinitely in both directions, which can be described using various forms. The parametric form expresses the line as a set of equations depending on a parameter, typically denoted as t.

Parametric Equations of a Line

A line in space can be expressed parametrically as:

$$
\begin{cases}
x = x_0 + a t \\
y = y_0 + b t \\
z = z_0 + c t
\end{cases}
$$

where:
- (x_0, y_0, z_0) is a point on the line (a point through which the line passes),
- $\vec{d} = (a, b, c)$ is the direction vector of the line,
- t is the parameter, which varies over all real numbers.

In two dimensions, the parametric form simplifies to:

$$
\begin{cases}
x = x_0 + a t \\
y = y_0 + b t
\end{cases}
$$

Formulating the Problem

Given:
- A point $P = (x_1, y_1, z_1)$ in space (or (x_1, y_1) in 2D),
- A line defined by parametric equations with a known point $Q_0 = (x_0, y_0, z_0)$ on the line and a direction vector $\vec{d} = \langle a, b, c \rangle$,

Goal: Find the shortest distance D from point P to the line.

Mathematical Approach to Finding the Distance

The shortest distance from a point to a line in space is the length of the perpendicular segment connecting the point to the line. In vector terms, this involves projecting the vector from Q_0 to P onto the line's direction vector and then calculating the magnitude of the component perpendicular to the line.

Step-by-step Procedure

1. Identify the vectors:

- $\vec{Q_0 P} = \vec{P} - \vec{Q_0} = (x_1 - x_0, y_1 - y_0, z_1 - z_0)$

- Direction vector $\vec{d} = (a, b, c)$

2. Project $\vec{Q_0 P}$ onto $\vec{d}$:

The projection scalar t that minimizes the distance is:

$$
t = \frac{\vec{Q_0 P} \cdot \vec{d}}{\|\vec{d}\|^2}
$$

3. Find the closest point Q on the line:

$$
Q = Q_0 + t \vec{d}
$$

4. Calculate the vector from Q to P:

$$
\vec{Q P} = \vec{P} - \vec{Q}
$$

5. Compute the distance:

$$
D = \|\vec{Q P}\|
$$

Explicit Distance Formula

Putting it all together, the shortest distance D from point P to the line, given the parametric equations, can be expressed as:

$$
\boxed{
D = \frac{\left\| (\vec{P} - \vec{Q_0}) \times \vec{d} \right\|}{\|\vec{d}\|}
}
$$

where:
- $\times$ denotes the cross product,
- $\|\cdot\|$ denotes the vector magnitude.

This formula is elegant because it provides a direct computation method without explicitly solving for the parameter t. It highlights the geometric intuition: the numerator measures the area of the parallelogram spanned by vectors $\vec{Q_0 P}$ and $\vec{d}$, and dividing by $\|\vec{d}\|$ yields the height, which is the shortest distance.

Applying the Formula: A Practical Example

Suppose you are given:

- Point $P = (3, 4, 5)$
- Line passing through $Q_0 = (1, 2, 3)$ with direction vector $\vec{d} = (2, -1, 4)$

Step 1: Calculate \(\vec{Q_0 P}\):

\[
\vec{Q_0 P} = (3 - 1, 4 - 2, 5 - 3) = (2, 2, 2)
\]

Step 2: Compute the cross product:

\[
\vec{Q_0 P} \times \vec{d} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
2 & 2 & 2 \\
2 & -1 & 4
\end{vmatrix}
\]

Calculating the determinant:

\[
\mathbf{i} (2 \times 4 - 2 \times (-1)) - \mathbf{j} (2 \times 4 - 2 \times 2) + \mathbf{k} (2 \times (-1) - 2 \times 2)
\]

\[
= \mathbf{i} (8 + 2) - \mathbf{j} (8 - 4) + \mathbf{k} (-2 - 4)
\]

\[
= \mathbf{i} (10) - \mathbf{j} (4) + \mathbf{k} (-6)
\]

Step 3: Find the magnitude of the cross product:

\[
\| \vec{Q_0 P} \times \vec{d} \| = \sqrt{10^2 + (-4)^2 + (-6)^2} = \sqrt{100 + 16 + 36} = \sqrt{152} \approx 12.33
\]

Step 4: Compute \(\|\vec{d}\|\):

\[
\|\vec{d}\| = \sqrt{2^2 + (-1)^2 + 4^2} = \sqrt{4 + 1 + 16} = \sqrt{21} \approx 4.58
\]

Step 5: Calculate the distance:

\[
D = \frac{12.33}{4.58} \approx 2.69
\]

Result: The shortest distance from point P to the line is approximately 2.69 units.

Special Cases and Additional Tips

While the above method covers most scenarios, here are some important considerations:

- Line and point coincide: If the point lies on the line, the distance is zero.
- Line is a point: When the direction vector $\vec{d}$ is the zero vector, the line degenerates to a point, and the distance reduces to the Euclidean distance between P and Q_0.
- Dimensionality: The same principles apply in 2D and 3D, but the formulas adapt accordingly.
- Accuracy: When dealing with very small or very large numbers, consider potential numerical errors and use appropriate precision.

Extensions and Applications

The ability to find the shortest distance from a point to a line using parametric equations is pivotal in numerous applications:

- Computer Graphics: Collision detection and rendering calculations.
- Robotics: Path planning and obstacle avoidance.
- Engineering: Structural analysis and design.
- Geospatial Analysis: Calculating proximity between features.

Additionally, understanding this concept lays the groundwork for more complex geometric computations, such as finding the distance between skew lines or the shortest distance between a point and a plane.

Conclusion

Finding the shortest distance between a point and a line given by parametric equations is a fundamental skill in analytic geometry. The key lies in leveraging vector operations—particularly the cross

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