

Find The Elementary Matrix E Such That $EA = B$

Where $A = \begin{bmatrix} 6 & 7 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 14 & 1 & 8 \\ 5 & -13 & -1 \end{bmatrix}$ And $\begin{bmatrix} 5 & -13 & -1 \end{bmatrix} \begin{bmatrix} 14$

Find The Elementary Matrix E Such That $EA = B$ Where $A = \begin{bmatrix} 6 & 7 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 14 & 1 & 8 \\ 5 & -13 & -1 \end{bmatrix}$, and $\begin{bmatrix} 14$

Understanding how elementary matrices function and their role in linear algebra is fundamental for solving systems of equations, understanding matrix transformations, and performing matrix decompositions. The problem of finding an elementary matrix E such that $EA = B$ involves exploring matrix operations, transformations, and the properties of elementary matrices. This article provides a comprehensive guide to solving this type of problem, focusing on the specific matrices provided.

Introduction to Elementary Matrices and Their Significance

Elementary matrices are special types of matrices that result from performing a single elementary row operation on an identity matrix. These matrices are essential tools in linear algebra for several reasons:

- They facilitate understanding of matrix invertibility.
- They are used to perform row operations efficiently.
- They are instrumental in matrix factorizations such as LU decomposition.
- They help in solving systems of linear equations systematically.

Any elementary matrix can be obtained from an identity matrix (I) by applying one of the three elementary row operations:

1. Row swapping: Interchanging two rows.
2. Row scaling: Multiplying a row by a non-zero scalar.
3. Row addition: Adding a multiple of one row to another row.

Given a matrix (A) and an elementary matrix (E) , the transformation (EA) corresponds to applying a single elementary row operation to (A) . Our goal is to find such an elementary matrix (E) that transforms (A) into the matrix (B) .

Problem Overview and Matrix Definitions

Let's clearly define the matrices involved:

- Matrix (A) :

$$A = \begin{bmatrix} 6 & 7 & 1 \end{bmatrix}$$

This is a 1×3 matrix, representing a single row vector.

- Matrix (B) :

$$B = \begin{bmatrix} 14 & 1 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 5 & -13 & -1 \\ 5 & -13 & -1 \\ \end{bmatrix}$$

This is a 3x3 matrix with three rows.

The problem states: Find the elementary matrix (E) such that

$$EA = B$$

However, note that the dimensions of (A) and (B) do not directly match for a straightforward multiplication unless we consider whether the problem involves transforming (A) into a matrix or if the context involves multiple steps or matrices.

Clarifying the Context and Assumptions

The key challenge is that (A) is a 1x3 row vector, and (B) is a 3x3 matrix. For the multiplication $(EA = B)$ to make sense, the dimensions must align:

- If (E) is an elementary matrix, typically (E) is square and of size compatible with (A) for the multiplication (EA) .
- Alternatively, if the problem involves transforming the row vector (A) into the matrix (B) , then perhaps the intention is to consider (A) as a row or column vector and analyze the transformation accordingly.

Given the provided matrices, a more plausible interpretation is that the goal is to find an elementary matrix (E) that when multiplied with a certain matrix (A) results in matrix (B) . But since (A) is a 1×3 matrix, and (B) is 3×3 , perhaps the problem is to find an elementary matrix (E) such that:

$$\begin{bmatrix} E \end{bmatrix} \cdot A^T = B^T$$

or similar.

Alternatively, perhaps the problem involves finding an elementary matrix (E) such that when it acts on a matrix (A) , it produces (B) .

Given the ambiguity, the most consistent interpretation is:

- (A) is a 1×3 matrix.
- (B) is a 3×3 matrix.
- The goal is to find an elementary matrix (E) such that:

$$\begin{bmatrix} E \end{bmatrix} \times A^T = B^T$$

which would allow us to relate the transformations more conveniently.

Step-by-Step Solution Approach

To systematically find the elementary matrix (E) , follow these steps:

1. Understand the transformation

Assuming the goal is to find (E) such that:

$$E \times A^T = B^T$$

then:

$$E \times \begin{bmatrix} 6 & 7 & 1 \end{bmatrix} = \begin{bmatrix} 14 & 1 & 8 \end{bmatrix}$$

$\quad \text{\textit{(for the first row of B)}}$

Similarly, for the second and third rows, the transformation would be different, indicating that perhaps the problem is to find an (E) such that:

$$E \times \text{\textit{some matrix}} = B$$

Alternatively, perhaps the problem is to find an elementary matrix (E) such that:

$$E \times A = B$$

but with matrix (A) being extended or transformed accordingly.

2. Recognize the key operations involved

Since elementary matrices correspond to elementary row operations, and (E) acts on (A) , transforming it into (B) , perhaps the goal is to find (E) such that:

$$E \times A = \text{some matrix} \quad \text{and} \quad \text{that matrix} = B$$

Given the inconsistency in dimensions, a plausible approach is:

- Treat (A) as a matrix, perhaps with more rows or columns.
- Alternatively, find a sequence of elementary matrices that transform (A) into a matrix with the same row space as (B) .

3. Constructing the elementary matrix (E)

If the goal is to find (E) such that:

$$E \times A = B$$

then, considering (A) is 1×3 and (B) is 3×3 , perhaps the problem involves stacking (A) to match the size of (B) , or treating (A) as a basis vector.

Practical Solution: Using Row Operations to Find (E)

Given the complexity and possible ambiguity, a practical way to proceed is:

- Assuming (A) is a row vector and (B) is a matrix obtained by elementary row operations on (A) .
- Alternatively, interpret the problem as finding an elementary matrix (E) that transforms (A) (considered as a matrix) into (B) .

For simplicity, let's consider the following scenario:

Suppose (A) is a 3×3 matrix, and the problem is to find an elementary matrix (E) such that:

$$E \times A = B$$

with the matrices:

$$A = \begin{bmatrix} 6 & 7 & 1 \\ \text{(other rows)} \\ \text{(other rows)} \end{bmatrix}$$

But since only one row of (A) is provided, and (B) is a 3×3 matrix, perhaps the problem involves transforming the single row (A) into the three rows of (B) through elementary row operations.

Step-by-Step Example: Finding (E) via Elementary Row Operations

Suppose the initial matrix (A) is:

$$A = \begin{bmatrix} 6 & 7 & 1 \end{bmatrix}$$

and the target matrix (B) :

$$B = \begin{bmatrix} 14 & 1 & 8 \\ 5 & -13 & -1 \\ 5 & -13 & -1 \end{bmatrix}$$

Our goal is to find an elementary matrix (E) such that:

$$E \times A' = B$$

where (A') is a matrix formed by row (A) or its multiples.

Alternative Approach: Using the Concept of Row Operations

Since the dimensions do not align for direct multiplication, perhaps the more meaningful approach is:

- Find the sequence of elementary row operations that transform (A) into each row of (B) , or
- Construct an elementary matrix that performs a specific transformation.

Let's consider the following:

Transform (A) into the first row of (B)

- $A = [6 \quad 7 \quad 1]$
- First row of (B) : $[14 \quad 1 \quad 8]$

To find an elementary matrix (E) such that:

$$E \times A^T = \begin{bmatrix} 14 & 1 & 8 \end{bmatrix}$$

we need to find a matrix (E) that acts on (A^T) to produce that vector.

Constructing an Elementary Matrix (E)

Frequently Asked Questions

What is the goal when finding the elementary matrix E such that $EA = B$?

The goal is to find an elementary matrix E that, when multiplied by matrix A , results in matrix B , i.e., $EA = B$.

Given $A = [6 \ 7 \ 1]$, $B = [[14, 1, 8], [5, -13, -1]]$, how do we set up the problem to find E ?

We set up the equation $EA = B$ and solve for E by considering how elementary row operations transform A into B .

Is matrix A a row vector or a matrix, and how does that affect finding E ?

Matrix A is given as a row vector $[6 \ 7 \ 1]$, which suggests the operation involves transforming this row vector into a new row or matrix B, but since B is a 2×3 matrix, the problem likely involves considering A as part of a larger context or transforming the entire matrix structure.

How can elementary matrices be used to solve for E in the equation $EA = B$?

Elementary matrices represent elementary row operations. To find E, you identify the sequence of operations transforming A into B and multiply the corresponding elementary matrices, or directly solve for E if possible.

Given the matrices, is the problem well-posed for finding a single elementary matrix E?

Since A is a 1×3 matrix and B is a 2×3 matrix, the problem

as stated is inconsistent unless A is part of a larger matrix or considering different contexts; clarifying the matrix dimensions is essential.

What steps should be taken to find E if A and B are compatible matrices?

Convert A into a matrix compatible with B (e.g., by stacking or considering a larger matrix), then identify the row operations needed to convert A into B , and construct E accordingly.

Are there any common pitfalls when trying to find an elementary matrix E such that $EA = B$?

Yes, common pitfalls include mismatched matrix dimensions, assuming E is unique when multiple elementary matrices may exist, and misinterpreting the role of A and B in the transformation.

What is the significance of the elementary matrix E in linear algebra?

Elementary matrices represent elementary row operations and are used to perform row operations via matrix multiplication, crucial for understanding matrix inverses, solving linear systems, and matrix factorizations.

Additional Resources

Elementary Matrix: Unlocking the Secrets to Matrix Transformation

Introduction

In the realm of linear algebra, the concept of elementary matrices plays a pivotal role in understanding matrix transformations, solving systems of equations, and performing matrix inversions. Whether you're a student, educator, or professional mathematician, grasping how to find an elementary matrix (E) such that $(EA = B)$ is fundamental for a variety of applications—from computational algorithms to theoretical proofs.

Today, we delve into a concrete example, exploring how to determine the elementary matrix (E) that transforms a given matrix (A) into another matrix (B) . Our specific matrices are:

$($

$$A = \begin{bmatrix} 6 & 7 & 1 \end{bmatrix}$$

$)$

and

$$B = \begin{bmatrix} 14 & 1 & 8 \\ 5 & -13 & -1 \end{bmatrix}$$

The goal: Find the elementary matrix (E) such that $(EA = B)$.

This process involves understanding the nature of elementary matrices, their properties, and the step-by-step method to derive (E) . Let's embark on this analytical journey.

Understanding Elementary Matrices: The Foundation

What Are Elementary Matrices?

Elementary matrices are special matrices obtained by performing a single elementary row operation on an identity matrix. They serve as building blocks for more complex matrix transformations.

Key features:

- Each elementary matrix corresponds to a specific elementary row operation (row swapping, scaling, or adding a multiple of one row to another).
- When an elementary matrix (E) multiplies another matrix (A) , it performs the same row operation on (A) .

Types of Elementary Row Operations and Their Matrices

1. Row swapping: Interchanging two rows.

2. Row scaling: Multiplying a row by a non-zero scalar.
3. Row addition: Adding a multiple of one row to another.

Corresponding elementary matrices are:

Operation	Elementary Matrix (E)	Effect on matrix (A)
Swap rows (i) and (j)	Swap the (i^{th}) and (j^{th}) rows of (I)	Swaps rows (i) and (j) of (A)
Scale row (i) by $(k \neq 0)$	Multiply (i^{th}) row of (I) by (k)	Scales row (i) of (A)
Add (m) times row (j) to row (i)	Add (m) times row (j) of (I) to row (i)	Adds (m) times row (j) of (A) to row (i)

The Strategy: How to Find (E) Such That $(EA = B)$

Given that:

$$(A = [6 \quad 7 \quad 1]$$

and

$$(B = \begin{bmatrix} 14 & 1 & 8 \\ 5 & -13 & -1 \end{bmatrix}$$

Our task is to find an elementary matrix (E) that, when

multiplied by (A) , produces (B) .

Recognizing the Dimensions and Nature of (A) and (B)

- (A) is a 1×3 matrix (a row vector).
- (B) is a 2×3 matrix.
- The multiplication (EA) must result in a 2×3 matrix, meaning (E) must be a 2×1 matrix (which is not square and thus not an elementary matrix in the classical sense).

Important note: Elementary matrices are always square matrices. Since (A) is a row vector (1×3), and (B) is a 2×3 matrix, the multiplication (EA) is only defined if (E) is a 2×1 matrix, which cannot be an elementary matrix (which are square).

Therefore, to proceed correctly, the problem likely involves the following interpretations:

- The goal is to find an elementary matrix (E) such that the multiplication $(E \times A^T = B^T)$, i.e., transforming the transpose of (A) into the transpose of (B) .

or

- Alternatively, perhaps the problem intends to find a matrix (E) such that $(E \times A^T = B^T)$, where:

$[$

$$A^T = \begin{bmatrix} 6 & 7 & 1 \end{bmatrix}$$

$]$

and

$[$

$$B^T = \begin{bmatrix} 14 & 5 & 1 & -13 & 8 & -1 \end{bmatrix}$$

$\end{bmatrix}$

$]$

which are 3×1 and 3×2 matrices respectively.

Given the matrices, the more logical approach is that the problem involves transforming the vector $\begin{pmatrix} A \end{pmatrix}$ into the first column of $\begin{pmatrix} B \end{pmatrix}$, and possibly the second column of $\begin{pmatrix} B \end{pmatrix}$ through similar processes, or it's a typo in the problem statement.

Clarification and Approach

Assumption: The problem aims to find an elementary matrix $\begin{pmatrix} E \end{pmatrix}$ such that when multiplied by matrix $\begin{pmatrix} A \end{pmatrix}$, it produces matrix $\begin{pmatrix} B \end{pmatrix}$. The matrices suggest that:

- $\begin{pmatrix} A \end{pmatrix}$ is a row vector.
- $\begin{pmatrix} B \end{pmatrix}$ is a 2×3 matrix.

Given this discrepancy, a more consistent interpretation is that the problem involves operations on (A) as a vector to produce multiple vectors in (B) , or perhaps the matrices are misrepresented.

Corrected Approach

Suppose the problem is to find a matrix (E) such that:

$$\begin{bmatrix} E \times A^T = \text{some relevant transformation} \end{bmatrix}$$

or,

Suppose the problem wants us to find the elementary matrix (E) such that:

$$E \times \text{(some matrix)} = B$$

Given the initial matrices, perhaps the intended problem is:

Reinterpreted Problem Statement

Find an elementary matrix E such that:

$$E \times \begin{bmatrix} 6 & 7 & 1 \end{bmatrix}^T = \text{the first column of } B$$

and

$$\begin{bmatrix} E \times \begin{bmatrix} 6 & 7 & 1 \end{bmatrix}^T = \begin{bmatrix} 14 & 5 \end{bmatrix} \end{bmatrix}$$

Similarly, for the second column of (B) , perhaps a different elementary matrix is needed.

Final Approach: Working with the Provided Data

Given the ambiguity, the most common and instructive scenario is:

Scenario: Find the elementary matrix (E) such that:

$$\begin{bmatrix}$$

$$E \times A = B$$

$\backslash]$

where:

$\backslash[$

$$A = [6 \quad 7 \quad 1]$$

$\backslash]$

and

$\backslash[$

$$B = \begin{bmatrix}$$

$$14 \quad 1 \quad 8 \quad \backslash \backslash$$

$$5 \quad -13 \quad -1$$

$$\end{bmatrix}$$

$\backslash]$

But note that:

- (A) is a 1×3 vector.
- (B) is 2×3 .

The multiplication $(E \times A)$ is only defined if (E) is a 2×1 matrix, which is not square and hence not an elementary matrix.

Key Insight: Transforming a Vector into Multiple Vectors

Since the initial data seems inconsistent for a straightforward elementary matrix application, a more practical approach is:

Transforming a vector (A) into a matrix (B)

- This can be achieved by defining a matrix (E) , possibly a 2×3 or 3×3 matrix, that acts on (A) or (A^T) to

produce (B) .

- Alternatively, the problem may involve finding a matrix (E) such that:

$$[E \times A^T = \text{first column of } B]$$

and similar for the second column.

Conclusion: Clarifying the Objective

Given the contradictions in the problem statement, I will proceed with the most logical and instructive scenario:

Scenario: Find an elementary matrix (E) such that

$$(E) \times A^T = \text{a target vector}$$

where:

- $(A^T = \begin{bmatrix} 6 & 7 & 1 \end{bmatrix})$

- (B) contains two vectors:

$$\text{Column 1 of } B: \begin{bmatrix} 14 \\ 5 \end{bmatrix}$$

and

$$\text{Column 2 of } B: \begin{bmatrix} 1 \\ -13 \end{bmatrix}$$

$\backslash]$

Step-by-Step Solution: Constructing an Elementary Matrix $\backslash(E \backslash)$

Step 1: Recognize the transformation

Suppose that $\backslash(E \backslash)$ is a 2×3 matrix that acts on $\backslash(A \backslash)$ (a 3×1 vector) to produce a 2×1 vector, matching the first column of $\backslash(B \backslash)$:

$\backslash[$

[Find The Elementary Matrix E Such That Ea = B Where \$\begin{bmatrix} 6 & 7 & 1 \\ 8 & 5 & 13 \end{bmatrix} = B\$ and \$\begin{bmatrix} 14 & 1 \\ 5 & 13 & 1 \end{bmatrix} = A\$](#)

Find The Elementary Matrix E Such That $Ea = B$ Where $a = \begin{bmatrix} 6 \\ 7 \\ 1 \\ 14 \end{bmatrix}$ And $B = \begin{bmatrix} 1 \\ 8 \\ 5 \\ 13 \end{bmatrix}$

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