Find The Equation Of The Line In Standard Form Ax+By=C That Has A Slope Of (-1)/(6) And Passes Through

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Understanding how to determine the equation of a straight line given a specific slope and a point it passes through is a fundamental concept in coordinate geometry. The standard form of a line's equation, Ax + By = C, provides a concise way to represent lines and analyze their properties. In this article, we will explore how to derive the standard form of a line with a given slope and passing through a particular point, focusing on the slope of -1/6. We will break down the process step-by-step, illustrate with examples, and discuss various methods to arrive at the desired equation.

Understanding the Components of a Line Equation

What Is the Standard Form of a Line?

The standard form of a linear equation is written as:

- Ax + By = C

where:
- A, B, and C are real numbers,
- A and B are not both zero,
- The coefficients are usually chosen to be integers with no common factors (for simplicity).

This form is particularly useful for quickly identifying intercepts, and for analyzing the relationship between x and y in a linear relationship.

What Is the Slope of a Line?

The slope (m) of a line indicates its steepness and direction. It is calculated as:

- **m = (change in y) / (change in x)**

Given the slope m = -1/6, it means:
- For every 6 units moved horizontally to the right, the line drops 1 unit vertically.

- The line is decreasing from left to right.

Given Data and Objective

Suppose the line passes through a specific point, say, (x_1, y_1). Our goal is to find the equation of the line in standard form, Ax + By = C, that:
- Has a slope $m = -\frac{1}{6}$,
- Passes through (x_1, y_1).

The process involves:
1. Using the point-slope form of the line equation.
2. Rearranging into standard form.

Method 1: Using the Point-Slope Form

Step 1: Write the Point-Slope Equation

The point-slope form of a line is:

- $y - y_1 = m(x - x_1)$

Given:
- $m = -\frac{1}{6}$,
- a point (x_1, y_1).

The equation becomes:

$$y - y_1 = -\frac{1}{6}(x - x_1)$$

Step 2: Clear the Fraction

Multiply both sides by 6 to eliminate the denominator:

$$6(y - y_1) = - (x - x_1)$$

which simplifies to:

$$6y - 6y_1 = -x + x_1$$

Step 3: Rearrange into Standard Form

Bring all terms to one side:

$$x + 6y = x_1 + 6 y_1$$

This is the line in standard form:

$$Ax + By = C$$

where:
- $A = 1$,
- $B = 6$,
- $C = x_1 + 6 y_1$.

Note: If specific point coordinates are provided, substitute them into the above to find the exact C.

Method 2: General Approach with a Specific Point

Suppose the line passes through the point $(x_1, y_1) = (2, 3)$. Let's find the equation step-by-step.

Step 1: Write the point-slope form

$$y - 3 = -\frac{1}{6}(x - 2)$$

Step 2: Multiply through by 6 to clear fractions

$$6(y - 3) = -(x - 2)$$

$$6y - 18 = -x + 2$$

Step 3: Rearrange to standard form

$$x + 6y = 20$$

This is the standard form of the line passing through (2, 3) with slope -1/6.

General Process for Any Point

If the point is (x_1, y_1), the general formula becomes:

$$x + 6 y = x_1 + 6 y_1$$

which directly provides the standard form once the specific point is substituted.

Additional Considerations

Ensuring the Standard Form is in the Correct Format

- Usually, A should be positive. If A is negative, multiply the entire equation by -1 to make A positive.
- Coefficients A, B, and C should be integers with no common factors for simplicity, if possible.

Example: Line Passing Through Multiple Points

Suppose you are asked to find the line passing through two points (x_1, y_1) and (x_2, y_2) with the given slope:

- Find the slope between the two points.
- Confirm that it matches the given slope (-1/6).
- Use the point-slope form with either point.
- Convert to standard form.

Practice Problem and Solution

Problem: Find the equation of a line in standard form that has a slope of -1/6 and passes through the point $(4, -2)$.

Solution:

1. Write point-slope form:

$$y - (-2) = -\frac{1}{6}(x - 4)$$

$$y + 2 = -\frac{1}{6}x + \frac{4}{6}$$

$$y + 2 = -\frac{1}{6}x + \frac{2}{3}$$

2. Multiply through by 6:

$$6 y + 12 = -x + 4$$

3. Rearrange to standard form:

$$x + 6 y = -12 + 4$$

$$x + 6 y = -8$$

This is the standard form of the line.

Summary and Key Takeaways

- To find the equation of a line in standard form given a slope and a point, start with the point-slope form.
- Clear fractions by multiplying through by the denominator.
- Rearrange the resulting equation to the form Ax + By = C.
- Ensure the coefficients are simplified and A is positive.

Conclusion

Deriving the equation of a line in standard form from a given slope and point involves a systematic process that starts with the point-slope form, simplifies to eliminate fractions, and rearranges into the desired standard form. Mastery of this method enables students and mathematicians to quickly analyze line equations, find intersections, and solve geometric problems efficiently. Whether working with theoretical problems or practical applications, understanding these steps is essential for a solid foundation in analytic geometry.

Frequently Asked Questions

How do I find the equation of a line in standard form with a slope of -1/6 passing through a specific point?

Use point-slope form: $y - y_1 = m(x - x_1)$, then rearrange to standard form Ax + By = C.

What is the process to convert a line's equation from slope-intercept form to standard form?

Rearrange the slope-intercept form y = mx + b into Ax + By = C by moving all terms to one side and clearing fractions if necessary.

If a line has a slope of -1/6 and passes through (x_1, y_1), how do I write its equation in standard form?

Start with point-slope form: $y - y_1 = (-1/6)(x - x_1)$, then multiply through by 6 to clear fractions and rearrange to Ax + By = C.

Can you give an example of finding the standard form of a line

with slope -1/6 passing through (2, 3)?

Yes. Using point-slope form: y - 3 = (-1/6)(x - 2). Multiply both sides by 6: 6(y - 3) = -1(x - 2). Simplify: 6y - 18 = -x + 2. Rearranged: x + 6y = 20.

What are common mistakes to avoid when deriving the standard form from a slope and a point?

Avoid forgetting to clear fractions by multiplying through by the denominator, and ensure all terms are moved to one side with proper signs.

How does knowing the slope (-1/6) help in constructing the standard form of the line?

The slope is used in the point-slope form to create the initial equation, which can then be rearranged into standard form with proper algebraic manipulation.

Is the standard form of a line unique for a given slope and point, and how do I ensure the coefficients are integers?

Yes, the standard form is unique up to multiplication by a non-zero constant. To have integer coefficients, multiply the entire equation by the least common denominator if necessary.

What is the general formula for the standard form of a line with a given slope passing through a specific point?

Start with $y - y_1 = m(x - x_1)$, then multiply both sides to clear fractions, and rearrange to Ax + By = C, ensuring A, B, C are integers.

How do I verify that my standard form equation correctly represents the line with the given slope and point?

Substitute the given point into your equation to check if it satisfies the equation, and verify that the slope derived from your equation matches the given slope.

Additional Resources

Find The Equation Of The Line In Standard Form Ax+By=C That Has A Slope Of (-1)/(6) And Passes Through

Understanding the equations of lines is a cornerstone of analytic geometry, a branch of mathematics that explores the relationships between algebraic equations and their geometric representations. Whether you're tackling high school algebra, preparing for college exams, or delving into advanced mathematics, being able to find the equation of a line given certain parameters is an essential skill. Today, we focus on a specific yet common problem: finding the equation of a line in standard form Ax+By=C that has a slope of \(-\frac{1}{6}\) and passes through a given point.

In this article, we will walk through the process step-by-step, explain key concepts, and provide practical examples to ensure clarity. Whether you're a student, educator, or math enthusiast, understanding this process will strengthen your grasp of line equations and their applications.

Understanding the Standard Form of a Line

Before we dive into the specific problem, it’s essential to understand what the standard form of a line represents and how it relates to other forms.

What is the Standard Form?

The standard form of a linear equation is expressed as:

$$ Ax + By = C $$

where:

- A, B, and C are constants,
- A and B are not both zero simultaneously.

This form is favored in many scenarios because:

- It clearly emphasizes the relationship between x and y,
- It is useful for identifying intercepts quickly,
- It provides a straightforward pathway for algebraic manipulations and solving systems.

Relationship to Slope-Intercept and Point-Slope Forms

Other common forms include:

- Slope-intercept form: $y = mx + b$, where m is the slope and b is the y-intercept.
- Point-slope form: $y - y_1 = m(x - x_1)$, where (x_1, y_1) is a point on the line, and m is the slope.

The standard form can be derived from these forms or used directly to analyze the line's properties, such as intercepts and slope.

Extracting the Key Parameters: Slope and Point

The problem provides two crucial pieces of information:

1. Slope $m = -\frac{1}{6}$
2. A point through which the line passes, say (x_1, y_1)

Without the point, the line’s position in the plane cannot be fully determined; the slope alone only indicates the line’s angle relative to the axes. Therefore, the first step is to understand how to incorporate the given point into the line equation.

Step 1: Using the Point-Slope Form

The point-slope form is particularly convenient for problems where the slope and a passing point are known:

$$
y - y_1 = m(x - x_1)
$$

Given:

- $m = -\frac{1}{6}$,
- (x_1, y_1) as the point of passage.

Suppose the point is (x_1, y_1). For illustration, assume the line passes through $(x_1, y_1) = (x_0, y_0)$. The specific coordinates are essential for an exact answer; otherwise, the solution remains in parametric form involving (x_1, y_1).

Example:

If the line passes through $(3, 2)$, then:

$$
y - 2 = -\frac{1}{6}(x - 3)
$$

Step 2: Converting to Slope-Intercept Form

Expanding the point-slope equation:

$$
y - y_1 = m(x - x_1)
$$

$$
y = m(x - x_1) + y_1
$$

Substituting the known slope:

$$
y = -\frac{1}{6}(x - x_1) + y_1
$$

Distribute:

$$

$$
y = -\frac{1}{6}x + \frac{1}{6}x_1 + y_1
$$

This is the slope-intercept form, from which you can identify b (the y-intercept):

$$
b = \frac{1}{6}x_1 + y_1
$$

Step 3: Converting to Standard Form $Ax + By = C$

To convert the equation into standard form, rearrange the slope-intercept form:

$$
y = -\frac{1}{6}x + \left(\frac{1}{6}x_1 + y_1\right)
$$

Multiply through by 6 to clear fractions:

$$
6y = -x + x_1 + 6y_1
$$

Rearranged:

$$
x + 6y = x_1 + 6y_1
$$

This is standard form with:

- $A = 1$,
- $B = 6$,
- $C = x_1 + 6y_1$.

Practice Example: Finding the Equation When a Specific Point Is Given

Suppose the line passes through $(4, 5)$ and has a slope of $-\frac{1}{6}$. Let's follow the steps:

Step 1: Write the point-slope form:

$$
y - 5 = -\frac{1}{6}(x - 4)
$$

Step 2: Expand to slope-intercept form:

$$
y = -\frac{1}{6}x + \frac{4}{6} + 5
$$

Simplify:

$$
y = -\frac{1}{6}x + \frac{2}{3} + 5
$$

Express 5 as $\frac{15}{3}$:

$$
y = -\frac{1}{6}x + \frac{2}{3} + \frac{15}{3} = -\frac{1}{6}x + \frac{17}{3}
$$

Step 3: Convert to standard form:

Multiply through by 6:

$$
6y = -x + 6 \times \frac{17}{3} = -x + 34
$$

Bring all to one side:

$$
x + 6y = 34
$$

Final Equation:

$$
x + 6y = 34
$$

This standard form represents the line passing through $(4, 5)$ with a slope of $-\frac{1}{6}$.

Handling the General Case: Unknown Passing Point

If the specific point isn't given, the problem remains a general one: find the standard form of a line with slope $-\frac{1}{6}$ passing through an arbitrary point (x_1, y_1). The general form can be written as:

$$
x + 6y = x_1 + 6 y_1
$$

This form emphasizes that for any point (x_1, y_1), the line equation in standard form can be expressed succinctly.

Additional Considerations

Ensuring the Standard Form is Properly Signed and Simplified

- The coefficient A is conventionally positive. If A turns out negative, multiply through by -1.
- The coefficients should be integers with no common factors (except possibly 1). Simplify if possible.

Verifying the Equation

Once the equation is established, verify:

- The point (x_1, y_1) satisfies the equation.
- The slope matches $-\frac{1}{6}$.

To verify the slope, you can also derive it directly from the standard form by rewriting it in slope-intercept form or using the formula for the slope:

$$
m = -\frac{A}{B}
$$

In the standard form $Ax + By = C$, the slope is $-A/B$. For example, in the equation $x + 6y = 34$:

$$
m = -\frac{1}{6}
$$

which confirms the slope.

Real-World Applications

The ability to find the equation of a line given a slope and passing point is not just an academic exercise. It has practical applications in various fields:

- Engineering: Designing roads or railways with specific inclines.
- Economics: Modeling cost-profit relationships.
- Physics: Describing motion with constant velocity.
- Computer Graphics: Rendering lines and curves.

In each context, precise equations enable accurate calculations, predictions, and visualizations.

Summary

To find the equation of a line in standard form \(Ax + By = C\) with a given slope \(-\frac{1}{6}\) passing through a known point \((x_1, y_1)\), follow these steps:

1. Use the point-slope form:

\[
y - y_1 = -\frac{1}{6}(x - x_1)
\]

2. Expand to slope-intercept form:

\[
y = -\frac{1}{6}x + \frac

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