Find The Equation Of The Line Through P=(9,8) Such That The Triangle Bounded By This Line And The Axes

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Understanding how to derive the equation of a line passing through a specific point and forming a bounded triangle with the coordinate axes is a fundamental concept in coordinate geometry. This problem involves several steps, including analyzing the geometric constraints, formulating the line's equation, and understanding how the intercepts on axes determine the shape and size of the triangle. In this article, we will explore these steps in detail, providing a comprehensive guide to solving such problems.

Understanding the Geometric Setup

The Point P=(9,8)

The point P=(9,8) lies in the first quadrant of the Cartesian coordinate system. Any line passing through this point will satisfy the line's equation at x=9, y=8.

The Triangle Bounded By The Line And The Axes

The triangle in question is formed by:
- The line itself
- The x-axis (where y=0)
- The y-axis (where x=0)

This triangle is bounded in the first quadrant, with its vertices on the axes and the line connecting these points.

Formulating the Line Equation

The Slope-Intercept Form

A general line equation in slope-intercept form is:

$$y = m x + c$$

where:
- m is the slope
- c is the y-intercept

Since the line passes through P=(9,8), it must satisfy:

$$8 = m \times 9 + c$$

which links m and c.

The Intercepts of the Line

The line intersects the axes at points:
- On the x-axis (where $y=0$), the x-intercept a satisfies:
$$
0 = m a + c \quad \Rightarrow \quad a = -\frac{c}{m}
$$
- On the y-axis (where $x=0$), the y-intercept is simply c.

The vertices of the triangle are thus:
- $A = (a, 0)$
- $B = (0, c)$
- $P = (9,8)$, which lies on the line.

Important: For the triangle to be bounded by the axes and the line, the line must intersect both axes in the first quadrant, meaning:
- $a > 0$
- $c > 0$

This constrains the values of m and c.

Determining the Equation of the Line

Expressing Intercepts in Terms of m and c

Given the point $P=(9,8)$, and the line passing through it:
$$
8 = 9m + c
$$
which rearranged gives:
$$
c = 8 - 9m
$$
The x-intercept:
$$
a = -\frac{c}{m} = -\frac{8 - 9m}{m} = -\frac{8}{m} + 9
$$

To ensure the intercepts are positive:
- $a > 0$, so:
$$
-\frac{8}{m} + 9 > 0
$$
- $c > 0$, so:
$$
8 - 9m > 0
$$
which implies:
$$
9m < 8 \quad \Rightarrow \quad m < \frac{8}{9}
$$

Similarly, from $a > 0$:
$$
-\frac{8}{m} + 9 > 0
$$

which can be rearranged to analyze the sign of m.

Note: Since the line must intersect both axes in the positive region, the slope m must satisfy certain inequalities.

Constraints on m and c

Let's analyze these inequalities:

1. $c = 8 - 9m > 0 \Rightarrow m < \frac{8}{9}$

2. $a = -\frac{8}{m} + 9 > 0$

Depending on the sign of m:

- If $m > 0$, then:
$$
a = -\frac{8}{m} + 9
$$
Since $\frac{8}{m} > 0$, $a > 0$ implies:
$$
-\frac{8}{m} + 9 > 0 \Rightarrow 9 > \frac{8}{m} \Rightarrow m > \frac{8}{9}
$$
which contradicts $m < 8/9$, so m cannot be greater than $8/9$. But since $m < 8/9$, the inequalities are incompatible for $m > 0$.

- If $m < 0$, then:
$$
a = -\frac{8}{m} + 9
$$
Since $m<0$, $\frac{8}{m}<0$, so:
$$
a = -(\text{negative}) + 9 > 0
$$
which is always true if $\frac{8}{m}<0$, i.e., $m<0$.

Also, for $c > 0$:
$$
8 - 9m > 0 \Rightarrow m < \frac{8}{9}
$$

Combining the constraints:
- $m<0$
- $m<\frac{8}{9}$

Therefore, the slope m must be negative and less than $8/9$. For the intercepts to be positive, a must also be positive, which is guaranteed if $m<0$.

Deriving the Equation of the Line

Final Form of the Equation

Using the relation:
$$
c = 8 - 9m

$$
\]
$$

the line's equation becomes:

$$
y = m x + (8 - 9m)
$$

or equivalently,

$$
y = m x + c
$$

The intercepts are:

- x-intercept:

$$
a = -\frac{c}{m} = -\frac{8 - 9m}{m}
$$

- y-intercept:

$$
c = 8 - 9m
$$

Given the constraints:

- $m<0$,
- $a>0$,
- $c>0$,

the entire family of lines satisfying these conditions is:

$$
\boxed{
\text{Equation: } y = m x + (8 - 9m), \quad \text{where } m<0
}
$$

Special Cases and Additional Constraints

- When $m \to -\infty$, the line approaches a vertical line at $x=0$, which isn't suitable for forming a bounded triangle with positive intercepts.
- When $m \to 0^-$, the line approaches the horizontal line $y=8$, which intersects the y-axis at 8, and the x-intercept becomes:

$$
a = -\frac{8 - 9 \times 0}{0} \rightarrow \text{undefined}
$$

indicating the intercept tends to infinity.

Thus, the valid range of m is:

$$
-\infty < m < 0
$$

with the corresponding intercepts ensuring the triangle is bounded in the first quadrant.

Calculating the Area of the Triangle

Vertices of the Triangle

The triangle's vertices are:
- $A = (a, 0) = \left(-\frac{8 - 9m}{m}, 0\right)$
- $B = (0, c) = (0, 8 - 9m)$
- $P = (9,8)$

Given that $a > 0$, $c > 0$, the vertices are in the first quadrant.

Area Formula

The area of the triangle bounded by axes and the line is:

$$
\text{Area} = \frac{1}{2} \times a \times c
$$

Substituting:

$$
a = -\frac{8 - 9m}{m}
$$

and

$$
c = 8 - 9m
$$

we get:

$$
\text{Area} = \frac{1}{2} \times \left(-\frac{8 - 9m}{m}\right) \times (8 - 9m)
$$

Simplify numerator:

$$
\text{Area} = \frac{1}{2} \times \frac{-(8 - 9m)(8 - 9m)}{m} = -\frac{1}{2} \times \frac{(8 - 9m)^2}{m}
$$

Since $m<0$, the denominator is negative, and the numerator is positive, the entire area is positive.

Summary:

\[
\boxed{
\text{Area}(m) = -\frac{

Frequently Asked Questions

How do I find the equation of a line passing through point P(9,8) that forms a triangle with the axes?

To find such a line, assume the line intersects the x-axis at (a, 0) and y-axis at (0, b). Using the point P(9,8), set up the equation y = m x + c and solve for m and c based on the intercepts, then verify the triangle formed with the axes.

What is the significance of the triangle bounded by the line through P(9,8) and the axes?

The triangle represents the area enclosed between the line and the coordinate axes, which can be calculated using the intercepts. Finding the line helps in understanding how the point P relates to the intercepts and the area of this triangle.

How can I determine the intercepts of the line passing through P(9,8) to form the triangle with axes?

Assuming the line intersects the x-axis at (a, 0) and y-axis at (0, b), you can find these intercepts by using the point P and the slope, then apply the line equation $y = m x + c$ to solve for intercepts.

If I want the triangle formed by the line through P(9,8) and the axes to have a specific area, how do I find the equation?

Set the area formula (Area = $\frac{1}{2} |a| |b|$) equal to the desired area. Express a and b in terms of the line's slope and intercepts, then solve for the line's equation parameters accordingly.

Can the line passing through P(9,8) be vertical or horizontal when forming the triangle with axes?

A vertical or horizontal line through P(9,8) can form a triangle with the axes, but the line's equation will be x=9 or y=8 respectively. These cases are special and can simplify the process of finding intercepts or analyzing the triangle.

Additional Resources

Find The Equation Of The Line Through P = (9,8) Such That The Triangle Bounded By This Line And The Axes

Understanding how to determine the equation of a line that passes through a specific point and forms a bounded triangle with the coordinate axes is a fundamental concept in analytic geometry. This problem combines the principles of line equations, intercepts, and geometric area calculations, providing a rich context for applying algebraic and geometric techniques. In this comprehensive review, we will explore the problem step-by-step, delve into the underlying concepts, and present methods to derive the required line equations.

Understanding the Geometric Context

Before diving into the algebraic formulations, it's crucial to understand the geometric setup:

- The Point P = (9,8):
The line we seek must pass through this specific point. This point lies in the first quadrant, indicating positive x and y coordinates, which influences the nature of the line's intercepts.

- The Triangle Bounded By The Line and the Axes:
The line intersects the x-axis and y-axis at points (a, 0) and (0, b) respectively, forming a triangle with vertices at these intercepts and the origin (0,0).

- The Triangle's Area and Constraints:
The area of this triangle can be calculated as $\frac{1}{2} |a \times b|$. Depending on additional constraints (if any were specified), such as maximizing or minimizing the area, or specific intercept lengths, the problem can take different directions.

General Equation of a Line Intercept Form

To find the line's equation, it's most convenient to express it in the intercept form:

$$
\frac{x}{a} + \frac{y}{b} = 1
$$

where:

- a is the x-intercept (the point where the line crosses the x-axis),
- b is the y-intercept (the point where the line crosses the y-axis).

This form is particularly useful because:

- It directly relates the line to its intercepts.
- The area of the triangle formed with axes is easy to compute as $\frac{1}{2} |a \times b|$.
- The line passes through any point (x, y) satisfying the equation.

Note: The intercepts a and b can be positive or negative depending on the line's position relative to the axes, but for the triangle in the first quadrant, positive intercepts are typical.

Deriving the Equation Using the Point (9,8)

Given the line passes through $P = (9,8)$, substitute $x=9$ and $y=8$ into the intercept form:

$$
\frac{9}{a} + \frac{8}{b} = 1
$$

This equation relates the intercepts a and b. It shows that for any pair (a, b) satisfying this relation, the corresponding line passes through P.

Rearranged:

$$
\frac{9}{a} + \frac{8}{b} = 1
$$

or equivalently,

$$
\frac{8}{b} = 1 - \frac{9}{a}
$$

which leads to:

$$
b = \frac{8a}{a - 9}
$$

Important considerations:

- The denominator $a - 9$ cannot be zero, so $a \neq 9$.

- The signs of a and b depend on the position of the line. For the triangle formed with axes in the first quadrant, $a > 0$ and $b > 0$.

Conditions for the Triangle Formed by the Line and the Axes

To ensure the line forms a triangle with the axes:

- Intercepts must be positive: $a > 0$ and $b > 0$.
- Line must cross both axes in the positive quadrant:
For the intercept form $\frac{x}{a} + \frac{y}{b} = 1$, positive intercepts imply the line crosses the axes at positive points.

Implication on a and b:

$$
b = \frac{8a}{a - 9}
$$

- For $b > 0$:

$$
\frac{8a}{a - 9} > 0
$$

- Since numerator $8a$ is positive if $a > 0$, the denominator $a - 9$ must also be positive for the entire fraction to be positive:

$$
a - 9 > 0 \Rightarrow a > 9
$$

Similarly, if $a < 0$, then numerator $8a < 0$, and for the fraction to be positive, denominator must also be negative:

$$
a - 9 < 0 \Rightarrow a < 9
$$

but then $a < 0$ and $a - 9 < 0$, so the ratio is positive.

However, for the triangle to be in the first quadrant:

- Both intercepts a, b should be positive, so the relevant domain for a is:

$$
a > 9 \quad \Rightarrow \quad b > 0
$$

Thus, the key feasible region is:

$$
a > 9, \quad b = \frac{8a}{a - 9} > 0
$$

Calculating the Area of the Triangle

The area A of the triangle bounded by the axes and the line is:

$$
A = \frac{1}{2} |a \times b|
$$

Substituting b:

$$
A(a) = \frac{1}{2} a \times \frac{8a}{a - 9} = \frac{1}{2} \times \frac{8a^2}{a - 9} = \frac{4a^2}{a - 9}
$$

This function describes how the area varies with a. To find the specific line that meets certain criteria (such as maximum area), we analyze this function.

Optimization: Finding the Maximum Area

Suppose the problem asks for the line passing through (9,8) that maximizes the triangle's area. To find such a line:

- Step 1: Consider $A(a) = \frac{4a^2}{a - 9}$, with $a > 9$.

- Step 2: Find the critical point by differentiating $A(a)$ with respect to a:

$$
A(a) = \frac{4a^2}{a - 9}
$$

Using the quotient rule:

$$
A'(a) = \frac{(8a)(a - 9) - 4a^2 \times 1}{(a - 9)^2} = \frac{8a(a - 9) - 4a^2}{(a - 9)^2}
$$

Simplify numerator:

$$
8a(a - 9) - 4a^2 = 8a^2 - 72a - 4a^2 = (8a^2 - 4a^2) - 72a = 4a^2 - 72a
$$

Set numerator equal to zero for critical points:

$$
4a^2 - 72a = 0
$$
$$
4a(a - 18) = 0
$$

Since $a > 9$, discard $a=0$. The critical point is at:

\[

$$
a = 18
$$

- Step 3: Verify that this corresponds to a maximum by examining the second derivative or the behavior of $A(a)$.

At $a=18$:

$$
b = \frac{8 \times 18}{18 - 9} = \frac{144}{9} = 16
$$

The area at this point:

$$
A(18) = \frac{4 \times 18^2}{18 - 9} = \frac{4 \times 324}{9} = \frac{1296}{9} = 144
$$

Thus, the maximum area of the triangle is 144, corresponding to the intercepts:

$$
a = 18, \quad b = 16
$$

and the line:

$$
\frac{x}{18} + \frac{y}{16} = 1
$$

which passes through $(9,8)$, as verified:

$$
\frac{9}{18} + \frac{8}{16} = \frac{1}{2} + \frac{1}{2} = 1
$$

This confirms the consistency and correctness of the solution.

Equation of the Line with Maximum Area

Consolidating the above:

$$
\boxed{
\frac{x}{18} + \frac{y}{16} = 1
}
$$

or equivalently,

\[
\frac{y}{16} = 1 - \frac{x}{

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