Find The Equation Of The Line Which Passes Through The Point (-1,-2), And Is Perpendicular To The Line

Find The Equation Of The Line Which Passes Through The Point (-1,-2), And Is Perpendicular To The Line is a common problem in coordinate geometry that involves understanding the relationships between lines, their slopes, and how to derive equations from given information. This type of problem typically tests your knowledge of the slope-intercept form, the concept of perpendicular lines, and your ability to manipulate algebraic equations to find the required line. In this article, we will explore the systematic approach to solving such problems, including understanding slopes, perpendicularity, and the step-by-step process to derive the equation.

Understanding the Problem Statement

Before diving into calculations, it's crucial to understand what is asked:

- The line must pass through a specific point, in this case, `(-1, -2)`.
- The line must be perpendicular to a given line, which is usually described by its equation or slope.

The key tasks are:

1. Identifying the slope of the given line.
2. Determining the slope of the line perpendicular to it.
3. Using the point-slope form to find the equation of the required line.

Let's explore each of these steps in detail.

Understanding Slopes and Perpendicularity

What is the Slope of a Line?

The slope of a line measures its steepness and is usually denoted by `m`. If the line passes through points `($x_1$, $y_1$)` and `($x_2$, $y_2$)`, then the slope is given by:

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

In the case of a line’s equation, the slope is the coefficient of `x` when expressed in slope-intercept form, `y = mx + c`.

Perpendicular Lines and Their Slopes

Two lines are perpendicular if they intersect at a right angle (90 degrees). The slopes of two perpendicular lines are negative reciprocals of each other:

$$
m_1 \times m_2 = -1
$$

This means:

$$
m_2 = -\frac{1}{m_1}
$$

If you know the slope of the given line, finding the slope of the perpendicular line involves taking its negative reciprocal.

Step-by-Step Approach to Find the Required Equation

The problem as commonly posed involves the following:

- The original line's equation (which is not explicitly given here).
- The point `(-1, -2)` through which the new line passes.
- The need to find the equation of the line passing through `(-1, -2)` and perpendicular to the original line.

For illustration, let's assume the original line's equation is provided, or if not, suppose the problem asks to find the line perpendicular to a known line, for example, `y = 2x + 3`.

Step 1: Find the Slope of the Given Line

Suppose the given line is:

\[
y = 2x + 3
\]

The slope, `$m_1$`, of this line is:

\[
m_1 = 2
\]

Step 2: Determine the Slope of the Perpendicular Line

Using the negative reciprocal rule:

\[
m_2 = -\frac{1}{m_1} = -\frac{1}{2}
\]

Thus, the slope of the line passing through `(-1, -2)` and perpendicular to the given line is `-1/2`.

Step 3: Use Point-Slope Form to Find the Equation

The point-slope form of a line's equation is:

\[
y - y_1 = m (x - x_1)
\]

Plugging in the point `(-1, -2)` and the slope `-1/2`:

\[
y - (-2) = -\frac{1}{2} (x - (-1))
\]

Simplify:

\[
y + 2 = -\frac{1}{2} (x + 1)
\]

Distribute:

\[

y + 2 = -\frac{1}{2} x - \frac{1}{2}
\]

Finally, solve for `y`:

\[
y = -\frac{1}{2} x - \frac{1}{2} - 2
\]

\[
y = -\frac{1}{2} x - \frac{1}{2} - \frac{4}{2}
\]

\[
y = -\frac{1}{2} x - \frac{5}{2}
\]

Thus, the equation of the line is:

\[
\boxed{y = -\frac{1}{2} x - \frac{5}{2}}
\]

or, in standard form:

\[
2y = -x - 5
\]

which simplifies to:

\[
x + 2y + 5 = 0
\]

Variations Based on Different Given Lines

The above example assumes the original line is `y = 2x + 3`. If the original line is different, the process remains the same:

1. Find the slope of the original line.

2. Calculate the negative reciprocal for the perpendicular slope.
3. Use the point $(-1, -2)$ and the new slope to write the equation.

Example: Original line $3x - 4y = 7$

Let's walk through this scenario:

Step 1: Convert to slope-intercept form:

$$
3x - 4y = 7
$$
$$
-4y = -3x + 7
$$
$$
y = \frac{3}{4} x - \frac{7}{4}
$$

Step 2: Slope of the original line:

$$
m_1 = \frac{3}{4}
$$

Step 3: Slope of the perpendicular line:

$$
m_2 = -\frac{1}{m_1} = -\frac{1}{\frac{3}{4}} = -\frac{4}{3}
$$

Step 4: Equation passing through $(-1, -2)$:

$$
y - (-2) = -\frac{4}{3} (x - (-1))
$$
$$
y + 2 = -\frac{4}{3} (x + 1)
$$

Distribute:

\[

y + 2 = -\frac{4}{3} x - \frac{4}{3}
\]

Solve for `y`:

\[
y = -\frac{4}{3} x - \frac{4}{3} - 2
\]
\[
y = -\frac{4}{3} x - \frac{4}{3} - \frac{6}{3}
\]
\[
y = -\frac{4}{3} x - \frac{10}{3}
\]

Standard form:

\[
3 y = -4 x - 10
\]
\[
4 x + 3 y + 10 = 0
\]

Additional Tips for Solving Such Problems

- Always convert the given line's equation to slope-intercept form to easily identify the slope.
- Remember that the negative reciprocal is key to perpendicular slopes.
- When working with fractions, clear denominators early to simplify calculations.
- Use the point-slope form for straightforward substitution of the point and slope.
- To write the final answer in standard form, move all terms to one side and simplify.

Summary

Finding the equation of a line passing through a given point and perpendicular to a known line involves understanding the concept of slopes and their relationships. The key steps include:

1. Determine the slope of the given line.
2. Calculate the slope of the perpendicular line as its negative reciprocal.

3. Use the point `( -1, -2 )` and the perpendicular slope in the point-slope form.
4. Simplify to get the line's equation in slope-intercept or standard form.

This methodical approach ensures accuracy and clarity in solving coordinate geometry problems involving perpendicular lines. Whether working with specific equations or general forms, these principles form the foundation for tackling such questions confidently.

Frequently Asked Questions

How do you find the equation of a line passing through the point (-1, -2) and perpendicular to a given line?

First, determine the slope of the given line, then find the negative reciprocal of that slope for the perpendicular line. Using the point (-1, -2) and the perpendicular slope, apply the point-slope form to find the equation.

What is the significance of the negative reciprocal in finding a perpendicular line?

The negative reciprocal of a slope ensures the two lines are perpendicular, as their slopes multiply to -1, indicating they are at right angles.

If the original line's equation is y = 3x + 4, what is the equation of the line passing through (-1, -2) and perpendicular to it?

The slope of the original line is 3, so the perpendicular slope is -1/3. Using point-slope form: y + 2 = -1/3(x + 1), which simplifies to y = -1/3 x - 7/3.

Can I use the slope-intercept form to write the equation of the perpendicular line?

Yes, once you find the slope of the perpendicular line and the point it passes through, you can use the slope-intercept form y = mx + b to write its equation.

What are common mistakes to avoid when finding the perpendicular line passing through a specific point?

Common mistakes include incorrectly calculating the negative reciprocal, using the wrong point in the point-slope formula, or forgetting to simplify the equation properly.

If the original line's equation is in standard form, how do I find the perpendicular line through a point?

Convert the standard form to slope-intercept form to find the slope, then proceed to find the negative reciprocal for the perpendicular line and use the point to write its equation.

Is it necessary to know the original line's equation to find the perpendicular line passing through a point?

Yes, because you need the original line's slope to determine the negative reciprocal for the perpendicular line. Without that, you cannot find the correct slope.

How do I verify that my perpendicular line is correct after finding its equation?

Check that the slopes of both lines are negative reciprocals and that the perpendicular line passes through the given point by substituting the point into your equation.

Additional Resources

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Introduction

In the realm of coordinate geometry, understanding how lines interact is fundamental. One common problem involves determining the equation of a line that passes through a specific point and is perpendicular to a given line. This task not only reinforces core concepts such as slope calculation and point-slope form but also exemplifies the practical use of perpendicularity in geometric and algebraic contexts.

This article delves into the systematic approach to find the equation of a line passing through the point (-1, -2) and perpendicular to a given line. We will explore the theoretical background, detailed step-by-step procedures, and illustrative examples to clarify the process, culminating in a comprehensive understanding suitable for students, educators, and professionals alike.

Theoretical Foundations

Understanding the Slope and Its Significance

The slope of a line, denoted as m, quantifies its steepness and direction. It is calculated as the ratio of the change in y-coordinates to the change in x-coordinates between any two points on the line:

$$
m = \frac{\Delta y}{\Delta x}
$$

A line's slope influences its orientation in the coordinate plane, and it is crucial in formulating its equation.

Perpendicular Lines and Their Slopes

Two lines are perpendicular if they intersect at a right angle (90 degrees). The slopes of perpendicular lines are negative reciprocals of each other, meaning:

$$
m_1 \times m_2 = -1
$$

or equivalently,

$$
m_2 = -\frac{1}{m_1}
$$

This relationship enables us to determine the slope of the line we seek once the slope of the given line is known.

Step-by-Step Methodology

Step 1: Identify the Equation of the Given Line

Before proceeding, the specific details of the original line are necessary. Typically, the line's equation is provided in forms such as:

- Slope-intercept form: $y = mx + c$
- Standard form: $Ax + By + C = 0$

Suppose the given line's equation is:

$$
L: y = m_L x + c
$$

Where:

- m_L is the slope of the given line
- c is the y-intercept

Note: If the line's equation is not explicitly provided, but points on the line are given, the slope can be calculated from those points.

Step 2: Find the Slope of the Perpendicular Line

Using the perpendicularity relationship:

$$
m_P = - \frac{1}{m_L}
$$

where:

- m_P is the slope of the perpendicular line we need to find

Important considerations:

- If $m_L = 0$ (the given line is horizontal), then the perpendicular line is vertical, and its equation is $x = \text{constant}$.
- If m_L is undefined (the given line is vertical), then the perpendicular line is horizontal, with equation $y = \text{constant}$.

Step 3: Formulate the Equation of the Perpendicular Line

Once the slope m_P is known, and the point (-1, -2) is given through which the line passes, the point-slope form of a line's equation is ideal:

$$
y - y_1 = m_P (x - x_1)
$$

Substituting:

$$
x_1 = -1, \quad y_1 = -2
$$

we get:

$$
y + 2 = m_P (x + 1)
$$

which can be rearranged into various forms (slope-intercept, standard, etc.) as needed.

Practical Example

Suppose the original line is:

$$
L: y = 2x + 3
$$

Step 1: Determine the slope of the given line

$$
m_L = 2
$$

Step 2: Find the slope of the perpendicular line

$$
m_P = -\frac{1}{2}
$$

Step 3: Write the equation passing through (-1, -2)

Using point-slope form:

$$
y - (-2) = -\frac{1}{2} (x - (-1))
$$

Simplify:

$$
y + 2 = -\frac{1}{2} (x + 1)
$$

Expand:

$$
y + 2 = -\frac{1}{2} x - \frac{1}{2}
$$

Subtract 2 from both sides:

$$
y = -\frac{1}{2} x - \frac{1}{2} - 2
$$

$$
y = -\frac{1}{2} x - \frac{1}{2} - \frac{4}{2}
$$

$$
y = -\frac{1}{2} x - \frac{5}{2}
$$

Final equation in slope-intercept form:

$$
\boxed{ y = -\frac{1}{2} x - \frac{5}{2} }
$$

Special Cases and Variations

Horizontal and Vertical Lines

- Original line is horizontal ($m_L = 0$): The perpendicular line is vertical, with equation $x = x_1$. For point (-1, -2), the line is:

$$
x = -1

\]

- Original line is vertical (m_L undefined): The perpendicular line is horizontal:

$$
y = y_1 = -2
$$

Multiple Lines and Generalizations

In more complex problems, the original line may be given in different forms or involve multiple segments. The approach remains similar: find the slope, determine the negative reciprocal, and use the point to find the line's equation.

Geometric Interpretation and Significance

Understanding the relationship between perpendicular lines offers vital insights in various fields such as physics, engineering, and computer graphics. For instance, in designing orthogonal coordinate systems, determining perpendicular trajectories, or analyzing geometric constraints, the ability to derive such equations is essential.

Conclusion

The process of finding the equation of a line passing through a specific point and perpendicular to a given line hinges on fundamental principles of slope and perpendicularity. The key steps involve identifying the original line’s slope, calculating its negative reciprocal, and applying the point-slope form with the given point.

This systematic approach not only simplifies the problem-solving process but also deepens comprehension of geometric relationships in the coordinate plane. Mastery of these techniques equips learners and professionals with the tools to analyze complex geometric configurations and solve related algebraic problems effectively.

References

- Stewart, J. (2015). Precalculus: Mathematics for Calculus. Cengage Learning.
- Larson, R., & Edwards, B. H. (2017). Precalculus with Limits. Cengage Learning.

- Katz, V. J. (2014). A History of Mathematics: An Introduction. Addison-Wesley.

Note: For specific or more complex lines, always adapt the methodology accordingly, considering the particular form of the original line's equation.

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