Find The Laplace Transform Of The Following Functions: 1. F(t) = 2t-4t F(s) = 2. F(t) = 4t3/2e-4t F(s)

Find The Laplace Transform Of The Following Functions: 1. $F(t) = 2t - 4t$ $F(s)$ = 2. $F(t) = 4t^{3/2}e^{-4t}$ $F(s)$

When delving into the realm of differential equations and systems analysis, the Laplace transform emerges as an indispensable mathematical tool. It transforms complex functions of time into simpler algebraic expressions in the complex frequency domain, making it easier to analyze, solve, and interpret linear time-invariant systems. Whether you're a student studying engineering, a researcher working on signal processing, or a professional in control systems, understanding how to compute the Laplace transform of various functions is crucial.

In this article, we will explore the process of finding the Laplace transform for specific functions, namely:

1. $F(t) = 2t - 4t$
2. $F(t) = 4t^{3/2}e^{-4t}$

We will analyze each function step-by-step, applying the fundamental rules and properties of Laplace transforms. By the end of this guide, you'll have a clear understanding of how to approach similar problems and interpret their results.

Understanding the Laplace Transform

The Laplace transform of a function F(t), defined for $t \geq 0$, is given by the integral:

$$\mathcal{L}\{F(t)\} = F(s) = \int_0^{\infty} e^{-st} F(t) \, dt$$

where:
- $F(t)$ is the original time-domain function,
- s is a complex variable $s = \sigma + j\omega$,
- $F(s)$ is the transformed function in the s-domain.

The Laplace transform converts differential equations into algebraic equations, simplifies convolution operations, and provides insights into system behavior such as stability and response.

Fundamental Laplace Transform Properties and Rules

Before analyzing specific functions, it's crucial to understand some basic properties and standard transforms:

- **Transform of a constant:** $\mathcal{L}\{a\} = \frac{a}{s}$ for a constant.
- **Transform of t^n:** $\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$, for $n > -1$.
- **Transform of exponential functions:** $\mathcal{L}\{e^{at}f(t)\} = F(s - a)$ if $F(s)$ is the transform of $f(t)$.
- **Transform of derivatives:** $\mathcal{L}\{\frac{d^n}{dt^n}f(t)\} = s^nF(s) - \text{initial conditions}$.

In addition, the linearity property allows us to compute the transform of sums and differences of functions easily.

Calculating the Laplace Transform of F(t) = 2t - 4t

At first glance, the function $F(t) = 2t - 4t$ simplifies to:

$$F(t) = (2t) - (4t) = -2t$$

This simplification makes the calculation straightforward. Let's proceed step-by-step.

Step 1: Simplify the Function

$$F(t) = -2t$$

Step 2: Use Linearity of the Laplace Transform

The Laplace transform of a linear combination:

$$\mathcal{L}\{a f(t) + b g(t)\} = a \mathcal{L}\{f(t)\} + b \mathcal{L}\{g(t)\}$$

Applying this:

$$ \mathcal{L}\{-2t\} = -2 \mathcal{L}\{t\} $$

Step 3: Recall the Standard Transform of t

From standard tables:

$$ \mathcal{L}\{t\} = \frac{1}{s^2} $$

Therefore,

$$ F(s) = -2 \times \frac{1}{s^2} = -\frac{2}{s^2} $$

Final Result:

$$ \boxed{F(s) = -\frac{2}{s^2}} $$

Key takeaway: The Laplace transform of the function $2t - 4t$ simplifies to $-2/s^2$.

Calculating the Laplace Transform of $F(t) = 4t^{3/2}e^{-4t}$

This function combines a power function $t^{3/2}$ with an exponential decay e^{-4t}. Computing its Laplace transform involves more advanced techniques, notably the use of the Laplace transform of functions involving powers of t multiplied by exponentials.

Step 1: Recognize the structure of the function

The function:

$$ F(t) = 4 t^{3/2} e^{-4t} $$

can be viewed as a scaled version of:

$$ f(t) = t^{3/2} e^{-4t} $$

The factor of 4 is a constant multiplier outside the transform.

Step 2: Recall the Laplace transform of $t^{\nu} e^{a t}$

The Laplace transform for $t^{\nu} e^{a t}$, where $\nu > -1$, is given by:

$$\mathcal{L}\{ t^{\nu} e^{a t} \} = \frac{\Gamma(\nu + 1)}{(s - a)^{\nu + 1}}$$

where $\Gamma(\cdot)$ is the Gamma function.

In our case:

- $\nu = \frac{3}{2}$
- $a = -4$

Thus,

$$\mathcal{L}\{ t^{3/2} e^{-4t} \} = \frac{\Gamma(\frac{3}{2} + 1)}{(s + 4)^{\frac{3}{2} + 1}}$$

Note: Since the exponential is e^{-4t}, the a parameter is -4, but the transform involves $s - a$. The standard formula involves $s - a$, so with $a = -4$, we get $s - (-4) = s + 4$.

Step 3: Compute the Gamma function $\Gamma(\frac{5}{2})$

Recall that:

$$\Gamma(n + 1) = n! \quad \text{for integer } n$$

and for half-integers:

$$\Gamma\left(n + \frac{1}{2}\right) = \frac{(2n)!}{4^n n!} \sqrt{\pi}$$

Alternatively, known values:

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

$$\Gamma\left(\frac{3}{2}\right) = \frac{\sqrt{\pi}}{2}$$

$$\Gamma\left(\frac{5}{2}\right) = \frac{3 \sqrt{\pi}}{4}$$

Since:

$$\Gamma\left(\frac{3}{2} + 1\right) = \Gamma\left(\frac{5}{2}\right) = \frac{3 \sqrt{\pi}}{4}$$

Step 4: Write the Laplace transform

Putting it all together:

$$ \mathcal{L}\{ t^{3/2} e^{-4t} \} = \frac{\frac{3 \sqrt{\pi}}{4}}{(s + 4)^{5/2}} $$

Multiplying by 4 (the scalar outside):

$$ \mathcal{L}\{ 4 t^{3/2} e^{-4t} \} = 4 \times \frac{3 \sqrt{\pi}}{4} \times \frac{1}{(s + 4)^{5/2}} = 3 \sqrt{\pi} \times \frac{1}{(s + 4)^{5/2}} $$

Final expression:

$$ \boxed{F(s) = \frac{3 \sqrt{\pi}}{(s + 4)^{5/2}}} $$

Summary of Results

Function	Laplace Transform $F(s)$	Explanation
$F(t) = 2t - 4t$	$-\frac{2}{s^2}$	Simplifies to $-2t$, standard transform applies.
$F(t) = 4 t^{3/2} e^{-4t}$	$\frac{3 \sqrt{\pi}}{(s + 4)^{5/2}}$	Uses the Gamma function and properties of

Frequently Asked Questions

What is the Laplace transform of the function F(t) = 2t?

The Laplace transform of F(t) = 2t is F(s) = 2 / s^2.

How do you find the Laplace transform of F(t) = 4t^{3/2} e^{-4t}?

The Laplace transform is F(s) = Γ(5/2) / (s + 4)^{5/2}, where Γ is the gamma function.

Why is the Laplace transform of F(t) = 2t equal to 2 / s^2?

Because the Laplace transform of t^n is n! / s^{n+1}; for n=1, it becomes 1! / s^2, so scaled by 2, it is 2 / s^2.

What is the Laplace transform of $F(t) = 4t^{3/2} e^{-4t}$?

It is $F(s) = (3/4)\sqrt{\pi} / (s + 4)^{5/2}$, derived using the Laplace transform of $t^{n} e^{at}$ functions.

How does the exponential term e^{-4t} affect the Laplace transform of the given functions?

The exponential term e^{-4t} shifts the Laplace transform by replacing s with s + 4, resulting in factors of (s + 4) in the denominator.

Additional Resources

Find The Laplace Transform Of The Following Functions: 1. $F(t) = 2t - 4t$, $F(s) =$ 2. $F(t) = 4t^{3/2} e^{-4t}$ $F(s)$

Understanding the process of finding the Laplace transform of various functions is fundamental in solving differential equations, analyzing systems, and transforming complex functions into algebraic expressions. This article provides a comprehensive review of how to determine the Laplace transform for specific functions, focusing on two illustrative examples: $F(t) = 2t - 4t$ and $F(t) = 4t^{3/2} e^{-4t}$. Through detailed explanations, step-by-step derivations, and feature analysis, readers will gain a clear understanding of the methods involved and the significance of these transforms in mathematical and engineering contexts.

Introduction to Laplace Transform

The Laplace transform is a powerful integral transform used to convert functions of time, typically denoting physical signals or system responses, into functions of complex frequency (s-plane). Defined by:

$$\mathcal{L}\{f(t)\} = F(s) = \int_{0}^{\infty} e^{-st}f(t)\, dt$$

it simplifies the process of solving linear differential equations by turning derivatives into algebraic terms. Its applications extend across engineering, physics, and mathematics, especially in control systems, circuit analysis, and signal processing.

Laplace Transform of Basic Functions

Before tackling the specific functions, it’s helpful to recall standard Laplace transforms:

- Transform of t^n: $\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$
- Transform of e^{at}: $\mathcal{L}\{e^{at}\} = \frac{1}{s - a}$
- Transform of t^n e^{at}: $\mathcal{L}\{t^n e^{at}\} = \frac{n!}{(s - a)^{n+1}}$

These basic formulas serve as building blocks for transforming more complex functions.

Transforming F(t) = 2t - 4t

Simplification of the Function

The given function simplifies straightforwardly:

$$F(t) = 2t - 4t = -2t$$

Thus, the problem reduces to finding the Laplace transform of $-2t$.

Applying the Linearity Property

The Laplace transform is linear, meaning:

$$\mathcal{L}\{a f(t) + b g(t)\} = a \mathcal{L}\{f(t)\} + b \mathcal{L}\{g(t)\}$$

Applying this:

$$\mathcal{L}\{-2t\} = -2 \mathcal{L}\{t\}$$

From the standard transforms,

$$\mathcal{L}\{t\} = \frac{1}{s^{2}}$$

therefore,

$$F(s) = -2 \times \frac{1}{s^{2}} = -\frac{2}{s^{2}}$$

Final Result for F(t) = 2t - 4t

$$
\boxed{
F(s) = -\frac{2}{s^{2}}
}
$$

Features:

- Simple to compute due to the linearity property.
- Useful in cases where the time domain function simplifies to a linear term.
- Demonstrates how basic functions translate into algebraic forms.

Pros & Cons:

- Pros:
- Straightforward calculation.
- Quick application of standard transforms.
- Cons:
- Limited to functions directly expressible as polynomials or exponentials.

Transforming F(t) = 4t^{3/2} e^{-4t}

Identifying the Function Type

This function combines a fractional power of t with an exponential decay:

$$ F(t) = 4 t^{3/2} e^{-4t} $$

This suggests the use of the generalized Laplace transform for functions involving powers of t multiplied by exponentials.

Recall of the Standard Formula

For functions of the form:

$$ t^{\alpha} e^{a t} $$

the Laplace transform is:

$$ \mathcal{L}\{ t^{\alpha} e^{a t} \} = \frac{\Gamma(\alpha + 1)}{(s - a)^{\alpha + 1}} $$

where $\Gamma(\cdot)$ is the Gamma function.

Applying the Formula

Given $\alpha = 3/2$ and $a = -4$, the transform becomes:

$$ \mathcal{L}\{ 4 t^{3/2} e^{-4t} \} = 4 \times \frac{\Gamma(3/2 + 1)}{(s + 4)^{3/2 + 1}} $$

Calculating $\Gamma(5/2)$:

$$
\Gamma\left(\frac{5}{2}\right) = \frac{3}{2} \times \frac{1}{2} \times \sqrt{\pi} = \frac{3}{4} \sqrt{\pi}
$$

Thus,

$$
F(s) = 4 \times \frac{\frac{3}{4} \sqrt{\pi}}{(s + 4)^{5/2}} = \frac{3 \sqrt{\pi}}{(s + 4)^{5/2}}
$$

Final Expression for F(s)

$$
\boxed{
F(s) = \frac{3 \sqrt{\pi}}{(s + 4)^{5/2}}
}
$$

Features:

- Incorporates the Gamma function, which extends factorials to non-integer values.
- Handles fractional powers of t, broadening the scope of Laplace transforms.
- Useful in modeling systems with non-integer power laws and exponential decay.

Pros & Cons:

- Pros:
- Capable of transforming complex, non-integer power functions.
- Provides a direct formula involving the Gamma function.
- Cons:
- Requires familiarity with the Gamma function.
- Slightly more complex calculation compared to elementary transforms.

Comparison and Practical Considerations

Understanding the differences in transforming these functions highlights the versatility of the Laplace transform:

- Linear functions like t are straightforward, with well-known transforms.
- Functions involving fractional powers require the Gamma function, expanding the transform's applicability.
- Exponential functions modify the s-plane shift, often simplifying the transforms.
- When combining these features, the transform formulas adapt accordingly.

Practical tips:

- Always simplify the time domain function first.
- Use linearity to split complex functions.
- Recall standard transforms and their extensions involving Gamma functions.
- Pay attention to the domain of the functions, especially when fractional powers are involved.
- For more complex functions, consider consulting tables or software tools like MATLAB or WolframAlpha.

Conclusion

Finding the Laplace transform of specific functions is a fundamental skill that unlocks powerful methods for solving differential equations and analyzing systems. In the case of $F(t) = 2t - 4t$, the transform simplifies neatly to $-2/s^{2}$, illustrating the ease of handling polynomial functions. Conversely, for $F(t) = 4t^{3/2} e^{-4t}$, the use of the Gamma function encapsulates the transform of fractional powers combined with exponentials, resulting in $\frac{3 \sqrt{\pi}}{(s + 4)^{5/2}}$.

Mastering these techniques enhances analytical capabilities across various scientific and engineering disciplines. The key is understanding when standard formulas apply and how to adapt them for more complex functions, leveraging properties like linearity and special functions such as the Gamma function. With practice, transforming diverse functions becomes an intuitive process, serving as a vital tool in mathematical analysis.

Note: For complex functions not directly covered, consider the use of Laplace transform tables, symbolic computation tools, or integral definitions to derive the transforms explicitly.

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