Find The Matrix A' For T Relative To The Basis B'. T: R2 R2, T(x, Y) = (4x Y, 3x), B' = {(-2, 1), (-1,

Find The Matrix A' For T Relative To The Basis B'. T: $R^2 \rightarrow R^2$, T(x, y) = (4x + y, 3x), B' = {(-2, 1), (-1, ?)}. (Note: The basis B' appears incomplete; assuming the second vector is (-1, 0) for clarity and completeness. If the original basis is different, please specify.)

Understanding the Problem: Finding the Matrix A' Relative To a Basis

Before diving into calculations, it’s essential to understand what it means to find a matrix relative to a basis. In linear algebra, a linear transformation T from R^2 to R^2 can be represented by a 2×2 matrix A when using the standard basis. However, when the basis is different, the matrix that represents T with respect to that new basis (here called B') is denoted as A'.

Key concepts:
- Linear transformation T: A function that maps vectors from R^2 to R^2 following specific rules.
- Standard basis: The usual basis for R^2, consisting of vectors (1, 0) and (0, 1).
- Alternative basis B': A set of vectors that form a basis for R^2, but are different from the standard basis.
- Change of basis: The process of translating the matrix representation from the standard basis to another basis B'.

Step 1: Clarifying the Basis B'

The problem states B' = {(-2, 1), (-1, ?)}. There's an incomplete vector; assuming the second vector is (-1, 0) for this explanation. If the actual basis vector differs, adjust accordingly.

Assumed basis B':
- v_1 = (-2, 1)
- v_2 = (-1, 0)

Why this assumption?

Because a basis in R^2 requires two linearly independent vectors. The first is clear; the second vector must be linearly independent from the first and in R^2.

Check for linear independence:
- Are v_1 and v_2 linearly independent?
- Compute the determinant: $(-2)(0) - (1)(-1) = 0 + 1 = 1 \neq 0$.
- Since the determinant is non-zero, the vectors are linearly independent, confirming they form a basis.

Step 2: Understanding the Transformation T

Given $T(x, y) = (4x + y, 3x)$.
- The transformation takes a vector (x, y) and maps it to a new vector in R^2.
- To express T relative to basis B', we need to determine how T acts on the basis vectors v_1 and v_2.

Step 3: Computing T on Basis Vectors

To find the matrix A' representing T relative to B', you need to express $T(v_1)$ and $T(v_2)$ in terms of the basis vectors B'.

Calculations:

1. $T(v_1)$:
- $v_1 = (-2, 1)$
- $T(-2, 1) = (4(-2) + 1, 3(-2)) = (-8 + 1, -6) = (-7, -6)$

2. $T(v_2)$:
- $v_2 = (-1, 0)$
- $T(-1, 0) = (4(-1) + 0, 3(-1)) = (-4, -3)$

Step 4: Expressing $T(v_1)$ and $T(v_2)$ in Terms of B'

To find the matrix A' relative to basis B', you need to express each T(v) as a linear combination of v_1 and v_2.

Expressing $T(v_1) = (-7, -6)$:
Find scalars a_1 and a_2 such that:
$(-7, -6) = a_1 (-2, 1) + a_2 (-1, 0)$

This gives the system:
- First component: $-7 = a_1 (-2) + a_2 (-1)$
- Second component: $-6 = a_1\ 1 + a_2\ 0$

From the second equation:
$a_1 = -6$

Substitute into the first:
$-7 = (-6)(-2) + a_2 (-1) \to$
$-7 = 12 - a_2 \to$
$a_2 = 12 + 7 = 19$

Expressing $T(v_2) = (-4, -3)$:
Find b_1 and b_2 such that:
$(-4, -3) = b_1 (-2, 1) + b_2 (-1, 0)$

Set up the system:
- First component: $-4 = b_1 (-2) + b_2 (-1)$
- Second component: $-3 = b_1\ 1 + b_2\ 0$

From the second:
$b_1 = -3$

Substitute into the first:
$-4 = (-3)(-2) + b_2 (-1) \to$
$-4 = 6 - b_2 \to$
$b_2 = 6 + 4 = 10$

Step 5: Constructing the Matrix A'

The matrix A' relative to basis B' has columns corresponding to the images of basis vectors expressed in basis B'.

Columns of A':
- First column: coefficients of $T(v_1)$ in basis B' = $[a_1, a_2]$ = [-6, 19]
- Second column: coefficients of $T(v_2)$ in basis B' = $[b_1, b_2]$ = [-3, 10]

Therefore, the matrix A' is:

$$
A' = \begin{bmatrix}
-6 & -3 \\
19 & 10
\end{bmatrix}
$$

This matrix allows you to compute T for any vector expressed in basis B' by multiplying the coordinate vector by A'.

Step 6: Validating the Result

To ensure correctness, verify that the transformation applied via A' matches T's action on vectors expressed in basis B'.

Example:
Express a vector in B' coordinates, for example, (1, 0) in B' basis corresponds to v_1.
- $T(v_1)$ in B' coordinates: multiply A' by $[1, 0]^T$:
$$
A' \times \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -6 \\ 19 \end{bmatrix}
$$
- This matches the coefficients found earlier for $T(v_1)$.

Similarly, for v_2:
$$
A' \times \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 10 \end{bmatrix}
$$
which matches the earlier calculations.

Summary and Final Remarks

- To find the matrix A' of a linear transformation T relative to a basis B', follow these steps:
1. Identify the basis vectors of B'.
2. Compute T on each basis vector.
3. Express the images T(v) as linear combinations of basis vectors in B'.
4. Formulate the matrix A' with columns as the coefficients from step 3.

- The process involves solving systems of linear equations to express transformed basis vectors back in terms of the basis B'.

- This approach generalizes to higher dimensions and other transformations, making it a fundamental skill in linear algebra.

Note: If the basis vectors or the transformation T are different from those assumed here, adjust calculations accordingly. The key is understanding how to express T(v) in the basis B' and then form the corresponding matrix A'.

Conclusion:

Finding the matrix A' for T relative to the basis B' involves a systematic process of applying the transformation to basis vectors, expressing the results in the basis, and constructing the matrix from these coordinate representations. Mastering this process enhances understanding of linear transformations and change of basis, essential concepts in linear algebra and its applications in various scientific and engineering fields.

Frequently Asked Questions

How do I find the matrix A of the linear transformation T relative to the basis B' = {(-2, 1), (-1, ?)}?

To find the matrix A, express T applied to each basis vector as linear combinations of the basis vectors, then form the columns of A from these coordinates.

What steps are involved in computing the matrix of T relative to basis

B'?

First, compute T for each basis vector, then solve for the coefficients of these images in terms of B', and finally arrange these coefficients as columns in the matrix A.

Given T(x, y) = (4x + y, 3x) and B' = {(-2, 1), (-1, ?)}, how do I determine the matrix A?

Calculate T applied to each basis vector, then express each result as a linear combination of the basis vectors. The coefficients form the columns of the matrix A.

How can I verify that the matrix A correctly represents T relative to B'?

Apply T to the basis vectors, express the results in terms of B', and check that these coordinate vectors match the columns of A.

Are there any special considerations when the basis vectors are not standard basis vectors?

Yes, you need to express T's output in terms of the given basis vectors, which involves solving linear equations to find the coordinate vectors relative to B'.

Can I use matrix transformation formulas to find A directly, or do I need to do coordinate expansion?

You need to compute the images of the basis vectors under T and then find their coordinates relative to the basis B'. This often involves solving linear systems unless the basis is standard.

Additional Resources

Understanding the Problem: Finding the Matrix \(A'\) for the linear transformation \(T\) relative to the basis \(B'\)

When dealing with linear transformations and bases in linear algebra, one of the fundamental tasks is to find the matrix representation of a linear transformation relative to a specific basis. This process involves understanding how the transformation acts on basis vectors and then expressing the images in terms of the basis vectors themselves. In this detailed review, we will explore this process step-by-step for the given transformation \(T\) and basis \(B'\).

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1. Overview of the Transformation T and Basis B'

1.1 The Linear Transformation T

Given:

$$
T: \mathbb{R}^2 \to \mathbb{R}^2, \quad T(x, y) = (4x + y, 3x)
$$

This transformation takes a vector (x, y) in $\mathbb{R}^2$ and maps it to another vector in $\mathbb{R}^2$. Let's analyze its properties:

- Linearity: T is linear because it satisfies the properties of additivity and scalar multiplication.
- Matrix form in standard basis: To find the matrix representation of T with respect to the standard basis, we evaluate T on the basis vectors $(1, 0)$ and $(0, 1)$:

$$
T(1, 0) = (4 \cdot 1 + 0, 3 \cdot 1) = (4, 3)
$$

$$
T(0, 1) = (4 \cdot 0 + 1, 3 \cdot 0) = (1, 0)
$$

So, in the standard basis, the matrix representation A is:

$$
A = \begin{bmatrix}
4 & 1 \\
3 & 0
\end{bmatrix}
$$

1.2 The Basis B'

Given basis:

$$
B' = \{ \mathbf{v}_1, \mathbf{v}_2 \} = \{ (-2, 1), (-1, \ast) \}
$$

Note: There appears to be an incomplete vector $(-1,)$. For completeness, let's assume the second vector is $(-1, 0)$. If the original problem specifies differently, please adjust accordingly. For this review, we'll proceed with:

$$
B' = \{ \mathbf{v}_1 = (-2, 1), \mathbf{v}_2 = (-1, 0) \}
$$

This basis is a set of two linearly independent vectors, which span $\mathbb{R}^2$.

---

2. The Goal: Find the Matrix A' of T Relative to B'

2.1 Conceptual Framework

The matrix A' represents the linear transformation T relative to the basis B'. Formally, for a linear transformation $T: V \to V$ and a basis $B' = \{\mathbf{v}_1, \mathbf{v}_2\}$:

$$
A' = [[T(\mathbf{v}_1)]_{B'} \quad [T(\mathbf{v}_2)]_{B'}]
$$

where $[T(\mathbf{v}_i)]_{B'}$ is the coordinate vector of $T(\mathbf{v}_i)$ expressed in the basis B'.

2.2 Step-by-step Approach

The process involves:

1. Compute $T(\mathbf{v}_1)$ and $T(\mathbf{v}_2)$ in $\mathbb{R}^2$.
2. Express these images as linear combinations of the basis vectors B'.
3. Identify the coordinate vectors of these images relative to B'.
4. Assemble these coordinate vectors as columns of A'.

---

3. Calculating $T(\mathbf{v}_1)$ and $T(\mathbf{v}_2)$

3.1 Image of $\mathbf{v}_1 = (-2, 1)$

Applying T:

$$
T(-2, 1) = (4 \cdot (-2) + 1, 3 \cdot (-2)) = (-8 + 1, -6) = (-7, -6)
$$

3.2 Image of $\mathbf{v}_2 = (-1, 0)$

Applying T:

$$
T(-1, 0) = (4 \cdot (-1) + 0, 3 \cdot (-1)) = (-4, -3)
$$

---

4. Expressing $T(\mathbf{v}_i)$ in Terms of B'

4.1 Expressing $T(\mathbf{v}_1) = (-7, -6)$

Find scalars a_1, a_2 such that:

$$
(-7, -6) = a_1(-2, 1) + a_2(-1, 0)
$$

This leads to the system:

$$
-7 = -2a_1 - a_2 \quad (1)
$$

$$
-6 = a_1 + 0 \quad (2)
$$

From equation (2):

$$
a_1 = -6
$$

Substitute into equation (1):

$$
-7 = -2(-6) - a_2 \Rightarrow -7 = 12 - a_2 \Rightarrow a_2 = 12 + 7 = 19
$$

Thus:

$$
T(\mathbf{v}_1) = -6 \mathbf{v}_1 + 19 \mathbf{v}_2
$$

4.2 Expressing $T(\mathbf{v}_2) = (-4, -3)$

Find scalars b_1, b_2 such that:

$$
(-4, -3) = b_1(-2, 1) + b_2(-1, 0)
$$

This leads to:

$$
-4 = -2b_1 - b_2 \quad (1)
$$
$$
-3 = b_1 + 0 \quad (2)
$$

From equation (2):

$$
b_1 = -3
$$

Substitute into (1):

$$
-4 = -2(-3) - b_2 \Rightarrow -4 = 6 - b_2 \Rightarrow b_2 = 6 + 4 = 10
$$

Therefore:

$$
T(\mathbf{v}_2) = -3 \mathbf{v}_1 + 10 \mathbf{v}_2
$$

\]

---

5. Constructing the Matrix A'

5.1 Coordinate Vectors of $T(\mathbf{v}_i)$

The coordinate vectors relative to B':

$$
[T(\mathbf{v}_1)]_{B'} = \begin{bmatrix} -6 \\ 19 \end{bmatrix}
$$

$$
[T(\mathbf{v}_2)]_{B'} = \begin{bmatrix} -3 \\ 10 \end{bmatrix}
$$

5.2 Forming A'

The matrix A' has these coordinate vectors as its columns:

$$
A' = \begin{bmatrix}
-6 & -3 \\
19 & 10
\end{bmatrix}
$$

This matrix represents the transformation T with respect to the basis B'.

---

6. Additional Considerations and Validations

6.1 Verifying the Basis and Transformation

- Linearity: Since T is linear, the process of expressing images in the basis B' is consistent.
- Invertibility of B': To ensure that the basis vectors are linearly independent, check the determinant:

$$
\det \begin{bmatrix}
-2 & -1 \\
1 & 0
\end{bmatrix} = (-2)(0) - (-1)(1) = 0 + 1 = 1 \neq 0
$$

Thus, B' is indeed a basis for $\mathbb{R}^2$.

6.2 Interpretation of the Result

The matrix A' provides a way to understand how T acts on vectors expressed in the basis B'. If you have a vector $\mathbf{v}$ in $\mathbb{R}^2$, and its coordinate vector in B' is $\mathbf{c}$, then the coordinate vector of $T(\mathbf{v})$ relative to B' is:

$$
A' \mathbf{c}
$$

This is especially useful in change-of-basis scenarios, simplifying the computation of transformations

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