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Understanding the Problem: Finding the Closest Point on a Line to a Given Point

When working with geometry and coordinate systems, a common problem involves finding the point on a specific line that is closest to a given external point. This problem not only tests your understanding of basic algebra but also involves concepts from coordinate geometry, such as distance calculations and optimization techniques.

In this article, we will explore how to find the point on the line $y = 6x + 9$ that is closest to the point $(3, 1)$. The process involves expressing the square of the distance between a generic point on the line and the given point, then minimizing this expression to find the optimal point.

Key Concepts and Terminology

Before diving into the calculations, let's clarify some key concepts:

- **Distance between two points:** Given points (x_1, y_1) and (x_2, y_2), the Euclidean distance is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
- **Closest point on a line:** The point on the line that minimizes the distance to a given external point.
- **Optimization:** The process of finding the minimum (or maximum) of a function, often by taking derivatives and setting them to zero.
- **Expressing the square of the distance:** To avoid dealing with square roots, we often minimize the squared distance, which simplifies calculations.

Step-by-Step Solution to Find the Closest Point

Let's now break down the process into manageable steps.

Step 1: Express the generic point on the line

Since the line is given by:

$$
y = 6x + 9
$$

any point P on the line can be represented as:

$$
P(x, y) = (x, 6x + 9)
$$

where x is a real number.

Step 2: Write the squared distance from P to $(3, 1)$

The point Q is $(3, 1)$. The squared distance D^2 between $P(x, 6x + 9)$ and $Q(3, 1)$ is:

$$
D^2(x) = (x - 3)^2 + \left((6x + 9) - 1 \right)^2
$$

Simplify the expression inside the second term:

$$
(6x + 9) - 1 = 6x + 8
$$

Thus,

$$
D^2(x) = (x - 3)^2 + (6x + 8)^2
$$

Step 3: Expand and simplify $D^2(x)$

Let's expand both terms:

$$
(x - 3)^2 = x^2 - 6x + 9
$$

$$
(6x + 8)^2 = 36x^2 + 96x + 64
$$

Adding these together:

$$
D^2(x) = x^2 - 6x + 9 + 36x^2 + 96x + 64
$$

Combine like terms:

$$
D^2(x) = (x^2 + 36x^2) + (-6x + 96x) + (9 + 64) = 37x^2 + 90x + 73
$$

Step 4: Minimize $D^2(x)$ to find the closest point

To find the value of x that minimizes $D^2(x)$, take the derivative with respect to x and set it to zero:

$$
\frac{d}{dx} D^2(x) = 74x + 90
$$

Set derivative equal to zero:

$$
74x + 90 = 0
$$

Solve for x:

$$
x = -\frac{90}{74} = -\frac{45}{37}
$$

Step 5: Find the corresponding y-coordinate

Recall that the line's equation is:

$$
y = 6x + 9

Substitute $x = -\frac{45}{37}$:

$$
y = 6 \times \left(-\frac{45}{37}\right) + 9 = -\frac{270}{37} + 9
$$

Express 9 as a fraction with denominator 37:

$$
9 = \frac{333}{37}
$$

Therefore,

$$
y = -\frac{270}{37} + \frac{333}{37} = \frac{63}{37}
$$

The Closest Point on the Line to (3, 1)

Putting it all together, the point P on the line $y = 6x + 9$ closest to $(3, 1)$ is:

$$
P\left(-\frac{45}{37}, \frac{63}{37} \right)
$$

This point minimizes the distance between the line and the external point.

Additional Insights and Visualization

Understanding the geometric interpretation can help solidify the concept. The closest point on a line to an external point is where the perpendicular from the point intersects the line. In this case, the process of minimizing the squared distance function effectively finds this intersection point.

Visual Explanation:

- The point $(3, 1)$ lies outside the line.
- The shortest distance from $(3, 1)$ to the line is along the perpendicular.
- The derived point $\left(-\frac{45}{37}, \frac{63}{37} \right)$ is exactly where this perpendicular intersects the line.

Graphical Representation:

Plotting the line $y = 6x + 9$ and the point $(3, 1)$, then drawing the perpendicular from $(3, 1)$ to the line, visually confirms the solution.

Summary of the Methodology

To summarize, the key steps involved in finding the closest point on a line to an external point are:

1. Parameterize the generic point on the line.
2. Express the squared distance between this point and the external point.
3. Expand and simplify the squared distance expression.
4. Minimize the squared distance by differentiating and solving for the critical point.
5. Substitute back to find the coordinates of the closest point.

This approach can be applied to various lines and external points, making it a versatile method in coordinate geometry.

Additional Tips for Solving Similar Problems

- Always consider expressing the squared distance to avoid dealing with square roots.
- Use derivatives to find minimum or maximum points efficiently.
- When possible, verify the solution by checking the perpendicular from the external point to the line.
- Visualize the problem graphically for better intuition.

Conclusion

Finding the point on a line closest to a given external point is a fundamental problem in coordinate geometry, combining algebraic manipulation and calculus techniques. By expressing the squared distance and minimizing it, we obtain a precise solution that can be easily verified and visualized.

In our specific example, the closest point on the line $y=6x+9$ to $(3,1)$ is:

$$
\boxed{\left(-\frac{45}{37}, \frac{63}{37} \right)}
$$

This method showcases a powerful strategy for tackling a wide range of similar geometric optimization problems.

Keywords: closest point on a line, coordinate geometry, squared distance minimization, optimization, algebra, calculus, geometric problem solving, line equation, external point, perpendicular distance.

Frequently Asked Questions

How do I find the point on the line y=6x+9 that is closest to the point (3,1)?

To find the closest point, you can minimize the distance between (3,1) and any point (x, y) on the line. Since y=6x+9, you set up the distance formula and minimize its square to avoid square roots, leading to solving for x.

What is the significance of expressing the square of the distance in solving for the closest point?

Expressing the square of the distance simplifies calculations by removing the square root, making differentiation easier and avoiding extraneous solutions, ultimately helping to find the minimum distance.

Can you outline the steps to find the closest point on y=6x+9 to (3,1)?

Yes. First, write the distance squared between (x, 6x+9) and (3,1). Then, differentiate with respect to x, set the derivative to zero to find critical points, and solve for x. Finally, substitute x back into the line equation to find the y-coordinate.

What is the formula for the distance squared between (x, y) on y=6x+9 and point (3,1)?

The squared distance $D^2 = (x - 3)^2 + (y - 1)^2$. Since y=6x+9, it becomes $D^2 = (x - 3)^2 + (6x + 9 - 1)^2 = (x - 3)^2 + (6x + 8)^2$.

What is the final closest point on y=6x+9 to (3,1)?

After solving, the closest point is approximately (0.5, 60.5 + 9) = (0.5, 12), which minimizes the distance to (3,1).

Additional Resources

Closest Point on the Line Y=6x+9 to the Point (3,1): An In-Depth Geometric Exploration

When it comes to analytical geometry, understanding the relationship between points and lines is fundamental. Whether you're a student seeking to excel in your coursework or a professional applying these principles in real-world scenarios, mastering how to find the closest point on a line to a given point is invaluable. Today, we delve into this classic problem: Find the point on the line y=6x+9 that is closest to the point (3,1). We'll explore the problem comprehensively, emphasizing the method of expressing the square of the distance, which simplifies calculations and enhances understanding.

Understanding the Geometric Context

Before diving into the solution, it's essential to grasp the geometric setup:

- Line Equation: $y = 6x + 9$
- External Point: $P = (3, 1)$
- Objective: Find point $Q = (x, y)$ on the line that minimizes the distance to P.

This problem is a classic example of finding the shortest distance from a point to a line, a task frequently encountered in fields such as computer graphics, navigation, and optimization.

Mathematical Foundations: Distance and Optimization

The crux of the problem lies in minimizing the distance between P and points on the line. The standard approach involves:

1. Expressing the squared distance between P and Q to avoid dealing with square roots, which simplifies differentiation.
2. Using calculus to find the minimum of this quadratic function.

Why squared distance?
The Euclidean distance between $P = (3,1)$ and $Q = (x, y)$ is:

$$ D = \sqrt{(x - 3)^2 + (y - 1)^2} $$

Minimizing D is equivalent to minimizing D^2:

$$ D^2 = (x - 3)^2 + (y - 1)^2 $$

This approach simplifies differentiation, as the square root complicates the derivative but squaring it yields a straightforward polynomial.

Step-by-Step Solution Process

1. Expressing y in terms of x

Given the line y = 6x + 9, the point Q on the line can be represented solely by x:

$$ y = 6x + 9 $$

Substituting into the squared distance formula:

$$ D^2(x) = (x - 3)^2 + (6x + 9 - 1)^2 $$

Simplify the second term:

$$ 6x + 9 - 1 = 6x + 8 $$

So,

$$ D^2(x) = (x - 3)^2 + (6x + 8)^2 $$

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2. Expanding and simplifying $D^2(x)$

Let's expand both parts:

- Expand $(x - 3)^2$:

$$ (x - 3)^2 = x^2 - 6x + 9 $$

- Expand $(6x + 8)^2$:

$$ (6x + 8)^2 = 36x^2 + 96x + 64 $$

Now, combine these:

$$
D^2(x) = x^2 - 6x + 9 + 36x^2 + 96x + 64
$$

Group like terms:

$$
D^2(x) = (x^2 + 36x^2) + (-6x + 96x) + (9 + 64)
$$

$$
D^2(x) = 37x^2 + 90x + 73
$$

This is a quadratic function in x that we want to minimize.

---

3. Finding the critical point by differentiation

To find the minimum of $D^2(x)$, differentiate with respect to x and set the derivative to zero:

$$
\frac{d}{dx} D^2(x) = 74x + 90
$$

Set equal to zero:

$$
74x + 90 = 0
$$

$$
x = -\frac{90}{74} = -\frac{45}{37}
$$

---

4. Calculating the corresponding y-coordinate

Recall the line equation:

$$
y = 6x + 9
$$

Substitute $x = -\frac{45}{37}$:

$$
y = 6 \times \left(-\frac{45}{37}\right) + 9
$$

Simplify:

$$
y = -\frac{270}{37} + 9
$$

Express 9 as a fraction with denominator 37:

$$
9 = \frac{333}{37}
$$

Now, combine:

$$
y = -\frac{270}{37} + \frac{333}{37} = \frac{63}{37}
$$

Final Answer: The Closest Point

The point on the line y=6x+9 that is closest to (3,1) is:

$$
Q = \left(-\frac{45}{37}, \frac{63}{37}\right)
$$

Expressed as decimals:

$$
x \approx -1.216, \quad y \approx 1.703
$$

Verifying the Solution

To confirm, we can compute the squared distance at this point:

$$
D^2\left(-\frac{45}{37}\right) = 37 \left(-\frac{45}{37}\right)^2 + 90 \left(-\frac{45}{37}\right) + 73
$$

Calculate step-by-step:

- Square $x = -\frac{45}{37}$:

$$
x^2 = \frac{2025}{1369}
$$

- Multiply by 37:

$$
37 \times \frac{2025}{1369} = \frac{37 \times 2025}{1369} = \frac{74,925}{1369}
$$

- Compute $90x$:

$$
90 \times -\frac{45}{37} = -\frac{4050}{37}
$$

Express all with common denominator 1369:

$$
-\frac{4050}{37} = -\frac{4050 \times 37}{1369} = -\frac{149,850}{1369}
$$

Now, sum all:

$$
D^2 = \frac{74,925}{1369} - \frac{149,850}{1369} + \frac{73 \times 1369}{1369}
$$

Note that $73 \times 1369 = 73 \times 1369$:

$$
73 \times 1369 = (70 + 3) \times 1369 = 70 \times 1369 + 3 \times 1369 = 95,830 + 4,107 = 99,937
$$

Thus:

$$
D^2 = \frac{74,925 - 149,850 + 99,937}{1369} = \frac{(74,925 + 99,937) - 149,850}{1369}
$$

Calculate numerator:

$$
74,925 + 99,937 = 174,862
$$

Subtract 149,850:

$$
174,862 - 149,850 = 25,012
$$

Therefore,

$$
D^2 = \frac{25,012}{1369} \approx 18.26
$$

Taking the square root:

$$
D \approx \sqrt{18.26} \approx 4.28
$$

This is the shortest distance from the point (3,1) to the line, confirming the correctness of the point.

Practical Implications and Applications

Understanding how to find the closest point on a line to a given point isn’t merely an academic exercise—it has real-world applications:

- Navigation Systems: Calculating the shortest route or closest approach to a route line.
- Robotics: Ensuring a robot maintains the shortest path to a trajectory.
- Computer Graphics: Determining the minimal distance from a cursor to a shape boundary.
- Data Analysis: Finding the closest data point on a regression line.

The method of expressing the squared distance function simplifies the calculus involved, making the problem more approachable and ensuring precise results.

Summary and Key Takeaways

- The problem involves finding the point on a line closest to an external point, a fundamental concept in geometry.
- Expressing the squared distance is an effective strategy to simplify differentiation.
- The solution involves:
- Substituting the line equation into the distance formula.
- Expanding and simplifying to obtain a quadratic function.
- Differentiating and setting the derivative to zero to find the minimum.
- Plugging the x-value back into the line's equation to find the y-coordinate.
- The closest point to (3,1) on $y=6x+9$ is $\left(-\frac{45}{37}, \frac{63}{37}\right)$.

This comprehensive exploration underscores the elegance and utility of analytical geometry techniques, fostering a deeper appreciation for their role in both theoretical and applied

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