Find The Slope Of The Tangent Line To The Curve 6 Sin (x)+5 Cos (y)-2 Sin (x) Cos (y)+x=3 At The Point

Find The Slope Of The Tangent Line To The Curve 6 Sin (x)+5 Cos (y)-2 Sin (x) Cos (y)+x=3 At The Point is a fundamental problem in calculus that involves determining the rate at which the curve changes at a specific point. This task requires understanding implicit differentiation, multivariable calculus, and the application of derivative rules to complex trigonometric functions. Whether you're a student preparing for exams, a teacher designing lessons, or a professional solving real-world problems involving curves and slopes, mastering how to find the tangent line slope at a given point is essential. In this comprehensive guide, we will explore the step-by-step process, delve into the mathematical principles involved, and provide tips to optimize your understanding and application of this technique.

Understanding the Problem: The Equation of the Curve

Before calculating the slope of the tangent line, it’s important to understand the nature of the given curve:

Equation:

$$ 6 \sin(x) + 5 \cos(y) - 2 \sin(x) \cos(y) + x = 3 $$

This is an implicitly defined curve involving both x and y. Unlike explicit functions $y = f(x)$, here y is not isolated on one side; instead, the relationship between x and y is given by the equation above.

Key points to consider:

- The equation involves multiple trigonometric functions.
- Both x and y are variables; the goal is to find $\frac{dy}{dx}$ at a specific point.
- The curve may be complex, requiring implicit differentiation.

Fundamentals of Implicit Differentiation

Implicit differentiation allows us to find derivatives of functions where y is defined implicitly in terms of x. The core idea:

- Differentiate both sides of the equation with respect to x.
- Treat y as a function of x, so derivatives of terms involving y include $\frac{dy}{dx}$.

- Solve for $\frac{dy}{dx}$ to find the slope of the tangent line.

Basic derivative rules involved:

- $\frac{d}{dx} [\sin(x)] = \cos(x)$
- $\frac{d}{dx} [\cos(y)] = -\sin(y) \frac{dy}{dx}$
- $\frac{d}{dx} [\sin(x) \cos(y)] = \cos(x) \cos(y) - \sin(x) \sin(y) \frac{dy}{dx}$

Step-by-Step Solution to Find the Slope

To find $\frac{dy}{dx}$ at a specific point, follow these steps:

Step 1: Differentiate Each Term with Respect to x

Given:
$$
6 \sin(x) + 5 \cos(y) - 2 \sin(x) \cos(y) + x = 3
$$

Differentiate term-by-term:

- $\frac{d}{dx} [6 \sin(x)] = 6 \cos(x)$
- $\frac{d}{dx} [5 \cos(y)] = -5 \sin(y) \frac{dy}{dx}$
- $\frac{d}{dx} [-2 \sin(x) \cos(y)]$

Use the product rule:

$$
\frac{d}{dx} [\sin(x) \cos(y)] = \cos(x) \cos(y) - \sin(x) \sin(y) \frac{dy}{dx}
$$

So,

$$
\frac{d}{dx} [-2 \sin(x) \cos(y)] = -2 [\cos(x) \cos(y) - \sin(x) \sin(y) \frac{dy}{dx}] = -2 \cos(x) \cos(y) + 2 \sin(x) \sin(y) \frac{dy}{dx}
$$

- $\frac{d}{dx} [x] = 1$
- The derivative of constant 3 is 0.

Step 2: Write the Differentiated Equation

Combining all derivatives:

$$
6 \cos(x) - 5 \sin(y) \frac{dy}{dx} - 2 \cos(x) \cos(y) + 2 \sin(x) \sin(y) \frac{dy}{dx} + 1 = 0
$$

$$

Step 3: Collect $\frac{dy}{dx}$ Terms and Solve

Group the $\frac{dy}{dx}$ terms:

$$
[-5 \sin(y) + 2 \sin(x) \sin(y)] \frac{dy}{dx} + [6 \cos(x) - 2 \cos(x) \cos(y) + 1] = 0
$$

Expressed as:

$$
\left(-5 \sin(y) + 2 \sin(x) \sin(y) \right) \frac{dy}{dx} = - \left(6 \cos(x) - 2 \cos(x) \cos(y) + 1 \right)
$$

Finally, solve for $\frac{dy}{dx}$:

$$
\boxed{
\frac{dy}{dx} = \frac{6 \cos(x) - 2 \cos(x) \cos(y) + 1}{-5 \sin(y) + 2 \sin(x) \sin(y)}
}
$$

or equivalently,

$$
\frac{dy}{dx} = \frac{2 \cos(x) (3 - \cos(y)) + 1}{- \sin(y) (5 - 2 \sin(x))}
$$

Calculating the Slope at a Specific Point

To find the slope of the tangent line at a particular point (x_0, y_0), substitute the coordinates into the derived formula:

$$
\frac{dy}{dx} \bigg|_{(x_0, y_0)} = \frac{6 \cos(x_0) - 2 \cos(x_0) \cos(y_0) + 1}{-5 \sin(y_0) + 2 \sin(x_0) \sin(y_0)}
$$

Important: Before substituting, verify the point satisfies the original equation to ensure it lies on the curve.

Example Calculation: Finding the Slope at a Given Point

Suppose the point provided is \((x, y) = (\pi/4, \pi/3)\). Let's verify and compute:

Step 1: Verify the point satisfies the original equation

Calculate:

\[
6 \sin(\pi/4) + 5 \cos(\pi/3) - 2 \sin(\pi/4) \cos(\pi/3) + \pi/4
\]

Using known values:

- \(\sin(\pi/4) = \frac{\sqrt{2}}{2}\)
- \(\cos(\pi/3) = \frac{1}{2}\)

Compute:

\[
6 \times \frac{\sqrt{2}}{2} + 5 \times \frac{1}{2} - 2 \times \frac{\sqrt{2}}{2} \times \frac{1}{2}
+ \frac{\pi}{4}
\]

\[
= 3 \sqrt{2} + 2.5 - \frac{\sqrt{2}}{2} + \frac{\pi}{4}
\]

Numerically approximately:

\[
= (3 \times 1.4142) + 2.5 - 0.7071 + 0.7854 \approx 4.2426 + 2.5 - 0.7071 + 0.7854 \approx 6.8209
\]

Since this sum does not equal 3, the point \((\pi/4, \pi/3)\) is not on the curve. For the purpose of the example, choose a point that satisfies the original equation, or proceed assuming the point is valid.

Step 2: Plug in \(x = \pi/4\), \(y = \pi/3\)

Calculate numerator:

\[
6 \cos(\pi/4) - 2 \cos(\pi/4) \cos(\pi/3) + 1
\]

\[
= 6 \times \frac{\sqrt{2}}{2} - 2 \times \frac{\sqrt{2}}{2} \times \frac{1}{2} + 1
\]

\[

= 3 \sqrt{2} - \frac{\sqrt{2}}{2} + 1
\]

Calculate denominator:

\[
-5 \sin(\pi/3) + 2 \sin(\pi/4) \sin(\pi/3)
\]

\[
= -5 \times \frac{\sqrt{3}}{2} + 2 \times \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2}
\]

\[
= -\frac{5 \

Frequently Asked Questions

What is the first step to find the slope of the tangent line to the curve 6 sin(x) + 5 cos(y) - 2 sin(x) cos(y) + x = 3 at a given point?

The first step is to differentiate the given equation implicitly with respect to x to find dy/dx, since y is a function of x.

How do you differentiate the term 6 sin(x) with respect to x?

The derivative of 6 sin(x) with respect to x is 6 cos(x).

How do you differentiate the term 5 cos(y) with respect to x?

Using the chain rule, the derivative is -5 sin(y) dy/dx.

What is the derivative of -2 sin(x) cos(y) with respect to x?

Apply the product rule: derivative of -2 sin(x) cos(y) is -2 [cos(x) cos(y) - sin(x) sin(y) dy/dx].

After differentiating the entire equation implicitly, how do you solve for dy/dx?

Collect all terms involving dy/dx on one side and solve algebraically to isolate dy/dx, which gives the slope of the tangent line.

How do you evaluate the slope at a specific point on the

curve?

Substitute the x and y coordinates of the point into the expression for dy/dx obtained from implicit differentiation.

Why is implicit differentiation necessary for this problem?

Because the equation involves both x and y in a way that y cannot be explicitly solved for, so we differentiate implicitly to find dy/dx.

What is the significance of finding the slope of the tangent line at a point on the curve?

The slope of the tangent line indicates the instantaneous rate of change of y with respect to x at that point, describing the curve's direction and steepness locally.

Additional Resources

Find The Slope Of The Tangent Line To The Curve 6 Sin (x)+5 Cos (y)-2 Sin (x) Cos (y)+x=3 At The Point

Understanding how to find the slope of the tangent line to a curve at a specific point is a fundamental concept in calculus. This process often involves implicit differentiation, especially when the curve is defined by an equation involving both x and y in a non-explicit way. The problem titled "Find The Slope Of The Tangent Line To The Curve 6 Sin (x)+5 Cos (y)-2 Sin (x) Cos (y)+x=3 At The Point" is a classic example that combines trigonometric functions with implicit differentiation, offering a rich opportunity to explore these techniques in depth. In this article, we will analyze the problem thoroughly, break down the necessary steps, and discuss the mathematical concepts involved to provide a comprehensive understanding of how to approach such problems.

Understanding the Problem Statement

Before diving into the solution, it’s crucial to interpret what the problem is asking. The goal is to find the slope of the tangent line to a given curve at a specific point. The curve is defined implicitly by the equation:

$$ 6 \sin(x) + 5 \cos(y) - 2 \sin(x) \cos(y) + x = 3 $$

The problem, as presented, indicates that there is a particular point on this curve where the tangent line's slope needs to be determined. However, in the provided statement, the specific point isn't explicitly given, which is common in such problems. Usually, the point is either provided or is to be found by solving the equation for known x or y values.

Key aspects to consider:

- The curve is implicitly defined; y is a function of x, denoted as y(x).
- The equation involves trigonometric functions of both x and y.
- The goal is to find $\frac{dy}{dx}$ at a specific point on the curve, which is the slope of the tangent line.

Mathematical Foundations Required

To effectively tackle this problem, certain mathematical concepts and techniques need to be understood:

Implicit Differentiation

- When y is not explicitly solved in terms of x, differentiation must be done implicitly.
- Differentiating both sides of the equation with respect to x, treating y as a function of x ($y=y(x)$), leads to expressions involving $\frac{dy}{dx}$.

Derivative of Trigonometric Functions

- $\frac{d}{dx} \sin(x) = \cos(x)$
- $\frac{d}{dx} \cos(y) = -\sin(y) \frac{dy}{dx}$
- $\frac{d}{dx} \sin(x) \cos(y) = \cos(x) \cos(y) - \sin(x) \sin(y) \frac{dy}{dx}$ (product rule combined with chain rule)

Chain Rule

- Essential for differentiating composite functions involving y.

Having these tools at hand allows for systematically differentiating the entire equation with respect to x.

Step-by-Step Solution Approach

Let's proceed through the solution methodically:

Step 1: Write down the original equation

$$
6 \sin(x) + 5 \cos(y) - 2 \sin(x) \cos(y) + x = 3
$$

Step 2: Differentiate both sides with respect to x

Since the right side is a constant (3), its derivative is 0.

Differentiate each term:

- $\frac{d}{dx}[6 \sin(x)] = 6 \cos(x)$
- $\frac{d}{dx}[5 \cos(y)] = 5 \times (-\sin(y)) \frac{dy}{dx} = -5 \sin(y) \frac{dy}{dx}$
- $\frac{d}{dx}[-2 \sin(x) \cos(y)]$

Using the product rule:

$$
\frac{d}{dx}[\sin(x) \cos(y)] = \cos(x) \cos(y) - \sin(x) \sin(y) \frac{dy}{dx}
$$

Multiplying by -2:

$$
-2 [\cos(x) \cos(y) - \sin(x) \sin(y) \frac{dy}{dx}] = -2 \cos(x) \cos(y) + 2 \sin(x) \sin(y) \frac{dy}{dx}
$$

- $\frac{d}{dx}[x] = 1$

Putting all derivatives together:

$$
6 \cos(x) - 5 \sin(y) \frac{dy}{dx} - 2 \cos(x) \cos(y) + 2 \sin(x) \sin(y) \frac{dy}{dx} + 1 = 0
$$

Step 3: Collect terms involving $\frac{dy}{dx}$

Group the $\frac{dy}{dx}$ terms:

$$
[-5 \sin(y) + 2 \sin(x) \sin(y)] \frac{dy}{dx} + [6 \cos(x) - 2 \cos(x) \cos(y) + 1] = 0
$$

Rewrite as:

$$
\left(-5 \sin(y) + 2 \sin(x) \sin(y)\right) \frac{dy}{dx} = - (6 \cos(x) - 2 \cos(x) \cos(y) + 1)
$$

Step 4: Solve for $\frac{dy}{dx}$

Divide both sides by the coefficient of $\frac{dy}{dx}$:

$$
\frac{dy}{dx} = \frac{6 \cos(x) - 2 \cos(x) \cos(y) + 1}{-5 \sin(y) + 2 \sin(x) \sin(y)}
$$

This expression provides the slope of the tangent line at any point (x, y) on the curve, provided we know the specific x and y values.

Finding the Specific Point and Slope

The initial problem states "at the point," implying a specific point exists. To proceed, we need to determine this point. Usually, the point is given or can be found by solving the original equation for specific values.

Suppose the problem provides a point, such as (x_0, y_0), or asks us to find the point where the curve intersects a particular x or y value. If not explicitly provided, you might need to:

- Solve the original equation for a specific x or y.
- Or, if the point is given, substitute into the expression for $\frac{dy}{dx}$.

For example, if the point is $(x, y) = (0, 0)$:

- Calculate the numerator:

$$
6 \cos(0) - 2 \cos(0) \cos(0) + 1 = 6(1) - 2(1)(1) + 1 = 6 - 2 + 1 = 5
$$

- Calculate the denominator:

$$
-5 \sin(0) + 2 \sin(0) \sin(0) = 0 + 0 = 0
$$

Since the denominator is zero, the slope is undefined at $(0, 0)$, indicating a vertical tangent line.

Features and Analysis of the Derived Formula

The expression for $\frac{dy}{dx}$:

$$
\frac{dy}{dx} = \frac{6 \cos(x) - 2 \cos(x) \cos(y) + 1}{-5 \sin(y) + 2 \sin(x) \sin(y)}
$$

has several noteworthy features:

Pros:

- General applicability: It can be used to find the slope at any point (x, y) satisfying the original equation.
- Involves standard functions: The formula employs common trigonometric functions, making it straightforward to evaluate numerically.
- Reveals geometric properties: The points where the denominator is zero correspond to vertical tangents, indicating potential points of interest on the curve.

Cons:

- Requires explicit x, y values: Without specific point data, the slope cannot be numerically computed.
- Complexity for solving: In some cases, solving the original equation for y explicitly may be challenging, complicating substitution.
- Potential singularities: The formula may have points where the denominator is zero, indicating undefined slope (vertical tangent), which requires special attention.

Practical Applications and Further Considerations

The ability to find the slope of the tangent line to such a curve has numerous practical applications:

- Physics: Determining instantaneous velocity or rate of change in systems described by complex equations.
- Engineering: Analyzing stress, strain, or other properties where relationships involve trigonometric dependencies.
- Mathematics: Exploring properties of implicit curves, such as identifying points of vertical tangent or curvature.

Furthermore, understanding the behavior of the slope

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