Find The Solution To Initial Value Problem Dt 2d2y2dt Dy+1y=0,y(0)=4,y (0)=1 Find The Solution Of Y 2y

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In the realm of differential equations, initial value problems (IVPs) play a crucial role in modeling real-world phenomena, from physics and engineering to biology and economics. The specific problem at hand involves solving a second-order differential equation with given initial conditions, which is fundamental for understanding system behaviors over time. This article provides a comprehensive guide to solving the initial value problem:

$$
\frac{d^2 y}{dt^2} + y = 0, \quad y(0) = 4, \quad y'(0) = 1
$$

and interpreting the solution, especially focusing on how to find y and then determine $2y$.

Understanding the Differential Equation

The Form of the Equation

The differential equation presented:

$$
\frac{d^2 y}{dt^2} + y = 0
$$

is a classic second-order linear homogeneous differential equation with constant coefficients. It commonly appears in oscillatory systems such as simple harmonic motion, electrical circuits, and mechanical vibrations. The general form:

$$
y'' + ky = 0
$$

where k is a constant, describes systems that oscillate with solutions involving sines and cosines.

Physical Interpretation

In physics, this equation models an idealized mass-spring system without damping, where the acceleration of the mass is proportional to its displacement but in the opposite direction, resulting in periodic motion.

Solving the Differential Equation

Step 1: Find the Characteristic Equation

To solve the differential equation, assume a solution of the form:

$$
y(t) = e^{rt}
$$

Substituting into the differential equation yields:

$$
r^2 e^{rt} + e^{rt} = 0
$$

Dividing through by e^{rt} (which is never zero):

$$
r^2 + 1 = 0
$$

This is the characteristic equation:

$$
r^2 + 1 = 0
$$

which has roots:

$$
r = \pm i
$$

Step 2: Write the General Solution

Since the roots are complex conjugates, the general solution for $y(t)$ is:

$$
y(t) = C_1 \cos t + C_2 \sin t
$$

where C_1 and C_2 are constants determined by initial conditions.

Applying Initial Conditions

Given Conditions:

$$
y(0) = 4, \quad y'(0) = 1
$$

Step 1: Find $y(0)$

Plugging $t = 0$ into the general solution:

$$
y(0) = C_1 \cos 0 + C_2 \sin 0 = C_1 \times 1 + C_2 \times 0 = C_1
$$

Thus:

$$
C_1 = 4
$$

Step 2: Find $y'(t)$ and evaluate at $t=0$

Differentiate $y(t)$:

$$
y'(t) = - C_1 \sin t + C_2 \cos t
$$

At $t = 0$:

$$

$$
y'(0) = - C_1 \times 0 + C_2 \times 1 = C_2
$$

Given:

$$
y'(0) = 1 \Rightarrow C_2 = 1
$$

Step 3: Write the Particular Solution

Substituting C_1 and C_2:

$$
y(t) = 4 \cos t + \sin t
$$

Finding $2y$: The Final Step

The problem asks to find the solution of $2y$, which is simply twice the particular solution:

$$
2y(t) = 2 \times (4 \cos t + \sin t) = 8 \cos t + 2 \sin t
$$

This expression represents the scaled oscillatory behavior of the system, and understanding it can be crucial for applications where amplitude or energy scaling is relevant.

Summary of Results

- The general solution to the differential equation $y'' + y = 0$ is:

$$
y(t) = C_1 \cos t + C_2 \sin t
$$

- Using initial conditions, the particular solution is:

$$
y(t) = 4 \cos t + \sin t
$$

- The scaled solution $2y(t)$ is:

$$
2y(t) = 8 \cos t + 2 \sin t
$$

Additional Insights and Applications

Oscillatory Nature and Amplitude

The solution $y(t) = 4 \cos t + \sin t$ describes a harmonic oscillator with specific initial displacement and velocity. The amplitude of oscillation can be found by rewriting the expression in a single sinusoidal form:

$$
y(t) = R \sin (t + \phi)
$$

where:

$$
R = \sqrt{C_1^2 + C_2^2} = \sqrt{4^2 + 1^2} = \sqrt{16 + 1} = \sqrt{17}
$$

and

$$
\phi = \arctan \left(\frac{C_2}{C_1}\right) = \arctan \left(\frac{1}{4}\right)
$$

This form provides a clearer picture of the oscillation's amplitude and phase shift.

Real-World Applications

Understanding solutions to such differential equations is vital in various fields:

- Mechanical Engineering: Designing systems with predictable oscillations
- Physics: Analyzing wave motion and vibrations
- Electrical Engineering: Studying RLC circuits
- Biology: Modeling biological rhythms

Conclusion

Solving the initial value problem $\frac{d^2 y}{dt^2} + y = 0$ with initial conditions $y(0) = 4$ and $y'(0) = 1$ involves identifying the characteristic roots, formulating the general solution, applying initial conditions, and then analyzing the solution's behavior. The particular solution $y(t) = 4 \cos t + \sin t$ accurately describes oscillatory motion starting from the given initial state. Scaling this solution by 2 yields $2y(t) = 8 \cos t + 2 \sin t$, which can be useful in applications requiring amplitude adjustments.

By mastering these steps, students and professionals can confidently approach similar differential equations, understand their physical implications, and apply solutions effectively across various scientific and engineering disciplines.

Keywords: differential equations, initial value problem, second-order linear homogeneous, harmonic oscillator, solving differential equations, initial conditions, sinusoidal solutions, oscillatory systems

Frequently Asked Questions

What is the initial value problem involving the differential equation Dt 2d2y2dt Dy+1y=0 with y(0)=4 and y'(0)=1?

The initial value problem is a second-order differential equation: Dt 2d^2y/dt^2 + y = 0, with initial conditions y(0) = 4 and y'(0) = 1.

How do you approach solving the differential equation Dt 2d^2y/dt^2 + y = 0 with given initial conditions?

You typically assume a solution of the form $y(t) = e^{rt}$, substitute into the differential equation to find the characteristic equation, solve for r, and then apply initial conditions to find the specific solution.

What is the general solution to the differential equation Dt 2d^2y/dt^2 + y = 0?

The general solution is $y(t) = C_1 \cos(t) + C_2 \sin(t)$, where C_1 and C_2 are constants determined by initial conditions.

How do you determine the particular solution using initial conditions y(0)=4 and y'(0)=1?

Substitute t=0 into the general solution to find $C_1 = 4$, then differentiate y(t) to find y'(t), evaluate at t=0 to solve for C_2, which turns out to be 1, giving $y(t) = 4 \cos(t) + \sin(t)$.

What is the final solution to the initial value problem y(t) from the given differential equation?

The solution is $y(t) = 4 \cos(t) + \sin(t)$, satisfying both the differential equation and the initial conditions.

Additional Resources

Find The Solution To Initial Value Problem Dt 2d2y2dt Dy+1y=0, y(0)=4, y'(0)=1 Find The Solution Of Y 2y

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Introduction

Mathematical modeling and differential equations are fundamental tools across various scientific and engineering disciplines. They allow us to describe dynamic systems, predict future behavior, and understand complex phenomena. Among the many differential equations encountered in these fields, second-order linear differential equations with initial conditions are particularly significant due to their widespread applications in physics, engineering, and applied mathematics.

This article explores an intricate initial value problem (IVP):

$$
\textbf{Dt 2d2y2dt Dy + 1 y = 0}, \quad y(0) = 4, \quad y'(0) = 1,
$$

which, upon interpretation, appears to involve a second-order differential equation with initial conditions. The goal is to thoroughly analyze the problem, derive its solution, and understand the implications of the solution, especially focusing on the equation's structure, solution methods, and the interpretation of the final expression involving y^2.

Deciphering the Differential Equation

Clarifying the Equation’s Structure

The notation in the problem statement appears somewhat unconventional, but it can be interpreted as a standard second-order linear differential equation of the form:

$$
\frac{d^2 y}{dt^2} + y = 0,
$$

which is a classic simple harmonic oscillator equation. The initial conditions are given as:

$$
y(0) = 4, \quad y'(0) = 1.

\]

The phrase "Dt 2d2y2dt Dy+1y=0" likely corresponds to the differential operator acting on $y(t)$. A plausible interpretation is:

$$
\frac{d^2 y}{dt^2} + y = 0,
$$

which is a homogeneous, second-order differential equation with constant coefficients.

Understanding the Equation's Origin

This form is common in modeling oscillatory systems such as springs, pendulums (for small angles), and electrical circuits with inductance and capacitance. Its solutions are well-understood and involve sine and cosine functions, making it an ideal candidate for analytical solution.

Solution Methodology

Step 1: Recognize the Type of Differential Equation

The differential equation:

$$
\frac{d^2 y}{dt^2} + y = 0,
$$

belongs to the class of linear, homogeneous differential equations with constant coefficients.

Step 2: Write the Characteristic Equation

Assuming solutions of the form $y(t) = e^{rt}$, substitute into the differential equation:

$$
r^2 e^{rt} + e^{rt} = 0,
$$

which simplifies to:

$$
e^{rt} (r^2 + 1) = 0.
$$

Since $e^{rt} \neq 0$, the characteristic equation is:

$$
r^2 + 1 = 0,
$$

yielding roots:

$$
r = \pm i,
$$

where i is the imaginary unit.

Step 3: General Solution

The roots are complex conjugates, so the general solution is:

$$
y(t) = C_1 \cos t + C_2 \sin t,
$$

where C_1 and C_2 are constants determined by initial conditions.

Application of Initial Conditions

Step 4: Apply $y(0) = 4$

At $t = 0$:

$$
y(0) = C_1 \cos 0 + C_2 \sin 0 = C_1 \times 1 + C_2 \times 0 = C_1,
$$

so:

$$
C_1 = 4.
$$

Step 5: Find $y'(t)$ and Apply $y'(0) = 1$

Differentiate $y(t)$:

$$
y'(t) = - C_1 \sin t + C_2 \cos t.
$$

At $t = 0$:

$$
y'(0) = - C_1 \times 0 + C_2 \times 1 = C_2,
$$

thus:

$$
C_2 = 1.
$$

Final solution:

$$
\boxed{
y(t) = 4 \cos t + \sin t.
}
$$

Exploring the Solution: The Behavior of $y(t)$

Characteristics of $y(t)$:

- It describes a harmonic oscillation with amplitude $\sqrt{4^2 + 1^2} = \sqrt{17}$.
- The oscillation frequency is 1 rad/sec, typical for simple harmonic motion.
- The phase shift can be expressed as a single sinusoid:

$$
y(t) = R \cos(t - \phi),
$$

where:

$$
R = \sqrt{C_1^2 + C_2^2} = \sqrt{17},
$$
$$
\phi = \arctan \left(\frac{C_2}{C_1} \right) = \arctan \left(\frac{1}{4} \right).
$$

The Expression "Find The Solution Of Y 2y"

Interpreting the Final Query

The phrase "Find The Solution Of Y 2y" appears to suggest exploring the solution for y^2 or perhaps analyzing y in relation to $2y$. Given the context, it’s logical to examine the square of the solution:

$$
Y(t) = y(t),
$$

and consider:

$$
Y^2(t) = \left(4 \cos t + \sin t \right)^2.
$$

Step 6: Derive $y^2(t)$

Expanding:

$$
y^2(t) = (4 \cos t + \sin t)^2 = 16 \cos^2 t + 8 \cos t \sin t + \sin^2 t.
$$

Using trigonometric identities:

- $\cos^2 t = \frac{1 + \cos 2t}{2}$,
- $\sin^2 t = \frac{1 - \cos 2t}{2}$,
- $\sin t \cos t = \frac{\sin 2t}{2}$,

we rewrite:

$$
y^2(t) = 16 \times \frac{1 + \cos 2t}{2} + 8 \times \frac{\sin 2t}{2} + \frac{1 - \cos 2t}{2}.
$$

Simplify each term:

$$
y^2(t) = 8 (1 + \cos 2t) + 4 \sin 2t + \frac{1 - \cos 2t}{2}.
$$

Combine like terms:

$$
y^2(t) = 8 + 8 \cos 2t + 4 \sin 2t + \frac{1}{2} - \frac{\cos 2t}{2}.
$$

Express as:

$$
y^2(t) = \left(8 + \frac{1}{2}\right) + \left(8 \cos 2t - \frac{\cos 2t}{2}\right) + 4 \sin 2t,
$$

which simplifies to:

$$
y^2(t) = \frac{17}{2} + \frac{16 - 1}{2} \cos 2t + 4 \sin 2t,
$$

or:

$$

\boxed{
y^2(t) = \frac{17}{2} + \frac{15}{2} \cos 2t + 4 \sin 2t.
}
\]

Significance of \(y^2(t) \)

This expression reveals the oscillatory nature of \(y^2(t) \), which oscillates between maximum and minimum values dictated by the amplitude of the combined sinusoidal terms.

Broader Implications and Applications

Physical Interpretation

The original differential equation models systems characterized by harmonic motion. The specific initial conditions specify the initial displacement and velocity, leading to a unique oscillatory solution. The analysis of \(y^2(t) \) can be essential in contexts where energy or squared amplitude is relevant — such as in calculating power in electrical circuits or energy stored in mechanical systems.

Mathematical Significance

- The problem exemplifies classical solution techniques for second-order linear ODEs.
- Using trigonometric identities simplifies complex expressions.
- The approach illustrates how initial conditions refine general solutions to particular solutions.

Extending the Analysis

Further investigations could involve:

- Analyzing the phase shift \(\phi \),
- Computing the maximum and minimum of \(y^2(t) \),
- Examining the integral or average values over a period,
- Considering damping or forcing functions to model more complex systems.

Conclusion

The carefully derived solution to the initial value problem:

\[
\frac{d^2 y}{dt^2} + y = 0, \quad y(0) = 4, \quad y'(0) = 1,
\]

is:

\[
\boxed{
y(t) = 4 \cos t +

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