Find The Third Term Of The Sequence Given By The Rule F(1) = 5 And F(n) = F(n-1) + 3 For N Bigger Than

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Understanding sequences and their rules is fundamental in mathematics, especially in algebra and arithmetic progression problems. One particular sequence that frequently appears in various mathematical contexts is defined recursively, meaning each term depends on the previous one. The sequence in question here is defined by the initial value and a recursive rule:

- F(1) = 5
- F(n) = F(n-1) + 3 for N > 1

The task is to find the third term of this sequence, but to do so effectively, it's crucial to grasp the underlying pattern and how the sequence progresses. This article will guide you through understanding recursive sequences, deriving explicit formulas, and applying these to find specific terms such as the third term. Additionally, we will explore related concepts, common mistakes, and practical applications to deepen your understanding.

Understanding Recursive Sequences

Recursive sequences are sequences where each term is expressed in terms of its preceding terms. They are fundamental in many areas of mathematics, computer science, and real-world modeling.

Definition of Recursive Sequences

A recursive sequence involves:
- A base case or initial term, often denoted as F(1), F(0), or similar.
- A recurrence relation that defines each subsequent term based on previous ones.

For example, the sequence:
F(1) = 5
F(n) = F(n-1) + 3 for n > 1
is an arithmetic sequence with a starting point of 5 and a common difference of 3.

Examples of Recursive Sequences

- The Fibonacci sequence: $F(1) = 0$, $F(2) = 1$, $F(n) = F(n-1) + F(n-2)$
- The sequence of factorials: $n!$ with $F(0) = 1$, $F(n) = n F(n-1)$
- Arithmetic sequence: as in our case, where each term increases by a constant.

Deriving the Explicit Formula

While recursive definitions describe how to generate terms sequentially, explicit formulas allow direct computation of the nth term without recursion. This simplifies calculations and helps analyze the sequence's behavior.

Understanding the Pattern

Given:
- $F(1) = 5$
- $F(n) = F(n-1) + 3$

We observe that starting from the first term:
- $F(2) = F(1) + 3$
- $F(3) = F(2) + 3$
- $F(4) = F(3) + 3$
- and so on...

This pattern suggests an arithmetic progression with:
- First term: $a_1 = 5$
- Common difference: $d = 3$

Formulating the Explicit Formula

The explicit formula for an arithmetic sequence is:
$F(n) = a_1 + (n - 1) d$

Applying this to our sequence:
$F(n) = 5 + (n - 1) 3$

This formula allows us to compute any term directly by plugging in the value of n.

Calculating the Third Term

Now, with the explicit formula at hand, finding the third term is straightforward.

Step-by-step Calculation

- $n = 3$
- $F(3) = 5 + (3 - 1) 3$
- $F(3) = 5 + 2 3$
- $F(3) = 5 + 6$
- $F(3) = 11$

Therefore, the third term of the sequence is 11.

Summary of Calculation

Term Number (n)	Calculation	Result
3	$5 + (3 - 1) 3$	11

This simple approach demonstrates how understanding the pattern and deriving an explicit formula simplifies the process of finding specific terms.

Additional Practice: Find Other Terms

To solidify your understanding, try calculating other terms in the sequence.

Examples:

1. First term (given): $F(1) = 5$

2. Second term: $F(2) = 5 + (2 - 1) 3 = 5 + 3 = 8$

3. Fourth term: $F(4) = 5 + (4 - 1) 3 = 5 + 9 = 14$

4. Fifth term: $F(5) = 5 + (5 - 1) 3 = 5 + 12 = 17$

Notice how each subsequent term increases by 3, confirming the arithmetic progression.

Understanding the Behavior of the Sequence

By analyzing the explicit formula, we can understand the long-term behavior of the sequence.

Growth Pattern

- The sequence increases linearly.
- The nth term depends directly on n, with the growth rate of 3 per step.
- As n increases, F(n) grows without bound, tending toward infinity.

Graphical Representation

Plotting the sequence:
- The points are (1, 5), (2, 8), (3, 11), (4, 14), (5, 17), ...
- The graph is a straight line with slope 3 and y-intercept at 5.

Common Mistakes and Misconceptions

Avoid these pitfalls when working with recursive sequences:

Confusing Recursive and Explicit Formulas

- Recursive: F(n) depends on previous terms.
- Explicit: F(n) is expressed directly in terms of n.
- Always derive or use the explicit formula for quick calculations.

Incorrect Initial Term

- Ensure the initial term matches the problem statement.
- Changing F(1) affects the entire sequence.

Misunderstanding the Common Difference

- Confirm the difference between successive terms.
- In this case, the difference is consistently 3.

Applications of Arithmetic Sequences

Arithmetic sequences appear in numerous real-life situations:

Financial Planning

- Calculating fixed periodic savings or payments.
- Example: Saving \$100 each month.

Physics and Engineering

- Uniform acceleration or constant rate scenarios.
- Example: objects moving with constant velocity.

Data Analysis

- Modeling linear trends over time.
- Example: Population growth with a constant rate.

Summary and Key Takeaways

- The sequence defined by $F(1) = 5$ and $F(n) = F(n-1) + 3$ is an arithmetic sequence.
- Its explicit formula is $F(n) = 5 + (n - 1) 3$.
- The third term is $F(3) = 11$.
- Understanding recursive and explicit formulas streamlines the process of calculating specific terms.
- Recognizing the pattern helps in solving related problems and applying the sequence in various contexts.

Final Thoughts

Mastering recursive sequences and their explicit formulas is essential for tackling a wide array of mathematical problems. Whether in algebra, calculus, or applied fields, being able to derive and apply these formulas empowers you to analyze and predict the behavior of sequences efficiently. Remember to verify initial conditions carefully, understand the pattern, and always consider deriving an explicit formula for ease of computation. With practice, identifying and working with such sequences will become an intuitive part of your mathematical toolkit.

Frequently Asked Questions

What is the third term in the sequence where $F(1) = 5$ and $F(n) = F(n-1) + 3$?

The third term is 11.

How do you find the third term of the sequence defined by $F(1) = 5$ and $F(n) = F(n-1) + 3$?

Calculate step by step: $F(2) = 5 + 3 = 8$, then $F(3) = 8 + 3 = 11$.

What is the recursive rule used to generate the sequence where F(1) = 5?

The rule is F(n) = F(n-1) + 3 for n > 1.

Can you find the formula for the nth term of the sequence given F(1) = 5 and F(n) = F(n-1) + 3?

Yes, the nth term is F(n) = 5 + 3(n - 1).

Using the explicit formula, what is the third term of the sequence?

F(3) = 5 + 3(3 - 1) = 5 + 6 = 11.

Is the sequence an arithmetic sequence, and how does that help find the third term?

Yes, it's arithmetic with a common difference of 3; so, F(3) = F(1) + 23 = 5 + 6 = 11.

If the first term is 5 and each subsequent term increases by 3, what is the third term?

The third term is 11.

Additional Resources

Find The Third Term Of The Sequence Given By The Rule F(1) = 5 And F(n) = F(n-1) + 3 For N Bigger Than

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Introduction

In the realm of mathematics, sequences serve as fundamental building blocks that help us understand patterns, relationships, and the nature of numbers themselves. Among the myriad types of sequences, arithmetic sequences stand out for their simplicity and wide-ranging applications across disciplines—from finance to computer science. The sequence in question, defined by the rule F(1) = 5 and F(n) = F(n-1) + 3 for N greater than 1, exemplifies an arithmetic progression.

This article delves into the detailed process of determining the third term of this sequence, explores the general formula governing the sequence, and discusses the broader implications of such sequences in mathematical and real-world contexts. Our aim is to provide a comprehensive, step-by-step analysis suitable for educators, students, and researchers interested in the mechanics of sequence generation and their applications.

Understanding the Sequence Definition

The Basic Components

The sequence is defined by two key components:

- Initial Term: $F(1) = 5$
- Recursive Rule: $F(n) = F(n-1) + 3$ for all $n > 1$

This recursive rule indicates that each term is obtained by adding 3 to the previous term, starting from the initial value of 5.

Significance of the Recursive Formula

Recursive definitions are common in sequences, capturing how each term relates to its predecessor. For this particular sequence:

- The sequence is arithmetic, characterized by a constant difference between consecutive terms.
- The common difference, denoted as d, is 3.

Understanding this structure is essential for deriving the nth term directly, without computing all prior terms.

Step-by-Step Calculation of the First Few Terms

To establish clarity, let us explicitly compute the first few terms:

- First term (n=1): $F(1) = 5$
- Second term (n=2): $F(2) = F(1) + 3 = 5 + 3 = 8$
- Third term (n=3): $F(3) = F(2) + 3 = 8 + 3 = 11$

Thus, the third term of the sequence is 11.

Deriving the General Formula for the Sequence

While recursive computation is straightforward for small n, it becomes cumbersome for larger values. Therefore, deriving a closed-form expression is beneficial.

Recognizing the Pattern

Given that the sequence is arithmetic with:

- First term: $a_1 = 5$
- Common difference: $d = 3$

The general term $F(n)$ of an arithmetic sequence is given by:

$$
F(n) = a_1 + (n - 1) \times d
$$

Applying our known values:

$$
F(n) = 5 + (n - 1) \times 3
$$

This formula allows us to compute any term directly.

Verifying the Formula

Let's verify this formula by calculating the third term:

$$
F(3) = 5 + (3 - 1) \times 3 = 5 + 2 \times 3 = 5 + 6 = 11
$$

This matches our earlier calculation, confirming the validity of the formula.

Exploring the Sequence’s Properties and Applications

Properties of the Sequence

- Arithmetic Progression: The sequence increases by a constant difference of 3.
- Linear Growth: The sequence’s terms grow linearly with n.
- Predictability: The general formula allows for quick computation of any term.

Practical Applications

Sequences like this are prevalent in various fields:

- Financial Planning: Modeling steady savings or payments increasing by fixed amounts.
- Computer Science: Iterative algorithms with linear step increments.
- Physics: Describing uniform acceleration scenarios.
- Education: Teaching fundamental concepts of arithmetic sequences.

Common Mistakes and Clarifications

While working with such sequences, students and practitioners often encounter misunderstandings:

- Misapplying the recursive rule: Remember that recursive definitions require initial conditions and step-by-step calculations.
- Confusing the sequence index: The sequence begins at n=1, so ensure that the formula aligns with this starting point.
- Forgetting the initial term: The formula must incorporate a_1 explicitly; otherwise, it might lead to incorrect results.

Summary of Key Points

- The sequence is defined by $F(1) = 5$ and $F(n) = F(n-1) + 3$.
- The third term, $F(3)$, is 11.
- The explicit formula for the nth term is $F(n) = 5 + (n - 1) \times 3$.
- Recognizing the sequence as an arithmetic progression simplifies calculations and analysis.

Broader Context and Final Remarks

Understanding how to find specific terms in a sequence is fundamental in mathematics. The process demonstrated here—starting from a recursive definition, computing initial terms, recognizing the pattern, and deriving a closed-form formula—is a core skill applicable across many areas of quantitative reasoning.

Sequences like the one examined serve as building blocks for more advanced topics, including geometric progressions, series summations, and discrete mathematics. They also underpin algorithms and models in sciences and engineering, showcasing the importance of mastering such foundational concepts.

In conclusion, the third term of the sequence given by the rule $F(1) = 5$ and $F(n) = F(n-1) + 3$ is 11, and the sequence's behavior can be succinctly captured through its explicit formula. This understanding not only solves the immediate problem but also enhances comprehension of linear patterns in mathematical structures.

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