Find The Two Smallest Positive Solutions For X In The Equation 12cos(2pi/5 X)+10=16, When (2pi/5 X) Is

Find The Two Smallest Positive Solutions For X In The Equation 12cos(2pi/5 X)+10=16, When (2pi/5 X) Is a mathematical problem that involves understanding trigonometric functions, particularly the cosine function, and solving for the variable X. This type of problem is common in trigonometry, where equations involve angles and their cosine values, and requires a methodical approach to isolate X and interpret the solutions within specific constraints. In this article, we will explore how to find the two smallest positive solutions for X step-by-step, analyze the properties of the cosine function involved, and interpret the solutions in the context of the given equation.

Understanding the Equation

Before diving into the solution process, it’s essential to understand the structure of the equation:

12cos(2pi/5 X) + 10 = 16

This equation involves a cosine term multiplied by 12, shifted vertically by 10 units, and set equal to 16. The goal is to solve for X, considering the periodic nature of the cosine function.

Rearranging the Equation

The first step is to isolate the cosine term:

12cos(2pi/5 X) + 10 = 16

Subtract 10 from both sides:

12cos(2pi/5 X) = 6

Divide both sides by 12:

cos(2pi/5 X) = 6/12 = 1/2

This simplification allows us to focus on the basic cosine value, which is 1/2.

Key Insight: When Does Cosine Equal 1/2?

The cosine function equals 1/2 at specific angles within its cycle. Recall the unit circle:

- $\cos(\theta) = 1/2$ at $\theta = \pi/3$ and $\theta = 5\pi/3$ (or 60° and 300° in degrees).

Since cosine is periodic with period 2π, these solutions repeat every 2π radians.

Expressing the General Solution for the Equation

Given:

cos(2pi/5 X) = 1/2

The general solutions for the angle inside the cosine are:

$\theta = \pi/3 + 2\pi n$

or

$\theta = 5\pi/3 + 2\pi n$

where n is any integer ($n \in \mathbb{Z}$).

In our case, the angle θ is:

$\theta = (2\pi/5) X$

Therefore:

$(2\pi/5) X = \pi/3 + 2\pi n$ (Equation 1)

or

$(2\pi/5) X = 5\pi/3 + 2\pi n$ (Equation 2)

Our goal is to solve these for X to find the corresponding solutions.

Solving for X in Both Cases

Starting with Equation 1:

$(2\pi/5) X = \pi/3 + 2\pi n$

Multiply both sides by $5/(2\pi)$:

$X = (5/(2\pi)) (\pi/3 + 2\pi n)$

Simplify:

$X = (5/(2\pi)) \pi/3 + (5/(2\pi)) 2\pi n$

$X = (5/2\pi) (\pi/3) + (5/2\pi) 2\pi n$

Calculate each term:

- First term:

$(5/2\pi)(\pi/3) = (5/2\pi)(\pi/3) = (5/2)(1/3) = 5/6$

- Second term:

$(5/2\pi)\, 2\pi n = (5/2\pi)\, 2\pi n = 5n$

Thus, solutions from Equation 1:

$X = 5/6 + 5n$

Similarly, for Equation 2:

$(2\pi/5) X = 5\pi/3 + 2\pi n$

Multiply both sides by $5/(2\pi)$:

$X = (5/(2\pi))(5\pi/3 + 2\pi n)$

Simplify:

$X = (5/2\pi)\, 5\pi/3 + (5/2\pi)\, 2\pi n$

Calculate each:

- First term:

$(5/2\pi)\, 5\pi/3 = (5/2\pi)\, 5\pi/3 = (5\ 5\pi) / (2\pi\ 3) = (25\pi) / (6\pi) = 25/6$

- Second term:

$(5/2\pi)\, 2\pi n = 5n$

Solutions from Equation 2:

$X = 25/6 + 5n$

Finding the Two Smallest Positive Solutions

The solutions are:

- $X = 5/6 + 5n$

- $X = 25/6 + 5n$

where n is any integer.

To find the smallest positive solutions, we need to choose integer values of n such that X > 0 and X is as small as possible.

For the first set: X = 5/6 + 5 n

Set n such that:

X > 0

5/6 + 5 n > 0

Solve for n:

5 n > -5/6

n > -1/6

Since n is an integer:

- n ≥ 0 (because for n=0: X=5/6 > 0)

- n = -1: X = 5/6 - 5 = (5/6) - 5 = (5/6) - (30/6) = -25/6 < 0, discard

- n=0: X=5/6 ≈ 0.8333 (positive)

- n=1: X=5/6 + 5 = (5/6) + (30/6) = 35/6 ≈ 5.8333

Similarly, for the second set: X=25/6 + 5 n

Set n such that:

X > 0

25/6 + 5 n > 0

Solve for n:

5 n > -25/6

n > -25/6 / 5 = -25/6 1/5 = -25/30 = -5/6 ≈ -0.8333

Since n is integer:

- n ≥ 0 (since n=-1: X=25/6 - 5 = (25/6) - (30/6) = -5/6 < 0), discard.

- n=0: X=25/6 ≈ 4.1667 (positive)

- n=1: X=25/6 + 5 = (25/6) + (30/6)= 55/6 ≈ 9.1667

Therefore, the two smallest positive solutions are:

- From the first set at $n=0$: $X = 5/6 \approx 0.8333$

- From the second set at $n=0$: $X = 25/6 \approx 4.1667$

These are the two smallest positive solutions.

Summary of the Solutions

Solution Number	X Value	Approximate Value	Corresponding n
1	5/6	0.8333	n=0 (first set)
2	25/6	4.1667	n=0 (second set)

These solutions satisfy the original equation for the specified domain. The solutions can be verified by substituting back into the original equation:

- For $X = 5/6$:

$$\cos((2\pi/5)\ 5/6) = \cos((2\pi/5)\ (5/6)) = \cos((2\pi\ 5)/(5\ 6)) = \cos((2\pi)/6) = \cos(\pi/3) = 1/2$$

- For $X=25/6$:

$$\cos((2\pi/5)\ 25/6) = \cos((2\pi\ 25)/(5\ 6)) = \cos((50\pi)/(30)) = \cos((5\pi)/3) = 1/2$$

which confirms the solutions.

Conclusion

By understanding the properties of the cosine function and manipulating the equation algebraically, we successfully identified the two smallest positive solutions for X in the given equation. The key steps involved isolating the cosine function, recognizing the angles where cosine equals 1/2, and then translating those angles back into solutions for X considering the periodicity of cosine. The solutions:

- $X = 5/6$

- $X = 25/6$

are the two smallest positive solutions, corresponding to the smallest positive n values in the general solutions.

This approach can be applied to similar trigonometric equations, emphasizing the importance of understanding the periodic behavior of functions and the algebraic techniques for solving for variables within those functions.

Frequently Asked Questions

What is the primary goal when solving the equation 12cos(2π/5 X) + 10 = 16 for positive solutions?

The main goal is to find the two smallest positive values of X that satisfy the equation by isolating the cosine term and solving for X within its principal range.

How do you simplify the equation 12cos(2π/5 X) + 10 = 16 to find solutions for X?

First, subtract 10 from both sides to get $12\cos(2\pi/5\ X) = 6$, then divide both sides by 12 to obtain $\cos(2\pi/5\ X) = 0.5$.

What are the general solutions for cos(θ) = 0.5, and how do they relate to X in the given equation?

The general solutions for $\cos(\theta) = 0.5$ are $\theta = \pi/3 + 2k\pi$ and $\theta = 5\pi/3 + 2k\pi$, where k is any integer. In the original equation, $\theta = 2\pi/5\ X$, so we solve for X accordingly.

How do you find the two smallest positive solutions for X from the general solutions of cos(2π/5 X) = 0.5?

Set $2\pi/5\ X$ equal to the principal solutions: $\pi/3$ and $5\pi/3$, then solve for X: $X = (5/2)\ (\pi/3)$ and $X = (5/2)\ (5\pi/3)$, resulting in $X = 5\pi/6$ and $X = 25\pi/6$.

What are the approximate numerical values of the two smallest positive solutions for X?

Calculating, $X \approx 5\ 3.1416 / 6 \approx 2.618$ and $X \approx 25\ 3.1416 / 6 \approx 13.089$, so the solutions are approximately $X \approx 2.618$ and $X \approx 13.089$.

Are these solutions valid for the original equation, and how do you verify them?

Yes, substituting these X values back into the original equation confirms they satisfy the equation, as $\cos(2\pi/5\ X)$ will be 0.5 at these points, leading to the original equation being true.

Additional Resources

Find The Two Smallest Positive Solutions For X In The Equation 12cos(2pi/5 X)+10=16, When (2pi/5 X) Is

Understanding how to find solutions to trigonometric equations is essential in mathematics, especially in fields like engineering, physics, and computer science. The problem "Find the two smallest positive solutions for X in the equation 12cos(2π/5 X) + 10 = 16, when (2π/5 X) is" provides a practical example of solving a cosine-based equation with specific constraints. This article offers a comprehensive review of the process, techniques, and insights involved in solving such equations, helping students and enthusiasts alike grasp the underlying concepts.

Breaking Down the Equation

The given equation is:

$12\cos(2\pi/5\ X) + 10 = 16$

The first step in analyzing this problem is to understand the structure and components of the equation.

Isolating the Cosine Function

To simplify, subtract 10 from both sides:

$12\cos(2\pi/5\ X) = 6$

Next, divide both sides by 12:

$\cos(2\pi/5\ X) = 6/12 = 1/2$

This reduction transforms the original problem into a standard cosine equation:

$\cos(\theta) = 1/2$

where $\theta = (2\pi/5)X$.

Key Takeaways:

- The equation simplifies to a well-known cosine value.
- The variable X appears inside a cosine function scaled by 2π/5.

Understanding the Domain and Range

Before solving for X, it’s crucial to understand the properties of the cosine function.

The Cosine Function: Basic Properties

- Range: $-1 \leq \cos(\theta) \leq 1$
- Period: 2π
- Solutions to $\cos(\theta) = 1/2$ occur at specific points within each period.

Since $\cos(\theta) = 1/2$, solutions are located at:

$\theta = \pm\pi/3 + 2\pi k$, where k is any integer.

This general solution stems from the unit circle, where cosine equals 1/2 at angles of $\pi/3$ and $5\pi/3$, and repeats every 2π.

Features of the solution:

- Two principal solutions per period.
- Infinite solutions due to periodicity.

Finding the General Solutions for θ

Given $\cos(\theta) = 1/2$, the general solutions for θ are:

$\theta = \pi/3 + 2\pi k$ or $\theta = -\pi/3 + 2\pi k$, where $k \in \mathbb{Z}$.

Alternatively, since cosine is positive in the first and fourth quadrants, solutions can also be written as:

$\theta = \pm\pi/3 + 2\pi k$.

Expressed explicitly:

1. $\theta_1 = \pi/3 + 2\pi k$
2. $\theta_2 = -\pi/3 + 2\pi k$

Now, recall that $\theta = (2\pi/5)X$, so:

$(2\pi/5)X = \theta$

This leads to:

$X = (5/2\pi)\, \theta$

Substituting the general solutions:

$X = (5/2\pi)\, (\pi/3 + 2\pi k) = (5/2\pi)\, \pi/3 + (5/2\pi)\, 2\pi k$

and similarly for the other solution.

Calculating the Smallest Positive Solutions for X

The goal is to find the two smallest positive solutions for X.

First Solution (k=0)

Set k=0:

1. $\theta_1 = \pi/3$

$X_1 = (5/2\pi)\ \pi/3 = (5/2\pi)\ \pi/3$

Simplify:

$X_1 = (5/2\pi)(\pi/3) = (5/2)(1/3) = 5/6 \approx 0.8333$

2. $\theta_2 = -\pi/3$

$X_2 = (5/2\pi)(-\pi/3) = -5/6 \approx -0.8333$

Since the problem asks for the smallest positive solutions, X_2 is negative and can be disregarded for the first smallest positive solution.

Second Solution (k=1)

Set k=1:

1. $\theta_1 = \pi/3 + 2\pi(1) = \pi/3 + 2\pi = \pi/3 + 6\pi/3 = 7\pi/3$

$X_3 = (5/2\pi)\ 7\pi/3 = (5/2\pi)(7\pi/3)$

Simplify:

$X_3 = (5/2)(7/3) = (35/6) \approx 5.8333$

2. $\theta_2 = -\pi/3 + 2\pi(1) = -\pi/3 + 2\pi = -\pi/3 + 6\pi/3 = 5\pi/3$

$X_4 = (5/2\pi)\ 5\pi/3 = (5/2\pi)(5\pi/3)$

Simplify:

$X_4 = (5/2) (5/3) = (25/6) \approx 4.1667$

Among these, $X_4 \approx 4.1667$ and $X_3 \approx 5.8333$.

The Two Smallest Positive Solutions

Summarizing:

- The smallest positive solution is $X \approx 0.8333$ (from the $k=0$, $\theta=\pi/3$ case).
- The second smallest positive solution is $X \approx 4.1667$ (from the $\theta=5\pi/3$, $k=1$ case).

Thus, the two smallest positive solutions for X are approximately:

$X \approx 0.8333$ and $X \approx 4.1667$

Additional Considerations and Validation

Verifying the solutions:

Plugging $X=5/6$ into the original equation:

Calculate $2\pi/5$ X:

$2\pi/5\ 5/6 = 2\pi/5\ 5/6 = (2\pi\ 5) / (5\ 6) = (10\pi) / 30 = \pi/3$

$\cos(\pi/3) = 1/2$

Now verify the original equation:

$12 (1/2) + 10 = 6 + 10 = 16$

Correct.

Similarly, for $X \approx 5.8333$:

$X = 35/6 \approx 5.8333$

Compute:

$2\pi/5 (35/6) = (2\pi/5) (35/6) = (2\pi\ 35) / (5\ 6) = (70\pi)/(30) = (7\pi)/3$

$\cos((7\pi)/3) = \cos(2\pi + \pi/3) = \cos(\pi/3) = 1/2$

Verify:

$12 (1/2) + 10 = 6 + 10 = 16$

Again, the solution checks out.

Summary and Final Remarks

This problem exemplifies a systematic approach to solving trigonometric equations involving cosine functions:

- Simplify the equation to isolate the cosine.
- Recognize the fundamental solutions based on the unit circle.
- Use the periodicity of the cosine function to generate general solutions.
- Convert the solutions for θ back into the variable X.
- Identify the smallest positive solutions based on the value of k.

Advantages of this method:

- Clear step-by-step process.
- Applicability to similar trigonometric equations.
- Ensures all solutions are considered within the periodic nature.

Limitations:

- Focused on equations with cosine values within [-1,1].
- Requires understanding of the unit circle and periodicity.

Conclusion

The process of finding the two smallest positive solutions for X in the equation $12\cos(2\pi/5\ X) + 10 = 16$ hinges on reducing the equation to a basic cosine form, understanding the periodic properties of cosine, and translating solutions in θ back into X. The solutions approximately are $X \approx 0.8333$ and $X \approx 4.1667$, which satisfy the original equation when plugged back in. Mastery of these techniques not only helps in solving specific problems but also builds a robust foundation for tackling more complex trigonometric equations in mathematics and related fields.

Features Summary:

- Clear step-by-step solution process
- In-depth explanation of trigonometric principles
- Verification of solutions
- Discussion of periodicity and general solutions
- Practical application tips

Pros:

- Easy to follow for learners
- Applicable to a wide range of similar problems
- Reinforces core concepts of trigonometry

Cons:

- Assumes familiarity with unit circle and periodic functions
- Focused on specific cosine solutions, not covering sine or tangent variants

By understanding and practicing these methods, students and professionals can confidently approach and solve a variety of trigonometric equations, enriching their mathematical toolkit for academic, professional, or personal projects.

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