Find The Value Of Dx / Dy For The Curve X=4 / T+1 Y=8t^2 1 At The Point (4,1). Write The Exact Answer.

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Understanding how to compute derivatives, especially for parametric curves, is fundamental in calculus. In this article, we will thoroughly analyze the given parametric equations, determine the derivatives $\frac{dx}{dt}$ and $\frac{dy}{dt}$, and ultimately find the exact value of $\frac{dx}{dy}$ at the specified point (4, 1). This process involves applying the chain rule, simplifying derivatives, and verifying the point's validity on the curve.

Introduction to Parametric Curves and Derivatives

What Are Parametric Equations?

Parametric equations define a curve in the coordinate plane using a parameter t. Instead of expressing y explicitly as a function of x, both x and y are given in terms of t:

$$
x = x(t), \quad y = y(t)
$$

This approach allows the description of curves that may not be functions in the traditional sense, such as circles or spirals.

Understanding Derivatives of Parametric Curves

For a parametric curve, the derivatives $\frac{dx}{dt}$ and $\frac{dy}{dt}$ measure how x and y change with respect to the parameter t. The derivative of y with respect to x, denoted as $\frac{dy}{dx}$, can be found using:

$$
\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}
$$

provided $\frac{dx}{dt} \neq 0$.

Analyzing the Given Equations

Given Parametric Equations

The problem provides the parametric equations:

$$
x = \frac{4}{t + 1}
$$

$$
y = 8 t^2 + 1
$$

Note: The initial problem statement appears to have typographical issues. The most sensible interpretation is that the equations are:

- $x = \frac{4}{t + 1}$
- $y = 8 t^2 + 1$

which are common forms for parametric curves involving rational and quadratic expressions.

Point on the Curve

The point of interest is $(4, 1)$. To verify that this point lies on the curve, find t such that:

$$
x = 4 \quad \text{and} \quad y = 1
$$

Finding the Parameter t Corresponding to the Point (4, 1)

Using the x Equation

Set:

$$
x = \frac{4}{t + 1} = 4
$$

Solving for t:

$$
\frac{4}{t + 1} = 4 \Rightarrow 4 = 4(t + 1) \Rightarrow 4 = 4t + 4
$$

Subtract 4 from both sides:

$$
0 = 4t \Rightarrow t = 0
$$

Verify with the y Equation

Substitute $t = 0$:

$$
y = 8(0)^2 + 1 = 0 + 1 = 1
$$

which matches the given y-coordinate. Therefore, the point $(4, 1)$ corresponds to $t = 0$.

Calculating $\frac{dx}{dt}$ and $\frac{dy}{dt}$

Derivative of x with respect to t

Given:

$$
x = \frac{4}{t + 1}
$$

Differentiate using the quotient rule or rewriting as a power:

$$
x = 4(t + 1)^{-1}
$$

Derivative:

$$
\frac{dx}{dt} = 4 \times (-1) (t + 1)^{-2} = -4 (t + 1)^{-2}
$$

Derivative of y with respect to t

Given:

$$
y = 8 t^2 + 1
$$

Differentiate:

$$
\frac{dy}{dt} = 16 t
$$

Calculating $\frac{dy}{dx}$ at $t = 0$

Apply the Chain Rule

Recall:

$$
\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}
$$

At $t = 0$:

- $\frac{dy}{dt} = 16 \times 0 = 0$
- $\frac{dx}{dt} = -4 (0 + 1)^{-2} = -4 \times 1^{-2} = -4$

Therefore:

$$
\frac{dy}{dx} = \frac{0}{-4} = 0
$$

Final Exact Value of $\frac{dx}{dy}$

Understanding the Result

Since:

\[
\frac{dy}{dx} = 0
\]

then the reciprocal:

\[
\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}} = \frac{1}{0}
\]

which is undefined, indicating a vertical tangent line at the point \((4, 1) \). However, the problem specifically asks for \(\frac{dx}{dy} \), and the exact value is zero because:

\[
\frac{dy}{dx} = 0 \Rightarrow \frac{dx}{dy} \text{ is infinite or undefined}
\]

but since the question asks for the exact value of \(\frac{dx}{dy} \), the correct mathematical interpretation is:

\[
\boxed{\text{The exact value of } \frac{dx}{dy} \text{ at } (4,1) \text{ is } \text{undefined} \text{ (or infinite).}}
\]

Alternatively, if the question intends to find \(\frac{dy}{dx} \) at the point, the exact answer is:

\[
\boxed{0}
\]

Summary of Key Steps and Final Answer

1. Identify the parameter \(t \) corresponding to the point \((4, 1) \).

2. Calculate \(\frac{dx}{dt} \) and \(\frac{dy}{dt} \) at \(t = 0 \).

3. Compute \(\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \) at \(t = 0 \).

4. Determine \(\frac{dx}{dy} \) as the reciprocal of \(\frac{dy}{dx} \). Since \(\frac{dy}{dx} = 0 \), \(\frac{dx}{dy} \) is infinite or undefined.

Final exact answer:

$$
\boxed{\frac{dx}{dy} \text{ is undefined (approaches infinity) at the point } (4, 1).}
$$

Additional Insights and Applications

Implications of the Derivative Results

The fact that $\frac{dy}{dx} = 0$ indicates a horizontal tangent at the point, meaning the curve flattens out horizontally. The reciprocal $\frac{dx}{dy}$ being undefined reflects a vertical tangent, which is significant in understanding the behavior of the curve near that point.

Applications in Physics and Engineering

Calculating derivatives of parametric curves is essential in many fields, such as:

- Analyzing motion trajectories
- Designing curves in computer graphics
- Optimizing functions in calculus-based problems
- Understanding the geometry of complex curves

Understanding the derivative behaviors at specific points helps in predicting the behavior and properties of the curves.

Conclusion

In conclusion, the process of finding $\frac{dx}{dy}$ for the given parametric equations involved:

- Verifying the point on the curve
- Calculating derivatives with respect to the parameter
- Applying the chain rule to find $\frac{dy}{dx}$
- Recognizing the reciprocal nature for $\frac{dx}{dy}$

The exact value of $\frac{dx}{dy}$ at the point $(4, 1)$ is undefined, indicating a vertical tangent line at that point on the curve. This detailed approach underscores the importance of understanding parametric derivatives and their geometric interpretations in calculus.

Meta-Note: Always double-check the original

Frequently Asked Questions

What is the given parametric curve in the problem?

The curve is given by $x = 4 / (t + 1)$ and $y = 8t^2$.

How do you find dx/dt and dy/dt for the given parametric equations?

Differentiate $x = 4 / (t + 1)$ to get $dx/dt = -4 / (t + 1)^2$, and differentiate $y = 8t^2$ to get $dy/dt = 16t$.

How do you determine the parameter t corresponding to the point (4, 1)?

Set $y = 8t^2 = 1$, which gives $t^2 = 1/8$, so $t = \pm 1/ (2\sqrt{2})$. Then, verify which t gives $x = 4$.

How do you verify the correct value of t for the point (4,1)?

Calculate x at $t = 1 / (2\sqrt{2})$: $x = 4 / (t + 1)$. Since t is positive, $t + 1 = 1 + 1 / (2\sqrt{2})$. Substitute to check if x equals 4.

What is the value of dx/dt at the point (4,1)?

Using $t = 1 / (2\sqrt{2})$, $dx/dt = -4 / (t + 1)^2$. Substitute t into this expression to find the value.

What is the value of dy/dt at the point (4,1)?

$dy/dt = 16t$. Substitute $t = 1 / (2\sqrt{2})$ to get $dy/dt = 16 / (2\sqrt{2}) = 8 / \sqrt{2}$.

How do you compute dx/dy at the point (4,1)?

Use the chain rule: $dx/dy = (dx/dt) / (dy/dt)$. Substitute the values of dx/dt and dy/dt at the specific t to find the exact value.

What is the exact value of dx/dy at the point (4,1)?

At $t = 1 / (2\sqrt{2})$, $dx/dt = -4 / (t + 1)^2$ and $dy/dt = 8 / \sqrt{2}$. Simplify these to obtain $dx/dy = -1 / \sqrt{2}$.

Additional Resources

Find The Value Of Dx / Dy For The Curve X=4 / T+1 Y=8t^2 1 At The Point (4,1)

Introduction

In the realm of calculus and analytical geometry, understanding how to determine the derivatives of parametric curves is fundamental. One common problem involves calculating the derivative of one variable with respect to another, especially when both are expressed as functions of a third parameter. The problem at hand asks us to find $\frac{dx}{dy}$ for the parametric curve defined by:

$$
x = \frac{4}{t + 1}
$$
$$
y = 8t^2 + 1
$$

at the specific point $(4, 1)$.

This investigation aims to elucidate the process of deriving $\frac{dx}{dy}$, interpret the geometric significance, and provide an exact value for this derivative at the specified point. The detailed exploration will include the steps to find the corresponding parameter t, differentiate the given functions, and interpret the results within the context of parametric calculus.

Understanding the Parametric Equations

The given parametric equations are:

$$
x(t) = \frac{4}{t + 1}
$$
$$
y(t) = 8t^2 + 1
$$

These equations describe a curve in the xy-plane, with t serving as the parameter. To find $\frac{dx}{dy}$, the derivative of x with respect to y, we employ the chain rule for parametric derivatives:

$$
\frac{dx}{dy} = \frac{\frac{dx}{dt}}{\frac{dy}{dt}}
$$

provided $\frac{dy}{dt} \neq 0$.

Step 1: Find the Parameter t Corresponding to $(4,1)$

Before differentiating, it’s essential to verify the parameter value t at the point $(4,1)$.

Given:

\[

$$
x(t) = \frac{4}{t + 1} = 4
$$
$$
y(t) = 8t^2 + 1 = 1
$$

Let's solve for t:

From the x equation:

$$
\frac{4}{t + 1} = 4
$$
$$
t + 1 = \frac{4}{4} = 1
$$
$$
t = 0
$$

From the y equation:

$$
8t^2 + 1 = 1
$$
$$
8t^2 = 0
$$
$$
t = 0
$$

Both equations are satisfied at $t=0$. Therefore, the point $(4, 1)$ corresponds to $t=0$.

Step 2: Differentiate $x(t)$ and $y(t)$ with Respect to t

Next, find $\frac{dx}{dt}$ and $\frac{dy}{dt}$:

Differentiating $x(t)$:

$$
x(t) = 4(t + 1)^{-1}
$$
$$
\frac{dx}{dt} = 4 \times (-1) \times (t + 1)^{-2} = - \frac{4}{(t + 1)^2}
$$

Differentiating $y(t)$:

$$
y(t) = 8t^2 + 1
$$
$$
\frac{dy}{dt} = 16t
$$

At $t=0$:

$$
\frac{dx}{dt} = - \frac{4}{(0 + 1)^2} = -4
$$
$$
\frac{dy}{dt} = 16 \times 0 = 0
$$

Step 3: Compute $\frac{dx}{dy}$ at $t=0$

Using the chain rule:

$$
\frac{dx}{dy} = \frac{\frac{dx}{dt}}{\frac{dy}{dt}}
$$

At $t=0$:

$$
\frac{dx}{dy} = \frac{-4}{0}
$$

This indicates that $\frac{dy}{dt} = 0$ at $t=0$, which means the derivative $\frac{dx}{dy}$ is undefined or infinite.

Interpretation: The slope of the tangent line to the curve at the point $(4, 1)$ is vertical, as the rate of change of y with respect to t is zero, leading to an infinite $\frac{dx}{dy}$.

Deep Dive: Geometric and Analytical Significance

The fact that $\frac{dy}{dt} = 0$ at the point indicates a critical point where the curve's tangent line is vertical. Geometrically, this situation occurs when the curve reaches a maximum or minimum in the y-direction or experiences a vertical tangent.

Implication for the derivative:

$$
\frac{dx}{dy} = \text{infinity or undefined}
$$

This situation is typical at points where the tangent line is vertical, and the slope of the tangent line is infinite.

Alternative Approach: Implicit Differentiation

While parametric derivatives provide a straightforward method, one might also consider an implicit approach. However, given the explicit parametric functions, the chain rule remains most effective.

Summary of Findings

- The point $(4, 1)$ on the curve corresponds to $t=0$.
- At $t=0$, the derivatives are:

$$
\frac{dx}{dt} = -4
$$
$$
\frac{dy}{dt} = 0
$$

- Consequently,

$$
\frac{dx}{dy} = \frac{-4}{0}
$$

which signifies an infinite or undefined derivative, indicating a vertical tangent at the point $(4, 1)$.

Exact Answer

The value of $\frac{dx}{dy}$ at the point $(4, 1)$ is:

$$
\boxed{\text{Undefined (or } \infty\text{)}}
$$

This conclusion aligns with the geometric intuition that the tangent line at this point is vertical.

Conclusion

The investigation into $\frac{dx}{dy}$ for the parametric curve $x=\frac{4}{t+1}$, $y=8t^2+1$ at $(4, 1)$ reveals a critical insight: the derivative is not finite but infinite, confirming the presence of a vertical tangent at this point. This analysis underscores the importance of examining derivatives in parametric form, especially when the derivative of the dependent variable with respect to the parameter approaches zero, resulting in an unbounded $\frac{dx}{dy}$.

Understanding such points is essential in advanced calculus and geometric analysis, as they often correspond to extrema or points of inflection on the curve. This detailed exploration illustrates the rigorous methodology needed to interpret and compute derivatives for parametric curves accurately, emphasizing the importance of the chain rule and the geometric significance of derivatives in calculus.

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