Find The Vector X Determined By The Given Coordinate Vector [x]g And The Given Basis B. B = { [-1 2 0]

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When working with vector spaces, a common problem involves determining a vector $\mathbf{x}$ given its coordinate vector $[\mathbf{x}]_g$ with respect to a specific basis B. In this case, the basis B consists of a single vector $\mathbf{b}_1 = \left[-1, 2, 0 \right]$, and we are provided with a coordinate vector $[\mathbf{x}]_g$. Understanding how to find $\mathbf{x}$ from this information is essential in linear algebra, especially in contexts involving change of basis, vector representations, and transformations.

This article will guide you through the process of finding the vector $\mathbf{x}$ determined by a given coordinate vector $[\mathbf{x}]_g$ and the basis B. We will explore the foundational concepts, step-by-step procedures, and practical examples to clarify this important operation.

Understanding the Basis and Coordinates

What Is a Basis?

- A basis of a vector space is a set of vectors that are linearly independent and span the entire space.
- In the context of $\mathbb{R}^3$, a basis typically consists of three linearly independent vectors.
- However, in this particular case, the basis $B = \{ \mathbf{b}_1 \}$ contains only a single vector $\mathbf{b}_1 = \left[-1, 2, 0 \right]$. This indicates we are considering a subspace or a scenario where the vector space is one-dimensional or a specific subspace spanned by $\mathbf{b}_1$.

Coordinate Vectors with Respect to a Basis

- The coordinate vector $[\mathbf{x}]_g$ expresses the vector $\mathbf{x}$ as a linear combination of the basis vectors.
- For a basis $B = \{ \mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_n \}$, any vector in the span of B can be written as:

$$
\mathbf{x} = c_1 \mathbf{b}_1 + c_2 \mathbf{b}_2 + \ldots + c_n \mathbf{b}_n
$$

- The coefficients $c_1, c_2, \ldots, c_n$ form the coordinate vector $[\mathbf{x}]_g = \left[c_1, c_2, \ldots, c_n \right]$.

Steps to Find the Vector $\mathbf{x}$ from $[\mathbf{x}]_g$ and Basis B

Step 1: Understand the Given Data

- Identify the basis vectors $\mathbf{b}_i$. In this case:
$$
\mathbf{b}_1 = \left[-1, 2, 0 \right]
$$
- Recognize the coordinate vector $[\mathbf{x}]_g$, which provides the scalar multiples in the linear combination. Suppose:
$$
[\mathbf{x}]_g = [c] \quad \text{(since there is only one basis vector)}
$$
- The problem reduces to finding $\mathbf{x}$ given c.

Step 2: Write the Vector as a Linear Combination

- Since the basis contains only $\mathbf{b}_1$, the vector $\mathbf{x}$ can be expressed as:
$$
\mathbf{x} = c \mathbf{b}_1
$$
- Substituting $\mathbf{b}_1$:
$$
\mathbf{x} = c \left[-1, 2, 0 \right]
$$
- This simplifies to:
$$
\mathbf{x} = \left[-c, 2c, 0 \right]
$$

Step 3: Find the Scalar c from the Coordinate Vector

- The coordinate vector $[\mathbf{x}]_g$ gives the value of c. For example, if:
$$
[\mathbf{x}]_g = [c] = [3]
$$
- Then the vector $\mathbf{x}$ is:

$$
\mathbf{x} = 3 \times \left[-1, 2, 0 \right] = \left[-3, 6, 0 \right]
$$

Step 4: Generalize for Any Coordinate Scalar c

- For an arbitrary scalar c, the vector $\mathbf{x}$ is:
$$
\mathbf{x} = c \times \mathbf{b}_1 = c \left[-1, 2, 0 \right]
$$
- This results in:
$$
\mathbf{x} = \left[-c, 2c, 0 \right]
$$
- Therefore, the process to find $\mathbf{x}$ involves multiplying the basis vector by the scalar from the coordinate vector.

Practical Example: Finding $\mathbf{x}$ When Given a Specific Coordinate Vector

Given Data

- Basis $B = \{ \mathbf{b}_1 \}$, with:
$$
\mathbf{b}_1 = \left[-1, 2, 0 \right]
$$
- Coordinate vector $[\mathbf{x}]_g = [4]$

Solution

- The vector $\mathbf{x}$ is:
$$
\mathbf{x} = 4 \times \left[-1, 2, 0 \right] = \left[-4, 8, 0 \right]
$$
- So, the vector $\mathbf{x}$ determined by the given coordinate vector and basis is $\left[-4, 8, 0 \right]$.

Alternate Scenario: Zero Coordinate Vector

- If $[\mathbf{x}]_g = [0]$, then:
$$
\mathbf{x} = 0 \times \left[-1, 2, 0 \right] = \left[0, 0, 0 \right]
$$

- This indicates that the vector $\mathbf{x}$ is the zero vector in the space spanned by B.

Extending to Multiple Basis Vectors

While the current basis contains only a single vector, most practical applications involve multiple basis vectors. Here's how to approach those situations:

Representation with Multiple Basis Vectors

- Suppose the basis $B = \{ \mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3 \}$:
$$
\mathbf{b}_1 = \left[-1, 2, 0 \right], \quad \mathbf{b}_2 = \left[a_2, b_2, c_2 \right], \quad \mathbf{b}_3 = \left[a_3, b_3, c_3 \right]
$$
- The coordinate vector:
$$
[\mathbf{x}]_g = \left[c_1, c_2, c_3 \right]
$$
- The vector $\mathbf{x}$ is then:
$$
\mathbf{x} = c_1 \mathbf{b}_1 + c_2 \mathbf{b}_2 + c_3 \mathbf{b}_3
$$

Computational Steps for Multiple Vectors

- Multiply each basis vector by its corresponding scalar coefficient.
- Sum the resulting vectors component-wise.
- The sum gives the vector $\mathbf{x}$.

Example Calculation

- For basis vectors:
$$
\mathbf{b}_1 = \left[-1, 2, 0 \right], \quad \mathbf{b}_2 = \left[1, 0, 1 \right], \quad \mathbf{b}_3 = \left[0, 1, -1 \right]
$$
- Given coordinate vector:
$$
[\mathbf{x}]_g = [2, -1, 3]
$$
- The vector:
$$
\mathbf{x} = 2 \times \mathbf{b}_1 + (-1) \times \mathbf{b}_2 + 3 \times \mathbf{b}_3

\]
- Calculation:
\[
\mathbf{x} = 2 \times \left[-1, 2, 0 \right] + (-1) \times \left[1, 0

Frequently Asked Questions

How do I find the vector X given the coordinate vector [x]g and the basis B = {[-1, 2, 0]}?

To find the vector X, multiply each basis vector by its corresponding coordinate in [x]g and then sum the results. For a single basis vector, X = x [-1, 2, 0].

What is the process to convert coordinates relative to basis B into the standard vector form?

Multiply each basis vector by its coordinate component from [x]g and sum them up to obtain the standard vector form of X.

Given the basis B = {[-1, 2, 0]}, what is the vector X corresponding to coordinate [x]g = [3]?

X = 3 [-1, 2, 0] = [-3, 6, 0].

Can the basis B = {[-1, 2, 0]} be used to find any vector in R^3?

No, since B contains only one vector, it spans only a one-dimensional subspace of R^3. To represent any vector in R^3, a basis with three vectors is needed.

What if the coordinate vector [x]g has multiple components, how does that affect the calculation?

If [x]g has multiple components, say [x1, x2, ..., xn], and the basis has corresponding vectors, then you multiply each basis vector by its respective coordinate and sum all to find X.

Is the basis B = {[-1, 2, 0]} linearly independent?

Yes, a single non-zero vector like [-1, 2, 0] is linearly independent by itself.

How do I verify if a vector X is represented correctly by

the coordinate [x]g over basis B?

Multiply the basis vector by the coordinate and check if the resulting vector matches X. If yes, the representation is correct.

What are the limitations of using a single vector basis like B = {[-1, 2, 0]}?

A single vector basis can only represent vectors along the line spanned by that vector, not the entire space R^3.

How do I find the coordinate vector [x]g if I know the vector X and the basis B?

Since the basis has only one vector, you can find [x]g by taking the dot product of X with the basis vector and dividing by the basis vector's magnitude squared, provided the basis vector is not zero.

What is the significance of the basis vector B = {[-1, 2, 0]} in representing vectors in R^3?

It serves as a reference vector along which vectors are projected or scaled to obtain their coordinates in that one-dimensional subspace.

Additional Resources

Find The Vector X Determined By The Given Coordinate Vector [x]g And The Given Basis B is a fundamental problem in linear algebra that involves expressing a vector in terms of a specified basis. This task not only reinforces understanding of basis vectors and coordinate systems but also has practical applications in areas such as computer graphics, engineering, and data science where coordinate transformations are essential. In this article, we will explore the process of determining the vector $\mathbf{X}$ from a given coordinate vector relative to a basis B, analyze the steps involved, and discuss related concepts and applications.

Understanding the Problem: Coordinates and Basis Vectors

What is a Basis in a Vector Space?

A basis B of a vector space V is a set of vectors that are both linearly independent and span the entire space V. In simple terms, every vector in V can be uniquely represented as a linear combination of the basis vectors.

For example, given the basis $B = \{ \mathbf{b}_1 \}$, where
$$
\mathbf{b}_1 = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}
$$
we are considering a one-vector basis in a three-dimensional space.

Note: The basis B contains only one vector, which suggests that the space V in question is one-dimensional, spanned by $\mathbf{b}_1$. Any vector $\mathbf{X}$ in this space can be expressed as a scalar multiple of $\mathbf{b}_1$.

Coordinate Vectors and Their Significance

A coordinate vector $[x]_g$ with respect to basis B contains the coefficients that, when multiplied by the basis vectors and summed, produce the vector $\mathbf{X}$. For example, if
$$
[x]_g = c
$$
then
$$
\mathbf{X} = c \mathbf{b}_1
$$

The problem simplifies to finding $\mathbf{X}$ given $[x]_g$ and B.

Step-by-Step Process to Find $\mathbf{X}$

Step 1: Identify the Given Data

- Basis vector $\mathbf{b}_1 = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$
- Coordinate vector $[x]_g = c$ (a scalar, which must be provided or assumed for calculation)

Note: Since the problem statement only gives the basis vector and refers to a coordinate vector $[x]_g$, it's essential to specify or assume the value of c. For demonstration, let's assume c is given as some scalar c.

Step 2: Express the Vector $\mathbf{X}$ as a Linear Combination

Given a basis B, any vector $\mathbf{X}$ can be written as:
\[

$$
\mathbf{X} = c \mathbf{b}_1
$$
where c is the coordinate scalar.

Step 3: Calculate $\mathbf{X}$

The vector $\mathbf{X}$ is obtained by multiplying the basis vector by the scalar c:
$$
\mathbf{X} = c \times \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -c \\ 2c \\ 0 \end{bmatrix}
$$

Example: If $[x]_g = 3$, then
$$
\mathbf{X} = 3 \times \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -3 \\ 6 \\ 0 \end{bmatrix}
$$

Implications and Applications

Understanding the Representation

Expressing vectors in terms of basis vectors simplifies many problems, allowing transformations between different coordinate systems. This is crucial in computer graphics, where objects are represented in local coordinate systems and then transformed into world coordinates.

Applications in Engineering and Data Science

- Signal Processing: Basis vectors can represent fundamental signals; finding $\mathbf{X}$ helps in reconstructing signals from coefficients.
- Machine Learning: Feature vectors are often expressed relative to basis sets for dimensionality reduction or feature extraction.
- Robotics: Coordinate transformations depend on basis representations to navigate and manipulate objects.

Features and Characteristics of the Process

Features:
- Linearity: The process of finding $\mathbf{X}$ is linear, involving scalar multiplication.

- Uniqueness: Given a basis, the coordinate vector uniquely determines the vector $\mathbf{X}$.
- Simplicity: For a single basis vector, the calculation involves straightforward scalar multiplication.

Pros:
- Clear and systematic approach to vector representation.
- Facilitates coordinate transformations and basis change.
- Fundamental in understanding vector spaces.

Cons:
- Limited to the context where the basis is known and linearly independent.
- Can become complex with higher-dimensional bases or multiple vectors.

Extensions and Related Concepts

Changing Bases

The process can be extended to changing from one basis to another, involving inverse matrices or transition matrices. This is key for coordinate transformations across different reference frames.

Orthogonal and Orthonormal Bases

When the basis vectors are orthogonal or orthonormal, calculations simplify further, especially for projections and decompositions.

Coordinate System in Higher Dimensions

For bases with multiple vectors, the coordinate vector becomes a tuple (or vector) of scalars, and the process involves solving systems of linear equations.

Summary and Conclusion

Finding the vector $\mathbf{X}$ determined by a given coordinate vector $[x]_g$ and a basis $B = \{ \mathbf{b}_1 \}$ is a fundamental operation in linear algebra. The process involves recognizing that the vector can be expressed as a scalar multiple of basis vectors, with the scalar provided by the coordinate vector. For the specific basis $\mathbf{b}_1 = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$, $\mathbf{X}$ is obtained simply by multiplying $\mathbf{b}_1$ by the scalar c.

This approach underscores the importance of understanding basis vectors and coordinate systems in various fields, offering a powerful tool for analysis, transformation, and representation of vectors. Whether in pure mathematics, engineering, or data science, mastering this fundamental concept enables more complex operations such as basis changes, projections, and coordinate transformations, which are essential for advanced applications.

In conclusion, the problem of determining \(\mathbf{X} \) from a coordinate vector relative to a basis is straightforward yet profound. It exemplifies the elegance of linear algebra and its utility in translating between different coordinate frameworks, thereby facilitating a deeper understanding of multidimensional spaces and their representations.

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