Find The Weighted Average Of The Numbers 1 And 5 With One Fourth Of The Weight On The First Number And

Find The Weighted Average Of The Numbers 1 And 5 With One Fourth Of The Weight On The First Number And how to calculate weighted averages efficiently. Understanding weighted averages is essential in various fields such as mathematics, economics, finance, and data analysis. Whether you're calculating grades, investment returns, or statistical measures, grasping the concept of weights assigned to different values can significantly improve your data interpretation skills.

In this article, we will explore the process of calculating the weighted average of the numbers 1 and 5, specifically focusing on the scenario where one-fourth of the total weight is assigned to the first number. We will break down the steps involved, explain the underlying principles, and provide practical examples to enhance your understanding.

What Is A Weighted Average?

Definition and Importance

A weighted average is a type of average where each number in a set is multiplied by a weight that indicates its relative importance or frequency. Unlike a simple arithmetic mean, which gives equal importance to all values, a weighted average considers the significance of each data point.

For example, in calculating your final grade, different assignments might carry different weights. An exam might be worth 50%, while homework contributes 20%. The overall grade is then a weighted average of these scores.

Basic Formula

The general formula for the weighted average of a set of numbers $x_1, x_2, ..., x_n$ with corresponding weights $w_1, w_2, ..., w_n$ is:

$$
\text{Weighted Average} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}
$$

This formula ensures that each value's contribution is proportional to its weight.

Calculating Weighted Average of 1 and 5 with Given Weights

Understanding the Problem

Let's consider the specific problem:

Find the weighted average of the numbers 1 and 5, where one-fourth of the total weight is assigned to the first number.

This means:

- The first number (1) has a weight that is one-fourth of the total weight.
- The second number (5) has the remaining weight, which is three-fourths of the total weight.

To compute the weighted average, we need to determine the weights for each number based on this information.

Assigning the Weights

Suppose the total weight is W. Then:

- Weight for 1: $w_1 = \frac{1}{4} W$
- Weight for 5: $w_2 = \frac{3}{4} W$

Since the weights are proportional, the actual value of W cancels out in the calculation, so we can set $W = 1$ for simplicity:

- $w_1 = \frac{1}{4}$
- $w_2 = \frac{3}{4}$

Alternatively, you could assign any total weight, and the ratio remains the same.

Step-by-Step Calculation

Step 1: Write Down the Formula

Using the weighted average formula:

$$
\text{Weighted Average} = \frac{w_1 \times x_1 + w_2 \times x_2}{w_1 + w_2}
$$

where:

- $x_1 = 1$
- $x_2 = 5$
- $w_1 = \frac{1}{4}$
- $w_2 = \frac{3}{4}$

Step 2: Plug in the Values

Substituting the values:

$$
\text{Weighted Average} = \frac{\left(\frac{1}{4}\right) \times 1 + \left(\frac{3}{4}\right) \times 5}{\frac{1}{4} + \frac{3}{4}}
$$

Simplify numerator and denominator:

$$
\text{Weighted Average} = \frac{\frac{1}{4} + \frac{15}{4}}{1}
$$

since:

$$
\left(\frac{1}{4} \times 1\right) = \frac{1}{4}
$$
$$
\left(\frac{3}{4} \times 5\right) = \frac{15}{4}
$$

and the sum of weights:

$$
\frac{1}{4} + \frac{3}{4} = 1
$$

Step 3: Simplify the Expression

Adding the numerators:

$$
\frac{1}{4} + \frac{15}{4} = \frac{16}{4} = 4
$$

Dividing by the total weight (which is 1):

$$
\frac{4}{1} = 4
$$

Therefore, the weighted average is 4.

Why Is This Calculation Useful?

Applications in Real Life

- Educational Grading: Assigning different weights to exams, quizzes, and projects to compute final grades.
- Finance: Calculating weighted average cost of capital (WACC) based on the proportion of debt and equity.
- Statistics: Combining data points with varying degrees of importance or reliability.
- Business Decisions: Analyzing sales data where certain products or regions have more impact.

Understanding Variations

The methodology used here can be adapted to more complex scenarios involving multiple data points and different weight distributions. The key is understanding the ratio of weights and ensuring they sum to the total.

Additional Examples and Practice Problems

Example 1: Weighted Average of Three Numbers

Suppose you want to find the weighted average of 2, 4, and 6, with weights 1, 2, and 3 respectively.

Solution:

$$
\text{Weighted Average} = \frac{1 \times 2 + 2 \times 4 + 3 \times 6}{1 + 2 + 3} = \frac{2 + 8 + 18}{6} = \frac{28}{6} \approx 4.67
$$

Practice Problem

Calculate the weighted average of the numbers 10 and 20, where the weight of 10 is one-third of the total weight.

Solution Steps:
1. Assign weights based on the ratio.
2. Use the formula to compute the weighted average.

Key Takeaways

- The weighted average considers the importance or frequency of each value.
- Assign weights proportionally and normalize if necessary.
- The formula simplifies calculations, especially when weights are expressed as ratios.
- Understanding weighted averages is vital for accurate data analysis across various disciplines.

Conclusion

Finding the weighted average of 1 and 5 with one-fourth of the weight on the first number is straightforward once you understand the principles of weight assignment and the basic formula. By assigning the appropriate weights and applying the weighted average formula, you can accurately compute combined metrics that reflect the relative importance of each data point. Mastering this concept enables you to handle more complex datasets and make informed decisions based on weighted data analysis.

Whether you're a student, professional, or data enthusiast, grasping how to find and interpret weighted averages enhances your analytical skills and improves your ability to interpret real-world data effectively.

Frequently Asked Questions

How do you calculate the weighted average of 1 and 5 when one-fourth of the total weight is assigned to 1?

First, assign weights: since one-fourth of the total weight is on 1, its weight is 1/4. The remaining weight is 3/4, which goes to 5. Then, calculate the weighted average: (1 1/4 + 5 3/4) / (1/4 + 3/4) = (1/4 + 15/4) / 1 = 16/4 = 4.

What is the formula to find the weighted average when given specific weights for numbers?

The formula is: Weighted Average = (Number1 Weight1 + Number2 Weight2) / (Weight1 + Weight2). In this case, with one-fourth weight on 1, the remaining weight on 5 is three-fourths.

If the total weight is 1, how much weight is assigned to each number in this problem?

One-fourth of the total weight (1/4) is assigned to 1, and the remaining three-fourths (3/4) is assigned to 5.

What is the significance of assigning weights in a weighted average problem like this?

Assigning weights reflects the relative importance or contribution of each number to the average. In this problem, the weight on 1 is less, so it influences the average less than 5.

Can you explain why the weighted average of 1 and 5 with one-fourth weight on 1 results in 4?

Yes. Since 1 has a weight of 1/4 and 5 has a weight of 3/4, the average is (1 1/4 + 5 3/4) = (1/4 + 15/4) = 16/4 = 4. The total weight sums to 1, so the weighted average is 4.

Additional Resources

Find The Weighted Average Of The Numbers 1 And 5 With One Fourth Of The Weight On The First Number And

In mathematics and statistics, calculating averages often goes beyond simply adding numbers and dividing by their count. When different elements contribute unequally to an overall measure, weighted averages come into play. This approach assigns various weights to each number based on their significance or proportion, offering a more nuanced understanding of the data set. Today, we delve into an illustrative example: determining the weighted average of the numbers 1 and 5, where one-fourth of the total weight is assigned to the first number. This problem not only demonstrates the mechanics of weighted averages but also underscores their practical importance across fields like finance, engineering, and data analysis.

Understanding the Concept of Weighted Averages

What Is a Weighted Average?

A weighted average is a type of mean where each data point contributes proportionally to its assigned weight. Unlike a simple average—which treats all numbers equally—a weighted average recognizes that some elements may be more influential or relevant than others. This method provides a more accurate reflection of the data's overall significance, especially when combining data from sources with varying importance levels.

Mathematically, the weighted average of a set of numbers $x_1, x_2, ..., x_n$, with corresponding weights $w_1, w_2, ..., w_n$, is expressed as:

$$
\text{Weighted Average} = \frac{w_1 x_1 + w_2 x_2 + \dots + w_n x_n}{w_1 + w_2 + \dots + w_n}
$$

Here, the numerator combines each number multiplied by its weight, summing to a total that reflects their weighted contributions, while the denominator sums all weights to normalize the result.

Importance of Weights in Calculations

Weights serve as indicators of the relative importance or contribution of each data point. For instance:

- In academic grading, homework, quizzes, and exams may carry different weights.
- In investment portfolios, different assets contribute variably to overall returns.
- In survey analysis, certain demographics may have higher representation, affecting overall results.

By assigning appropriate weights, analysts and decision-makers can derive averages that more precisely mirror the significance of each component within the larger context.

Applying the Concept: The Given Problem

Restating the Problem

The problem asks: "Find the weighted average of the numbers 1 and 5, with one-fourth of the total weight assigned to the first number." This statement implies that:

- There are two numbers: 1 and 5.
- The total weight is distributed such that the first number (1) receives one-fourth ($\frac{1}{4}$) of the total weight.
- The remaining three-fourths ($\frac{3}{4}$) of the weight is allocated to the second number (5).

This setup ensures that the weights are proportional to these fractions, which sets the stage for our calculations.

Determining the Weights

Since the total weight is divided into parts, and the first number receives one-fourth of the total weight, the weights can be represented as:

- Weight on 1: $w_1 = \frac{1}{4} W$
- Weight on 5: $w_2 = \frac{3}{4} W$

Where W is the total weight, which can be assigned an arbitrary value (commonly 1 for simplicity). Choosing $W = 1$ simplifies calculations without affecting the ratio, because the weighted average depends on the ratios of weights, not their absolute values.

Thus,

$$
w_1 = \frac{1}{4} \times 1 = \frac{1}{4}
$$
$$
w_2 = \frac{3}{4} \times 1 = \frac{3}{4}
$$

Calculating the Weighted Average

Step-by-Step Calculation

Using the formula for weighted average:

\[

\text{Weighted Average} = \frac{w_1 \times x_1 + w_2 \times x_2}{w_1 + w_2}
\]

Substituting the values:

\[
= \frac{\left(\frac{1}{4}\right) \times 1 + \left(\frac{3}{4}\right) \times 5}{\frac{1}{4} + \frac{3}{4}}
\]

Calculating numerator:

\[
= \frac{\frac{1}{4} + \frac{15}{4}}{\frac{4}{4}}
\]

Adding the numerators:

\[
= \frac{\frac{1 + 15}{4}}{1}
\]

\[
= \frac{\frac{16}{4}}{1}
\]

Simplifying numerator:

\[
= \frac{4}{1} = 4
\]

Since the denominator sums to 1, the weighted average is simply 4.

Interpretation of the Result

The computed weighted average of 4 indicates that, given the specified weighting scheme, the combined measure of the two numbers leans heavily towards the value 5, as expected due to its larger weight. Despite the lower contribution of 1, the higher weight assigned to 5 pulls the overall average closer to itself, reflecting the weighted influence.

Practical Implications and Variations

Why Use Weighted Averages?

Weighted averages are essential when data points are not equally significant. For example:

- Academic Grading: Final grades often depend more heavily on exam scores than on homework.
- Financial Analysis: Portfolio returns consider the proportion of investment in each asset.
- Manufacturing: Quality metrics may weigh more heavily on critical components.

In all these cases, understanding how to assign and manipulate weights is crucial for accurate analysis.

Adjusting the Weight Distribution

Suppose the problem changes, and instead of one-fourth, the weight on the first number is half (½). The process would be similar:

- Assign weights: $w_1 = \frac{1}{2}$, $w_2 = \frac{1}{2}$.
- Calculate the weighted average:

$$
= \frac{\frac{1}{2} \times 1 + \frac{1}{2} \times 5}{\frac{1}{2} + \frac{1}{2}} = \frac{\frac{1}{2} + \frac{5}{2}}{1} = \frac{\frac{6}{2}}{1} = 3
$$

This demonstrates how changing weights affects the overall average, shifting it closer to the more heavily weighted number.

Common Mistakes and Clarifications

Misinterpreting the Weight Fractions

A common mistake is to incorrectly assign weights based on fractions that do not sum to 1. Remember, the weights should reflect the relative importance, and their sum can be any value—often normalized to 1 for simplicity, but not mandatory.

Ensuring Correct Application of the Formula

Always verify:

- Correctly multiply each number by its weight.
- Sum all weighted products in the numerator.
- Sum all weights in the denominator.
- Simplify carefully to avoid arithmetic errors.

Conclusion: The Power of Weighted Averages

Calculating the weighted average of two numbers, such as 1 and 5, with a specified weight distribution, exemplifies a fundamental yet powerful statistical method. In our case, assigning one-fourth of the total weight to 1 and three-fourths to 5 yields an average of 4, illustrating how weights influence the combined measure. Mastery of weighted averages enables analysts, students, and professionals across various domains to derive more accurate insights, tailor calculations to context-specific importance, and make informed decisions based on nuanced data interpretations.

By understanding the underlying principles—how weights are assigned, how they influence the overall average, and how to perform the calculations—readers can confidently approach similar problems in academic, professional, or everyday scenarios. Whether evaluating grades, financial portfolios, or survey results, the concept of weighted averages remains an indispensable tool for meaningful data analysis.

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