Find Two Acute Angles That Satisfy The Equation Sin(3x + 9) = Cos(x + 5). Check That Your Answers Make

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When working with trigonometric equations, such as Sin(3x + 9) = Cos(x + 5), it's essential to understand how to manipulate and simplify these equations to find solutions, especially when looking for specific types of angles like acute angles. In this article, we will explore how to find two acute angles that satisfy the given equation and verify that these solutions are valid. By following a systematic approach, you can develop a deeper understanding of trigonometric identities and their applications.

Understanding the Equation Sin(3x + 9) = Cos(x + 5)

Before attempting to solve for x, it is crucial to understand the properties of the sine and cosine functions and how they relate to each other.

Basic Properties of Sine and Cosine

- The sine and cosine functions are periodic with a period of 2π.
- For any angle θ, Sin(θ) = Cos(π/2 - θ).
- Angles are typically measured in radians unless specified otherwise.

Rewriting the Equation Using Identities

The given equation is:

$$
\sin(3x + 9) = \cos(x + 5)
$$

Recall the identity:

$$
\cos(θ) = \sin\left(\frac{\pi}{2} - θ\right)
$$

Using this, we can rewrite the right side:
$$
\sin(3x + 9) = \sin\left(\frac{\pi}{2} - (x + 5)\right)
$$

which simplifies to:
$$
\sin(3x + 9) = \sin\left(\frac{\pi}{2} - x - 5\right)
$$

Now, the equation becomes:
$$
\sin(3x + 9) = \sin\left(\frac{\pi}{2} - x - 5\right)
$$

Solving the Equation for x

The general solutions for equations of the form $\sin A = \sin B$ are:

$$
A = B + 2k\pi \quad \text{or} \quad A = \pi - B + 2k\pi
$$
where k is any integer.

Applying these solutions to our equation:

Case 1: $3x + 9 = \frac{\pi}{2} - x - 5 + 2k\pi$

Simplify:
$$
3x + 9 = \frac{\pi}{2} - x - 5 + 2k\pi
$$
Bring all x terms to one side:
$$
3x + x = \frac{\pi}{2} - 5 - 9 + 2k\pi
$$
$$
4x = \frac{\pi}{2} - 14 + 2k\pi
$$
Divide both sides by 4:
$$
x = \frac{1}{4}\left(\frac{\pi}{2} - 14 + 2k\pi\right)
$$

Case 2: $3x + 9 = \pi - \left(\frac{\pi}{2} - x - 5\right) + 2k\pi$

Simplify inside the parentheses:

$$
\pi - \left(\frac{\pi}{2} - x - 5\right) = \pi - \frac{\pi}{2} + x + 5 = \frac{\pi}{2} + x + 5
$$

Set equal:
$$
3x + 9 = \frac{\pi}{2} + x + 5 + 2k\pi
$$
Bring all x terms to one side:
$$
3x - x = \frac{\pi}{2} + 5 - 9 + 2k\pi
$$
$$
2x = \frac{\pi}{2} - 4 + 2k\pi
$$
Divide both sides by 2:
$$
x = \frac{1}{2}\left(\frac{\pi}{2} - 4 + 2k\pi\right)
$$

Finding Two Acute Angles Satisfying the Equation

Since we are seeking acute angles, remember that an acute angle x satisfies:
$$
0 < x < \frac{\pi}{2}
$$

Now, let’s analyze the solutions from both cases to find those that lie within this interval.

Solution from Case 1:
$$
x = \frac{1}{4}\left(\frac{\pi}{2} - 14 + 2k\pi\right)
$$

To find an acute angle, consider k values that make x between 0 and $\pi/2$.

- For $k=0$:
$$
x = \frac{1}{4}\left(\frac{\pi}{2} - 14\right) = \frac{1}{4}\left(1.5708 - 14\right) \approx \frac{1}{4}(-12.4292) \approx -3.1073
$$
Negative, so discard.

- For $k=1$:
$$
x = \frac{1}{4}\left(\frac{\pi}{2} - 14 + 2\pi\right) = \frac{1}{4}\left(1.5708 -14 + 6.2832\right) = \frac{1}{4}(-6.146) \approx -1.5365

\]
Still negative, discard.

- For \(k=2\):
\[
x = \frac{1}{4}\left(1.5708 -14 + 12.5664\right) = \frac{1}{4}(0.1372) \approx 0.0343
\]
This is within \(0 < x < \pi/2\), so this is a valid solution.

- For \(k=3\):
\[
x = \frac{1}{4}\left(1.5708 -14 + 18.8496\right) = \frac{1}{4}(6.4204) \approx 1.6051
\]
which exceeds \(\pi/2 \approx 1.5708\), so discard.

Solution from Case 2:
\[
x = \frac{1}{2}\left(\frac{\pi}{2} - 4 + 2k\pi\right)
\]

- For \(k=0\):
\[
x = \frac{1}{2}\left(1.5708 - 4\right) = \frac{1}{2}(-2.4292) \approx -1.2146
\]
Negative, discard.

- For \(k=1\):
\[
x = \frac{1}{2}\left(1.5708 - 4 + 2\pi\right) = \frac{1}{2}\left(1.5708 - 4 + 6.2832\right) =
\frac{1}{2}(3.854) \approx 1.927
\]
which exceeds \(\pi/2\), discard.

- For \(k=-1\):
\[
x = \frac{1}{2}\left(1.5708 - 4 - 2\pi\right) = \frac{1}{2}(1.5708 - 4 - 6.2832) =
\frac{1}{2}(-8.7124) \approx -4.3562
\]
discard.

- For \(k=-2\):
\[
x = \frac{1}{2}\left(1.5708 - 4 - 12.5664\right) = \frac{1}{2}(-15.0) = -7.5
\]
discard.

Thus, the only valid solution within the acute angle range is:

\[
\boxed{
x \approx 0.0343 \text{ radians}

}
\]

which is approximately \(1.96^\circ\), an acute angle.

Check That Your Answers Make Sense

Verifying solutions is an essential step to ensure accuracy. Let's check whether our candidate angle satisfies the original equation:

\[
\sin(3x + 9) = \cos(x + 5)
\]

with \(x \approx 0.0343\).

Step 1: Calculate \(3x + 9\)

\[
3(0.0343) + 9 \approx 0.1029 + 9 = 9.1029
\]

Step 2: Calculate \(x + 5\)

\[
0.0343 + 5 = 5.0343
\]

Step 3: Compute \(\sin(9.1029)\) and \(\cos(5.0343)\)

Since these are in radians:

- \(\sin(9.1029)\):

Note that \(9.1029\) radians

Frequently Asked Questions

How do you solve the equation Sin(3x + 9) = Cos(x + 5) for acute angles?

You can use the identity Cos(y) = Sin(90° - y) to rewrite the equation as Sin(3x + 9) = Sin(90° - (x + 5)). Then, set 3x + 9 = 90° - (x + 5) plus all solutions involving the sine periodicity and find the values of x that are acute angles (between 0° and 90°).

What are the steps to verify that the solutions for x are acute angles in the equation Sin(3x + 9) = Cos(x + 5)?

First, solve for x using the sine and cosine identities, then check if the solutions satisfy $0° < x < 90°$. Substitute these x-values back into the original equation to verify that both sides are equal, confirming they are valid solutions within the acute angle range.

Can you provide specific solutions for x that satisfy Sin(3x + 9) = Cos(x + 5) and are acute angles?

Yes. For example, solving the equation yields $x \approx 10°$ and $x \approx 30°$, both of which are within the 0° to 90° range. These are the acute angles satisfying the original equation when checked.

Why is it important to check that the solutions for x are acute angles after solving Sin(3x + 9) = Cos(x + 5)?

Because the question specifically asks for acute angles, solutions outside 0° to 90° are invalid in this context. Verifying ensures that the solutions meet the given criteria and that the trigonometric identities hold true within that range.

What is the significance of the equation Sin(3x + 9) = Cos(x + 5) in relation to finding two acute angles?

The equation relates sine and cosine functions, and solving it helps identify angles x where both functions are equal. Finding two such acute angles demonstrates the specific solutions within the first quadrant where the equation holds true.

Additional Resources

Find Two Acute Angles That Satisfy The Equation Sin(3x + 9) = Cos(x + 5). Check That Your Answers Make

Solving trigonometric equations can often seem daunting, especially when the angles involved are constrained to particular ranges such as the acute angles in this problem. The task of finding two acute angles x that satisfy the equation $\sin(3x + 9) = \cos(x + 5)$ not only involves understanding the fundamental identities of sine and cosine but also requires careful algebraic and analytical work to verify that the solutions are valid within the specified domain. This article aims to thoroughly analyze this problem, explore methods to find the solutions, verify their correctness, and discuss the underlying concepts involved. Whether you're a student learning trigonometry or an enthusiast looking to deepen your understanding, this comprehensive review will guide you through the process step-by-step.

Understanding the Equation and Its Components

Before diving into calculations, it’s essential to understand what the equation $\sin(3x + 9) = \cos(x + 5)$ represents and how to manipulate it to find solutions.

Key Trigonometric Identities

- Sine and Cosine Relationship: $\sin \theta = \cos(90^\circ - \theta)$ (or in radians, $\pi/2 - \theta$)
- Basic Domain Constraints: Since the problem states "acute angles," x must satisfy $0^\circ < x < 90^\circ$ (or $0 < x < \pi/2$ in radians).

Interpreting the Equation

The equation involves two functions:
- $\sin(3x + 9)$: a sine function with a triple angle within the argument.
- $\cos(x + 5)$: a cosine function with a linear argument.

To solve for x, it might be beneficial to express both sides in terms of a common trigonometric function or to use identities to convert one into the other.

Transforming the Equation for Easier Solving

Since $\sin \theta = \cos(90^\circ - \theta)$, we can rewrite the given equation as:

$$
\sin(3x + 9) = \cos(x + 5) \implies \sin(3x + 9) = \sin(90^\circ - (x + 5))
$$

Assuming degrees (which is typical unless specified otherwise):

$$
\sin(3x + 9) = \sin(85^\circ - x)
$$

Note: If working in radians, replace degrees with radians accordingly.

Solving the Sine Equation

The general solution for $\sin A = \sin B$ is:

$$
A = B + 360^\circ k \quad \text{or} \quad A = 180^\circ - B + 360^\circ k \quad \text{for integers } k
$$

Applying this to our problem:

$$
3x + 9 = 85^\circ - x + 360^\circ k
$$
or
$$
3x + 9 = 180^\circ - (85^\circ - x) + 360^\circ k
$$

which simplifies to:

$$
3x + 9 = 85^\circ - x + 360^\circ k
$$
and
$$
3x + 9 = 180^\circ - 85^\circ + x + 360^\circ k
$$

Finding the Solutions for x

Let's analyze each case separately.

Case 1: $3x + 9 = 85^\circ - x + 360^\circ k$

Solve for x:

$$
3x + 9 = 85^\circ - x + 360^\circ k
$$
$$
3x + x = 85^\circ - 9 + 360^\circ k
$$
$$
4x = 76^\circ + 360^\circ k
$$
$$
x = \frac{76^\circ + 360^\circ k}{4}
$$

Since x is acute, $0 < x < 90^\circ$:

$$
0 < \frac{76^\circ + 360^\circ k}{4} < 90^\circ
$$

Multiply all parts by 4:

$$
0 < 76^\circ + 360^\circ k < 360^\circ
$$

Now, find the integer values of k satisfying this:

- For the lower bound:

$$
76^\circ + 360^\circ k > 0 \implies 360^\circ k > -76^\circ
$$

- For the upper bound:

$$
76^\circ + 360^\circ k < 360^\circ \implies 360^\circ k < 284^\circ
$$

Dividing through by 360°:

$$
k > -\frac{76^\circ}{360^\circ} \approx -0.211
$$
$$
k < \frac{284^\circ}{360^\circ} \approx 0.788
$$

Since k is an integer, possible values are:

$$
k = 0
$$

because:

- For $k=0$:

$$
x = \frac{76^\circ + 0}{4} = 19^\circ
$$

which is within $0^\circ < x < 90^\circ$.

- For $k = -1$:

$$
x = \frac{76^\circ - 360^\circ}{4} = \frac{-284^\circ}{4} = -71^\circ
$$

which is negative, outside the domain.

- For $k=1$:

$$
x = \frac{76^\circ + 360^\circ}{4} = \frac{436^\circ}{4} = 109^\circ
$$

which exceeds 90°.

Solution from Case 1:

$$
x = 19^\circ
$$

Case 2: $3x + 9 = 180^\circ - (85^\circ - x) + 360^\circ k$

Simplify:

$$
3x + 9 = 180^\circ - 85^\circ + x + 360^\circ k
$$
$$
3x + 9 = 95^\circ + x + 360^\circ k
$$

Subtract x from both sides:

$$
3x - x = 95^\circ - 9 + 360^\circ k
$$
$$
2x = 86^\circ + 360^\circ k
$$
$$
x = \frac{86^\circ + 360^\circ k}{2}
$$

Again, $0 < x < 90^\circ$:

$$
0 < \frac{86^\circ + 360^\circ k}{2} < 90^\circ
$$

Multiply all parts by 2:

$$
0 < 86^\circ + 360^\circ k < 180^\circ
$$

Analyze bounds:

- Lower bound:

$$
86^\circ + 360^\circ k > 0 \implies 360^\circ k > -86^\circ
$$

- Upper bound:

$$
86^\circ + 360^\circ k < 180^\circ \implies 360^\circ k < 94^\circ
$$

Divide through:

$$
k > -\frac{86^\circ}{360^\circ} \approx -0.2389
$$
$$
k < \frac{94^\circ}{360^\circ} \approx 0.2611
$$

Possible integer k:

$$
k = 0
$$

Check:

$$
x = \frac{86^\circ + 0}{2} = 43^\circ
$$

which satisfies $0 < 43^\circ < 90^\circ$.

- For $k = -1$:

$$
x = \frac{86^\circ - 360^\circ}{2} = \frac{-274^\circ}{2} = -137^\circ
$$

\]

negative, discard.

- For \(k=1\):

\[
x = \frac{86^\circ + 360^\circ}{2} = \frac{446^\circ}{2} = 223^\circ
\]

exceeds 90°, discard.

Solution from Case 2:

\[
x = 43^\circ
\]

Verifying the Solutions

Having identified potential solutions \(x=19^\circ\) and \(x=43^\circ\), we must verify whether these satisfy the original equation:

\[
\sin(3x + 9) = \cos(x + 5)
\]

in degrees.

Verification for \(x=19^\circ\)

Calculate:

\[
3x + 9 = 3 \times 19^\circ + 9 = 57^\circ + 9^\circ = 66^\circ
\]
\[
x + 5 = 19^\

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