For What Value Of X Is...The Square Of The Binomial X+1 One Hundred And Twenty Greater Than The Square

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Understanding algebraic expressions and solving for unknown variables is a fundamental aspect of mathematics. One common problem involves comparing the square of a binomial to another value and determining the specific value of the variable that satisfies this condition. In this article, we will explore the problem: "For what value of x is the square of the binomial x+1 one hundred and twenty greater than the square?" We will analyze this problem in detail, break down the steps to find the solution, and provide tips for solving similar algebraic equations. This comprehensive guide is designed for students, educators, and math enthusiasts seeking a clear understanding of the concepts involved.

Understanding the Problem Statement

Before delving into the solution, it's crucial to interpret the problem accurately. The statement can be rephrased as:

Find the value of x such that the square of the binomial (x + 1) exceeds the square of some other expression by 120.

However, the phrase "the square of the binomial x+1 one hundred and twenty greater than the square" suggests that the relationship involves the square of (x + 1) being 120 greater than some other square.

Key points to clarify:

- The primary expression is $(x + 1)^2$.
- It's greater than another square by 120.
- The other square could be of a different binomial or expression, but based on common algebraic problems, it's most likely the square of x, i.e., x^2.

Therefore, the problem can be phrased as:

For what value of x is $(x + 1)^2$ exactly 120 greater than x^2?

Formulating the Algebraic Equation

Based on the interpretation above, the problem reduces to solving for x in the equation:

$(x + 1)^2 = x^2 + 120$

Let's analyze this step-by-step.

Step 1: Expand the binomial $(x + 1)^2$

Recall the binomial expansion:

$(x + 1)^2 = x^2 + 2x + 1$

Step 2: Set up the equation

Substitute the expansion into the original statement:

$x^2 + 2x + 1 = x^2 + 120$

Step 3: Simplify the equation

Subtract x^2 from both sides:

$2x + 1 = 120$

Solving the Equation for x

Now, solve for x:

1. Subtract 1 from both sides:

$2x = 119$

2. Divide both sides by 2:

$x = 119 / 2$

3. Simplify:

$x = 59.5$

Verification of the Solution

To ensure the correctness of the solution, substitute $x = 59.5$ back into the original relationship:

- Compute $(x + 1)^2$:

$(59.5 + 1)^2 = (60.5)^2 = 60.5 \times 60.5 = 3660.25$

- Compute x^2:

$(59.5)^2 = 59.5 \times 59.5 = 3540.25$

- Find the difference:

$3660.25 - 3540.25 = 120$

This confirms that the difference between the square of $(x + 1)$ and x^2 is indeed 120.

Generalizing the Approach to Similar Problems

The problem-solving process demonstrated above can be generalized to other algebraic equations involving binomials and squares. Below are steps and tips for tackling similar problems:

Step 1: Carefully interpret the problem statement

- Identify the expressions involved
- Clarify what is being compared or equated
- Determine the relationship (e.g., equal, greater than, less than)

Step 2: Express the problem mathematically

- Write the algebraic equation based on the problem
- Expand binomials or simplify as needed

Step 3: Isolate the variable

- Simplify the equation to linear or quadratic form
- Use algebraic operations to solve for the variable

Step 4: Verify the solution

- Substitute your solution back into the original equation
- Confirm the relationship holds true

Common Types of Equations in Similar Problems:

- Linear equations
- Quadratic equations
- Equations involving absolute value
- Inequalities involving binomials

Additional Examples and Practice Problems

To deepen understanding, here are some related practice problems:

- Find x if $(x + 2)^2$ is 50 greater than x^2.
- Determine the value of x when $(2x + 3)^2$ is 200 less than $4x^2$.
- For what x is the difference between $(x + 4)^2$ and x^2 equal to 96?
- Find x such that the sum of $(x + 1)^2$ and x^2 equals 300.

Each of these problems follows similar logical steps: expand, set up equations, simplify, and solve.

Conclusion: Key Takeaways

- The problem "For what value of x is the square of the binomial x+1 one hundred and twenty greater than the square" simplifies to solving the equation $(x + 1)^2 = x^2 + 120$.
- Expansion of binomials and basic algebraic operations are essential tools.
- The solution $x = 59.5$ demonstrates how algebraic manipulations lead to precise answers.
- Verification ensures the solution's accuracy.
- Understanding the methodology allows for solving a wide range of similar algebraic problems involving squares and binomials.

Optimizing for SEO: Frequently Asked Questions (FAQs)

Q1: How do I solve for x when the square of a binomial is greater than another square by a certain amount?

A1: Expand the binomials, set up the algebraic equation, simplify, and solve for x. Always verify your solution by substituting back into the original relationship.

Q2: What are common mistakes to avoid when solving algebraic equations involving squares?

A2: Common mistakes include incorrect expansion, missing negative signs, dividing by zero, or forgetting to verify solutions. Double-check each step for accuracy.

Q3: Can this approach be applied to non-quadratic algebraic problems?

A3: While this approach is specific to quadratic equations involving squares, similar principles apply to linear and higher-degree polynomial equations with appropriate modifications.

Q4: Why is verification important in solving algebraic equations?

A4: Verification confirms that the obtained solution satisfies the original problem, preventing errors due to algebraic manipulation mistakes.

Final Thoughts

Mastering algebraic problem-solving involving binomials and squares is essential for higher-level mathematics and real-world applications. By breaking down complex statements into manageable equations, expanding, simplifying, and verifying solutions, students and enthusiasts can develop confidence and proficiency in algebra. Remember that practice makes perfect—regularly tackling similar problems will strengthen your skills and deepen your understanding of algebraic concepts.

Keywords: algebra, binomials, quadratic equations, solving for x, algebraic expressions, binomial expansion, quadratic solutions, algebra practice, mathematical problem-solving, verifying solutions

Frequently Asked Questions

What is the value of x in the equation $(x + 1)^2 = 120 + x^2$?

The value of x is 11.

How do you solve for x when $(x + 1)^2$ is 120 greater than x^2?

Expand $(x + 1)^2$ to get $x^2 + 2x + 1$, then set up the equation $x^2 + 2x + 1 = x^2 + 120$, subtract x^2 from both sides, and solve for x, resulting in $x = 119$.

Is the equation $(x + 1)^2 = x^2 + 120$ a linear or quadratic equation?

It is a quadratic equation because it involves x^2 terms.

What steps are involved in solving for x in the equation $(x + 1)^2 = x^2 + 120$?

First, expand $(x + 1)^2$ to get $x^2 + 2x + 1$, then subtract x^2 from both sides to get $2x + 1 = 120$, then solve for x: $2x = 119$, so $x = 59.5$.

What is the significance of the 120 in the equation $(x + 1)^2 = x^2 + 120$?

It represents the constant difference between the square of the binomial and the square of x, indicating how much larger $(x + 1)^2$ is compared to x^2.

Can you verify the solution x = 59.5 in the original equation?

Yes. Substitute $x = 59.5$: $(59.5 + 1)^2 = 60.5^2 = 3660.25$, and $x^2 + 120 = (59.5)^2 + 120 = 3540.25 + 120 = 3660.25$, confirming the solution is correct.

Are there multiple solutions for x in the given equation?

No, solving the equation yields a unique solution, $x = 59.5$.

What is the general approach to solving equations of the form $(x + a)^2 = x^2 + b$?

Expand $(x + a)^2$, simplify the equation, and solve for x, often resulting in a linear equation in x after simplifying.

Additional Resources

For What Value Of X Is...The Square Of The Binomial X+1 One Hundred And Twenty Greater Than The Square is a fascinating algebraic problem that invites us to delve into the fundamental principles of quadratic equations, binomial expansion, and problem-solving strategies. Such problems are common in mathematics education because they test one's understanding of algebraic identities, manipulation skills, and logical reasoning. This particular question, with its intriguing phrasing and numerical detail, serves as an excellent example to explore the steps involved in translating word problems into algebraic expressions, solving for unknowns, and understanding the underlying concepts involved.

In this article, we will systematically analyze this problem, break down the steps necessary to find the value of X, and discuss the broader significance of such problems in the context of mathematical learning. By approaching the problem methodically, learners can develop a deeper appreciation for algebraic techniques and their applications.

Understanding the Problem Statement

Before diving into calculations, it is crucial to interpret the problem accurately. The question reads: "For what value of X is the square of the binomial X+1 one hundred and twenty greater than the square". Although the statement appears somewhat fragmented, it can be understood as a comparison involving the square of the binomial $X+1$ and another quadratic expression, with the difference being 120.

The key components are:
- The binomial: $X+1$
- Its square: $(X+1)^2$
- The other square: likely referring to X^2, as that is the most straightforward interpretation.
- The numerical difference: 120.

Rephrasing, the problem asks:
"Find the value of X such that $(X+1)^2$ is 120 greater than X^2."

Expressed algebraically:

$$
(X+1)^2 = X^2 + 120
$$

This interpretation aligns with typical algebraic problem structures, where a binomial squared is compared to a quadratic expression with a specified difference.

Translating the Problem into an Equation

The core step in solving algebraic problems is translating verbal descriptions into precise equations. Based on the understanding above, the equation becomes:

$$
(X+1)^2 = X^2 + 120
$$

Expanding $(X+1)^2$:

$$
X^2 + 2X + 1 = X^2 + 120
$$

Subtract X^2 from both sides:

$$
2X + 1 = 120
$$

Now, solve for X:

$$
2X = 119
$$

$$
X = \frac{119}{2} = 59.5
$$

The solution indicates that when $X = 59.5$, the square of the binomial $X+1$ exceeds X^2 by 120.

Verifying the Solution

Verification is essential in algebra to ensure the derived solution satisfies the original problem statement.

Calculate $(X+1)^2$ at $X=59.5$:

$$
(59.5 + 1)^2 = (60.5)^2
$$

Calculate:

$$
60.5^2 = (60 + 0.5)^2 = 60^2 + 2 \times 60 \times 0.5 + 0.5^2 = 3600 + 60 + 0.25 = 3660.25
$$

Calculate X^2:

$$
(59.5)^2 = (60 - 0.5)^2 = 60^2 - 2 \times 60 \times 0.5 + 0.5^2 = 3600 - 60 + 0.25 = 3540.25
$$

Find the difference:

$$
3660.25 - 3540.25 = 120
$$

The difference matches the problem statement, confirming the solution $X=59.5$.

Alternative Interpretations and Variations of the Problem

While the most straightforward interpretation is as above, it's worth exploring other possible versions or extensions of the problem:

1. The Square of the Binomial Is 120 Greater Than the Square of X

This is essentially the same as the above, reaffirming the solution.

2. The Square of the Binomial X+1 Is 120 Greater Than Some Other Square

If the problem intended to compare $(X+1)^2$ with a different quadratic, such as $(X-1)^2$, then the equation would look different, for example:

$$
(X+1)^2 = (X-1)^2 + 120
$$

Expanding:

$$
X^2 + 2X + 1 = X^2 - 2X + 1 + 120
$$

\]

Simplify:

$$
2X = -2X + 120
$$

$$
4X = 120
$$

$$
X = 30
$$

Verify:

$$
(30+1)^2 = 31^2= 961
$$

$$
(30-1)^2= 29^2= 841
$$

Difference:

$$
961 - 841= 120
$$

Again, the solution confirms the interpretation.

Broader Educational Significance

This problem exemplifies several important concepts in algebra:

1. Binomial Expansion

Understanding how to expand $(X+1)^2$ is foundational, as binomial squares frequently appear in algebra and calculus.

2. Equation Formulation

Translating word problems into algebraic equations is a crucial skill that enhances problem-solving abilities.

3. Solving Linear Equations

The solution reduces to a simple linear equation, emphasizing the importance of isolating variables.

4. Verification

Verifying solutions ensures correctness and deepens understanding of the relationships involved.

Pros and Cons of the Approach

Pros:

- Clarity and simplicity: The problem reduces to a straightforward algebraic equation after interpretation.
- Educational value: Reinforces fundamental algebraic techniques like expansion, simplification, and solution verification.
- Versatility: The approach can be adapted to more complex problems involving quadratic and higher-degree equations.

Cons:

- Initial ambiguity: The wording may be confusing without careful reading, especially if the original problem statement is not precise.
- Limited complexity: The problem is relatively simple; more complex problems might involve multiple steps or identities.
- Potential for misinterpretation: Variations in phrasing can lead to different equations, so clarity is essential.

Conclusion

The problem, "For what value of X is the square of the binomial X+1 one hundred and twenty greater than the square," is fundamentally about understanding the relationship between a binomial square and a quadratic expression, with a specific numerical difference. Through systematic interpretation, algebraic translation, and verification, the solution reveals that \(X = 59.5\).

This exercise underscores the importance of careful reading, precise translation of words into equations, and verifying solutions. Such problems are not only excellent practice for mastering algebra but also serve as stepping stones toward more advanced mathematical concepts. Whether in education, competitive exams, or real-world applications, the skills gained from solving such problems are invaluable. By mastering these techniques, learners develop critical thinking and problem-solving skills that are essential across countless domains.

In summary, understanding and solving the problem about the square of the binomial \(X+1\) being 120 greater than \(X^2\) demonstrates key algebraic principles and problem-solving strategies. It highlights the importance of clarity in mathematical communication, the power of systematic approaches, and the satisfaction of arriving at verified solutions.

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