Functions G And H Are Invertible Functions. G(x)=(x+8)/(5) And H(x)=5(x-8) Answer Two Questionis About

Functions G And H Are Invertible Functions. G(x)=(x+8)/(5) And H(x)=5(x-8) Answer Two Questions About

Understanding the properties of functions is fundamental in mathematics, especially when analyzing their invertibility. In this article, we'll examine the functions G(x) = (x + 8)/5 and H(x) = 5(x - 8), demonstrating why they are invertible and exploring two key questions related to their inverses. Whether you're a student studying algebra or a math enthusiast, this comprehensive guide aims to clarify the concepts with detailed explanations and examples.

1. Determining the Invertibility of G(x) and H(x)

Before delving into the questions, it's important to establish why G(x) and H(x) are invertible functions.

1.1 What Does It Mean for a Function to Be Invertible?

A function is invertible if there exists another function, called its inverse, that "undoes" its operation. Formally:
- For a function f(x), an inverse function f^(-1)(x) satisfies:
- f(f^(-1)(x)) = x for all x in the domain of f^(-1)
- f^(-1)(f(x)) = x for all x in the domain of f

In simpler terms, applying the function and then its inverse returns you to your original value.

1.2 Conditions for Invertibility

- One-to-one (Injective): Each output corresponds to exactly one input.
- Onto (Surjective): The function covers its entire range (not always necessary for invertibility, depending on the context).
- Strict Monotonicity: The function is either strictly increasing or decreasing across its domain.

Since we're dealing with functions defined on real numbers, the key is to verify whether they are one-to-one.

1.3 Analyzing G(x) and H(x)

Both functions are linear:

- G(x) = (x + 8)/5

- H(x) = 5(x - 8)

Linear functions with non-zero coefficients are always one-to-one over the real numbers because their slopes are non-zero, meaning they are strictly monotonic.

Conclusion: Both G(x) and H(x) are invertible functions over the real numbers.

2. Finding the Inverses of G(x) and H(x)

Having established their invertibility, the next step is to explicitly find their inverse functions.

2.1 Finding the Inverse of G(x) = (x + 8)/5

Step-by-step process:

1. Replace G(x) with y: y = (x + 8)/5
2. Interchange x and y to find the inverse: x = (y + 8)/5
3. Solve for y:
   - Multiply both sides by 5: 5x = y + 8
   - Subtract 8 from both sides: y = 5x - 8

Result:
The inverse function of G(x) is:
G^(-1)(x) = 5x - 8

2.2 Finding the Inverse of H(x) = 5(x - 8)

Step-by-step process:

1. Replace H(x) with y: y = 5(x - 8)
2. Interchange x and y: x = 5(y - 8)
3. Solve for y:
   - Divide both sides by 5: x/5 = y - 8

- Add 8 to both sides: $y = x/5 + 8$

Result:
The inverse function of H(x) is:
$H^{-1}(x) = x/5 + 8$

3. Analyzing the Two Key Questions About the Inverse Functions

Now, with the inverse functions explicitly determined, we can address two significant questions:

- Question 1: How do these inverse functions relate to the original functions in terms of composition?
- Question 2: What are the practical applications and significance of these inverse functions?

Let's explore each in detail.

4. Question 1: How Do G^(-1)(x) and H^(-1)(x) Relate to G(x) and H(x) in Terms of Composition?

Understanding the relationship between a function and its inverse through composition is crucial in grasping their inverse nature.

4.1 Composition of a Function and Its Inverse

- For any invertible function f(x), the following holds:
- $f^{-1}(f(x)) = x$ for all x in the domain of f
- $f(f^{-1}(x)) = x$ for all x in the domain of f^{-1}

This means:

- Composing a function with its inverse (in either order) results in the identity function, which outputs the same value as input.

4.2 Applying Composition to G and G^(-1)

Calculate $G^{-1}(G(x))$:

- $G(x) = (x + 8)/5$
- $G^{-1}(x) = 5x - 8$

Calculate:

G^(-1)(G(x)) = 5 [(x + 8)/5] - 8 = (x + 8) - 8 = x

Similarly, G(G^(-1)(x)):

- G^(-1)(x) = 5x - 8
- G(G^(-1)(x)) = [(5x - 8) + 8]/5 = (5x)/5 = x

Conclusion: The inverse function acts as a perfect "undo" for the original function.

4.3 Applying Composition to H and H^(-1)

Calculate H^(-1)(H(x)):

- H(x) = 5(x - 8)
- H^(-1)(x) = x/5 + 8

Calculate:

H^(-1)(H(x)) = (5(x - 8))/5 + 8 = (x - 8) + 8 = x

Similarly, H(H^(-1)(x)):

- H^(-1)(x) = x/5 + 8
- H(H^(-1)(x)) = 5[(x/5 + 8) - 8] = 5(x/5) = x

Conclusion: Both functions satisfy the fundamental property that composing with their inverse yields the identity, confirming their invertibility.

5. Question 2: What Are the Practical Applications and Significance of These Inverse Functions?

Understanding inverse functions isn't just an academic exercise; they have real-world applications across various fields.

5.1 Reversing Transformations in Engineering and Physics

- Signal Processing: Inverse functions are used to recover original signals after transformations like filtering or modulation.
- Kinematics: Inverse functions help determine initial conditions or original positions from final measurements.
- Control Systems: Reversing system responses or calibrations often involves inverse functions.

5.2 Data Transformation and Calibration

- When data is transformed through a function (e.g., scaling or shifting), inverse functions allow reversing the transformation.
- Example: Converting temperature between Celsius and Fahrenheit involves linear functions with known inverses.

5.3 Computer Graphics and Image Processing

- Inverse functions are used to map pixel coordinates back to original image coordinates, crucial for image transformations and editing.

5.4 Financial Mathematics

- Calculating the original principal or interest rates from compounded data often involves inverse functions.

5.5 Solving Equations and Modeling

- Inverse functions facilitate solving for variables in equations where the original function models a relationship.
- For example, if a function models the growth of a population over time, its inverse can determine the time needed to reach a certain population size.

6. Summary and Key Takeaways

- Both G(x) = (x + 8)/5 and H(x) = 5(x - 8) are linear functions with non-zero slopes, making them invertible over the set of real numbers.
- Their inverses are G^(-1)(x) = 5x - 8 and H^(-1)(x) = x/5 + 8, respectively.
- The composition of each function with its inverse results in the identity function, confirming their invertibility.
- Inverse functions have extensive applications in science, engineering, finance, and computer science, enabling the reversal of transformations and solving complex problems.

Final thoughts: Recognizing the invertibility of functions like G and H, deriving their inverse functions, and understanding their practical significance are essential skills in mathematics. These concepts form the foundation for more advanced topics in algebra, calculus,

Frequently Asked Questions

Are functions G(x) = (x + 8)/5 and H(x) = 5(x - 8) invertible?

Yes, both functions G(x) and H(x) are invertible because they are linear functions with non-zero slopes, which makes them one-to-one and therefore invertible.

What is the inverse of the function G(x) = (x + 8)/5?

To find $G^{-1}(x)$, replace G(x) with y: y = (x + 8)/5. Solving for x gives x = 5y - 8. Therefore, $G^{-1}(x)$ = 5x - 8.

What is the inverse of the function H(x) = 5(x - 8)?

Let y = 5(x - 8). Then, x = (y/5) + 8. So, the inverse function $H^{-1}(x)$ = (x/5) + 8.

How do the graphs of G(x) and H(x) relate to their inverse functions?

The graphs of G and H are reflections of their inverse functions across the line y = x, illustrating the inverse relationship between them.

Are the inverse functions $G^{-1}(x)$ and $H^{-1}(x)$ also linear?

Yes, both $G^{-1}(x)$ = 5x - 8 and $H^{-1}(x)$ = (x/5) + 8 are linear functions, maintaining the linearity of the original functions.

Could the functions G and H be considered bijective?

Yes, since both are linear with non-zero slopes, they are both injective (one-to-one) and surjective (onto), making them bijective and thus invertible.

Additional Resources

Invertibility of Functions G and H: An In-Depth Analysis

When exploring the world of functions in mathematics, understanding whether a function is invertible—or bijective—is fundamental. Invertible functions allow us to reverse the process: given an output, we can find the original input. This property is crucial across various disciplines, from calculus to computer science, as it facilitates solving equations, modeling real-world systems, and understanding the structure of mathematical relationships.

In this article, we will examine two functions:

- $G(x) = \frac{x + 8}{5}$
- $H(x) = 5(x - 8)$

Our goal is to analyze whether these functions are invertible, understand their inverse functions, and answer two key questions regarding their properties.

Understanding the Functions G and H

Before delving into invertibility, it's essential to understand the structure and nature of the functions themselves.

Function G: $G(x) = \frac{x + 8}{5}$

Type of Function:
G is a linear function, characterized by a slope and intercept. Its general form resembles $y = mx + b$, where m is the slope and b is the y-intercept.

Analysis:
- The numerator $x + 8$ shifts the input by 8 units.
- Dividing by 5 scales the value down by a factor of 5.
- Graphically, G is a straight line with a slope of $\frac{1}{5}$ and y-intercept at $\frac{8}{5}$.

Domain and Range:
- Domain: All real numbers $\mathbb{R}$.
- Range: All real numbers $\mathbb{R}$, since a linear function with a non-zero slope is continuous and unbounded.

Function H: $H(x) = 5(x - 8)$

Type of Function:
H is also a linear function, with a form similar to $y = mx + b$.

Analysis:
- The expression $x - 8$ shifts the input by 8 units.
- Multiplying by 5 scales the output by a factor of 5.
- Graphically, H is a straight line with a slope of 5 and y-intercept at -40.

Domain and Range:
- Domain: All real numbers $\mathbb{R}$.
- Range: All real numbers $\mathbb{R}$, for the same reasons as G.

Are G and H Invertible? A Mathematical Perspective

Invertibility in functions hinges on the concept of bijectivity—being both injective (one-to-one) and surjective (onto). For functions defined over the real numbers, linear functions with non-zero slopes are typically invertible because they are strictly monotonic (either always increasing or decreasing).

Criteria for Invertibility of Linear Functions

- Non-zero slope:
A linear function $f(x) = mx + b$ is invertible iff $m \neq 0$.

- Strict Monotonicity:
If $m > 0$, the function is strictly increasing; if $m < 0$, it is strictly decreasing.

- Inverse Function:
The inverse of $f(x)$ can be found algebraically.

Applying these criteria:

- For $G(x) = \frac{x + 8}{5}$, the slope is $\frac{1}{5} \neq 0$, so G is invertible.
- For $H(x) = 5(x - 8)$, the slope is 5 $\neq 0$, so H is also invertible.

Deriving the Inverse Functions

Understanding that both functions are invertible, we proceed to find their inverses explicitly.

Inverse of G(x): $G^{-1}(x)$

Step 1: Set $y = G(x)$:

$$
y = \frac{x + 8}{5}
$$

Step 2: Swap x and y to reflect the inverse:

$$
x = \frac{y + 8}{5}
$$

Step 3: Solve for y:

$$
5x = y + 8
$$

$$
y = 5x - 8
$$

Result:

\[
\boxed{
G^{-1}(x) = 5x - 8
}
\]

Interpretation:
The inverse function of G is \(G^{-1}(x) = 5x - 8 \).

Inverse of H(x): \(H^{-1}(x) \)

Step 1: Set \(y = H(x) \):

\[
y = 5(x - 8)
\]

Step 2: Swap \(x \) and \(y \):

\[
x = 5(y - 8)
\]

Step 3: Solve for \(y \):

\[
\frac{x}{5} = y - 8
\]

\[
y = \frac{x}{5} + 8
\]

Result:

\[
\boxed{
H^{-1}(x) = \frac{x}{5} + 8
}
\]

Interpretation:
The inverse of H is \(H^{-1}(x) = \frac{x}{5} + 8 \).

Analysis of the Inverse Functions and Their Relationships

Having found explicit forms of the inverse functions, it becomes clear that G and H are inverse functions of each other, as they are algebraic inverses.

Verification of Inverse Relationship

To confirm, verify that:

- $G(G^{-1}(x)) = x$
- $G^{-1}(G(x)) = x$
- $H(H^{-1}(x)) = x$
- $H^{-1}(H(x)) = x$

Test for G:

$$
G(G^{-1}(x)) = G(5x - 8) = \frac{(5x - 8) + 8}{5} = \frac{5x}{5} = x
$$

$$
G^{-1}(G(x)) = G^{-1}\left(\frac{x + 8}{5}\right) = 5 \cdot \frac{x + 8}{5} - 8 = (x + 8) - 8 = x
$$

Test for H:

$$
H(H^{-1}(x)) = H\left(\frac{x}{5} + 8\right) = 5 \left(\frac{x}{5} + 8 - 8 \right) = 5 \left(\frac{x}{5}\right) = x
$$

$$
H^{-1}(H(x)) = H^{-1}(5(x - 8)) = \frac{5(x - 8)}{5} + 8 = (x - 8) + 8 = x
$$

The tests confirm the inverse relationship.

Key Takeaways and Implications

1. Both G and H are invertible functions because they are linear with non-zero slopes, ensuring they are bijective over $\mathbb{R}$.

2. Their inverse functions are straightforward to derive, highlighting the elegance of linear functions:

- $G^{-1}(x) = 5x - 8$
- $H^{-1}(x) = \frac{x}{5} + 8$

3. The inverse functions reflect each other, with G and H being functional inverses:

$$
H = G^{-1} \quad \text{and} \quad G = H^{-1}
$$

This reciprocal relationship exemplifies a core concept in function analysis.

Answering the Two Key Questions

Question 1: Are functions G and H invertible?

Answer:
Yes. Both G and H are linear functions with non-zero slopes ($\frac{1}{5}$ and 5 respectively), making them bijective over the real numbers. As a result, each has an inverse function, which can be explicitly calculated.

Question 2: What are the inverse functions of G and H?

Answer:
The inverse functions are:

- $G^{-1}(x) = 5x - 8$
- $H^{-1}(x) = \frac{x}{5} + 8$

These inverse functions confirm the reciprocal relationship between G and H, and they can be used to reverse the transformations applied by the original functions.

Final Remarks and Practical Significance

Understanding the invertibility of functions like G and H is not merely an academic exercise—it has practical implications in solving equations, modeling phenomena, and designing systems where reversible transformations are essential. The straightforward derivation of their inverses demonstrates the power of algebraic manipulation and the importance of linearity in function analysis.

Whether you're a student mastering the fundamentals or a professional applying these concepts in complex systems, recognizing when and

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