Give Two Examples Of A Function From Z To Z That Is: One-to-one But Not Onto. Onto But Not One-to-one.

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Understanding functions from the set of integers ($\mathbb{Z}$) to itself is a fundamental part of discrete mathematics and helps to clarify key concepts like injectivity (one-to-one functions) and surjectivity (onto functions). In this article, we will explore two illustrative examples: one function from $\mathbb{Z}$ to $\mathbb{Z}$ that is one-to-one but not onto, and another that is onto but not one-to-one. These examples will deepen your understanding of how these properties manifest in functions between integers, a common setting in combinatorics, number theory, and computer science.

Understanding the Concepts: One-to-one and Onto Functions

Before diving into the examples, it’s essential to clarify what it means for a function to be one-to-one (injective) and onto (surjective).

One-to-one (Injective) Functions

A function $f: \mathbb{Z} \to \mathbb{Z}$ is called one-to-one or injective if different inputs produce different outputs. Formally,

- For all $x, y \in \mathbb{Z}$, if $f(x) = f(y)$, then $x = y$.
- Equivalently, no two distinct integers map to the same integer.

Example: The function $f(x) = 2x + 3$ is injective because if $f(x) = f(y)$, then $2x + 3 = 2y + 3$, which implies $x = y$.

Onto (Surjective) Functions

A function $f: \mathbb{Z} \to \mathbb{Z}$ is onto or surjective if every element in the codomain has at least one pre-image in the domain. In other words,

- For every $z \in \mathbb{Z}$, there exists some $x \in \mathbb{Z}$ such that $f(x) = z$.

Example: The function $f(x) = x$ is surjective because for any integer z, choosing $x = z$ yields $f(x) = z$.

Examples of Functions From $\mathbb{Z}$ To $\mathbb{Z}$: One-to-one But Not Onto

Example 1: A One-to-one but Not Onto Function

Consider the function:

$$f(x) = 2x$$

Analysis:

- Injectivity:
Suppose $f(x) = f(y)$. Then,

$$2x = 2y \Rightarrow x = y$$

Thus, f is injective because different inputs give different outputs.

- Surjectivity:
Is f onto? For $f(x) = 2x$, the range is all even integers. For any even integer z, there exists $x = z/2$, which is an integer only if z is divisible by 2. But for odd integers z, there is no integer x such that $2x = z$. Therefore, f is not onto because it does not produce all integers — only the even ones.

Summary:
$f(x) = 2x$ is one-to-one but not onto over $\mathbb{Z}$.

Example 2: A Function That Is Onto But Not One-to-one

Now, consider the function:

$$g(x) = \begin{cases}
x/2, & \text{if } x \text{ is even} \\
(x-1)/2, & \text{if } x \text{ is odd}
\end{cases}$$

Or, more simply, define the function:

$$g(x) = \left\lfloor \frac{x}{2} \right\rfloor$$

but since we're working within $\mathbb{Z}$ and want a clear example, let's choose:

$$g(x) = \left\lfloor \frac{x}{2} \right\rfloor$$

which is the greatest integer less than or equal to $x/2$. Since the domain and codomain are both $\mathbb{Z}$, this is well-defined.

Alternatively, a straightforward example is:

$$g(x) = \left\lfloor \frac{x}{2} \right\rfloor$$

but to stay within $\mathbb{Z}$, better to define:

$$g(x) = \begin{cases}
x/2, & \text{if } x \text{ is even} \\
x//2, & \text{if } x \text{ is odd}
\end{cases}$$

But better yet, here's a more concrete and classic example:

$$g(x) = \left\{
\begin{array}{ll}
x/2, & \text{if } x \text{ is even} \\
x/2, & \text{if } x \text{ is odd}
\end{array}
\right.$$

which simplifies to:

$$g(x) = \lfloor x/2 \rfloor$$

and since $\lfloor x/2 \rfloor$ always outputs integers, the function is:

$$g(x) = \left\lfloor \frac{x}{2} \right\rfloor$$

But because $\lfloor x/2 \rfloor$ maps every integer x to an integer, it covers all integers in the codomain.

However, to have a function from $\mathbb{Z}$ to $\mathbb{Z}$ that is onto but not one-to-one, a simple example is:

$$g(x) = \left\lfloor \frac{x}{2} \right\rfloor$$

which maps:

- $x = 0, 1$ to 0,

- $x = 2, 3$ to 1,
- $x = -2, -1$ to -1, etc.

Analysis:

- Surjectivity:
For any integer z, choosing $x = 2z$ yields:

$$g(2z) = \left\lfloor \frac{2z}{2} \right\rfloor = z$$

Therefore, every integer z in the codomain has a pre-image.

- Injectivity:
But $g(x)$ is not injective because multiple inputs map to the same output. For example,

$$g(0) = 0, \quad g(1) = 0$$

so different inputs 0 and 1 both produce 0, violating injectivity.

Summary:
The function $g(x) = \left\lfloor \frac{x}{2} \right\rfloor$ is onto but not one-to-one.

Visualizing and Understanding These Functions

Visual representations can help to clarify these properties:

- For $f(x) = 2x$:

- Domain: all integers.
- Range: all even integers.
- The function “skips” odd integers, hence not onto.

- For $g(x) = \left\lfloor x/2 \right\rfloor$:

- Domain: all integers.
- Range: all integers.
- Multiple domain elements (like 0 and 1) map to the same value, so not injective but surjective.

Practical Implications and Applications

Understanding these types of functions has practical importance in various fields:

- Cryptography:
Injective functions ensure unique mappings, which are crucial for encryption schemes where invertibility is essential.

- Number Theory:
Surjective functions help analyze the coverage of certain number sets, such as quadratic residues.

- Computer Science:
Hash functions often aim for injectivity but may not always achieve surjectivity, affecting collision resistance.

- Combinatorics:
Analyzing functions with these properties assists in counting and constructing specific mappings.

Summary of Key Points

- A function from $\mathbb{Z}$ to $\mathbb{Z}$ can be injective but not surjective, exemplified by $f(x) = 2x$. This function is one-to-one but only maps onto even integers.

- Conversely, a function can be surjective but not injective, exemplified by $g(x) = \left\lfloor x/2 \right\rfloor$. It covers all integers but maps multiple inputs to the same output.

- Recognizing these properties helps in understanding the structure of functions, their invertibility, and their applications across mathematics and computer science.

By studying these examples, you gain a clearer picture of how injectivity and surjectivity operate in the context of functions from integers to integers. These insights are fundamental in advanced topics like group theory, number theory, and algorithm design.

Frequently Asked Questions

Can you provide an example of a function from Z to Z that is one-to-one but not onto?

Yes. Consider the function f(n) = 2n. This function is one-to-one because if f(n1) = f(n2), then 2n1 = 2n2, which implies n1 = n2. However, it is not onto because odd integers in Z are not mapped to by any integer n; thus, not all elements in Z are images of f.

What is an example of a function from Z to Z that is onto but not one-to-one?

An example is the function g(n) = |n|. This function is onto because every integer m in Z has a pre-image (either m or -m), but it is not one-to-one because g(n) = g(-n), so different inputs can produce the same output.

Why is the function f(n) = 2n from Z to Z one-to-one but not onto?

Because distinct integers n produce distinct outputs 2n, ensuring injectivity, but since odd integers in Z are never output, the function is not surjective (not onto).

Why is the function g(n) = |n| from Z to Z onto but not one-to-one?

Since every integer m in Z has a pre-image (either m itself if m ≥ 0 or -m if m < 0), g is onto. However, g(n) = g(-n) for all n, so different inputs produce the same output, making it not one-to-one.

Can you explain the importance of these examples in understanding functions from Z to Z?

These examples illustrate the distinctions between injective (one-to-one) and surjective (onto) functions, helping clarify that a function can be one without being the other, especially within the set of integers.

Are functions like f(n) = 2n and g(n) = |n| common examples used in teaching properties of functions from Z to Z?

Yes, they are classic examples that effectively demonstrate the concepts of injectivity and surjectivity in the context of functions from integers to integers, making them useful in educational settings.

Additional Resources

Give Two Examples Of A Function From Z To Z That Is: One-to-one But Not Onto. Onto But Not One-to-one.

In the fascinating realm of mathematical functions, understanding the nuanced distinctions between different types of functions is crucial for both students and professionals alike. Among the foundational concepts are injective (one-to-one) and surjective (onto) functions—terms that describe how elements from one set relate to elements in another. When the domain and codomain are both the set of integers, denoted as Z, these concepts become particularly intriguing due to the infinite nature of integers and the rich variety of functions that can be constructed.

This article aims to shed light on this subject by providing clear, concrete examples of functions from Z to Z that embody the following properties:

- One-to-one but not onto (injective but not surjective)
- Onto but not one-to-one (surjective but not injective)

Through detailed explanations, formal definitions, and illustrative examples, we will explore these two categories, enhancing understanding of the core principles that govern functions between integers.

Understanding the Core Concepts: Injective and Surjective Functions

Before diving into examples, it’s essential to clarify what it means for a function to be injective or surjective.

Injective (One-to-one) Functions

A function $f: Z \to Z$ is injective if distinct inputs produce distinct outputs. Formally:

> For all $x_1, x_2 \in Z$, if $f(x_1) = f(x_2)$, then $x_1 = x_2$.

This means no two different integers map to the same integer in the codomain. An example of an injective function would be:

$$
f(x) = 2x + 1
$$

since different integers x produce different outputs.

Surjective (Onto) Functions

A function $f: Z \to Z$ is surjective if every element in the codomain has at least one pre-image in the domain. Formally:

> For every $y \in Z$, there exists some $x \in Z$ such that $f(x) = y$.

An example is:

$$
f(x) = x^3
$$

since every integer y can be expressed as x^3 for some integer x (not necessarily every integer is a perfect cube, but in the case of $f(x) = x^3$, every integer y is indeed a cube of some integer).

Example 1: A Function From Z to Z That Is One-to-one But Not Onto

Let's now explore a concrete example of a function that is injective but not surjective.

The Function $f(x) = 2x + 1$

This function is a classic example to demonstrate an injective but not surjective function from the integers to the integers.

Formal Definition:

$$
f: Z \to Z,\quad f(x) = 2x + 1
$$

Why Is This Function One-to-one?

- Suppose $f(x_1) = f(x_2)$, which implies:

$$
2x_1 + 1 = 2x_2 + 1
$$

- Subtracting 1 from both sides:

$$
2x_1 = 2x_2
$$

- Dividing both sides by 2:

$$
x_1 = x_2
$$

- Therefore, the function is injective because equal outputs imply equal inputs.

Why Is It Not Onto?

- To be surjective, for every $y \in Z$, there must exist some $x \in Z$ such that:

$$
f(x) = y \Rightarrow 2x + 1 = y
$$

- Solving for x:

$$
x = \frac{y - 1}{2}
$$

- Since x must be an integer, $y - 1$ must be divisible by 2.

- This implies:

$$
y - 1 \equiv 0 \pmod{2}
$$

- So, y must be odd.

- Therefore, all odd integers are in the range of f.

- But even integers cannot be obtained as $f(x)$ because:

$$
f(x) \text{ is always odd}
$$

- Consequently, the function is not onto because not every integer (specifically, even integers) has a pre-image.

Summary

Property	Explanation
Injective	Yes; different inputs produce different outputs.
Surjective	No; only odd integers are in the range.

This example underscores that a function can be one-to-one but fail to cover the entire codomain.

Example 2: A Function From Z to Z That Is Onto But Not One-to-one

Next, we'll examine a function that is surjective but not injective.

The Function $g(x) = \lfloor \frac{x}{2} \rfloor$

This function maps each integer x to the integer division of x by 2, rounding down to the nearest integer.

Formal Definition:

$$
g: Z \to Z,\quad g(x) = \left\lfloor \frac{x}{2} \right\rfloor
$$

In programming terms, this is akin to integer division.

Why Is This Function Onto?

- For any $y \in Z$, we need to find some $x \in Z$ such that:

$$
g(x) = y
$$

- Since:

$$
g(x) = y \Rightarrow \left\lfloor \frac{x}{2} \right\rfloor = y
$$

- We analyze the possible x:

$$
y \leq \frac{x}{2} < y + 1
$$

- Multiplying through by 2:

$$
2y \leq x < 2y + 2
$$

- The integers x that satisfy this are:

$$
x \in \{ 2y, 2y + 1 \}
$$

- Both are integers, so for each y, we can choose $x = 2y$ or $x = 2y + 1$. For simplicity, pick $x = 2y$.

- Therefore, for every $y \in Z$, there exists at least one $x \in Z$ (specifically, $x = 2y$) such that $g(x) = y$.

- Hence, g is onto.

Why Is It Not One-to-one?

- Check if distinct inputs can produce the same output:

- For example, compare $x = 0$ and $x = 1$:

$$
g(0) = \left\lfloor 0/2 \right\rfloor = 0
$$
$$
g(1) = \left\lfloor 1/2 \right\rfloor = 0
$$

- Here, two different inputs produce the same output, demonstrating lack of injectivity.

- Similarly, $x = -1$ and $x = -2$:

$$
g(-1) = \left\lfloor -1/2 \right\rfloor = -1
$$
$$
g(-2) = \left\lfloor -2/2 \right\rfloor = -1
$$

- Again, different inputs map to the same output.

Summary

Property	Explanation
Onto	Yes; every integer y can be obtained from some x.
One-to-one	No; multiple inputs map to the same output.

This example illustrates that functions can cover the entire codomain but not be injective, especially when multiple inputs are grouped or "collapsed" into the same output.

Deeper Insights and Implications

Understanding these examples provides wider insights into the structure and behavior of functions over the integers.

The Infinite Nature of \(\mathbb{Z}\)

- The set of all integers is infinite, which allows for functions with diverse behaviors.
- For injective functions, the critical property is that no two elements in the domain map to the same element in the codomain.
- For surjective functions, it's about coverage—every element in the codomain is "hit" at least once.

Constructing Functions with Specific Properties

Mathematically, it's relatively straightforward to craft functions with desired properties:

- To create an injective but not surjective function, design one that skips some elements in the codomain.
- To craft a surjective but not injective function, design one where multiple domain elements map to the same codomain element.

Practical Significance

These concepts are not just theoretical:

- In computer science, understanding injective functions is essential for encoding data uniquely.
- In number theory and cryptography, functions that are injective or surjective underpin many

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