# Given That The Seventh Term And Fifth Term Of A Geometric Series Are 27 And 9 Respectively. If The Sum

**Given That The Seventh Term And Fifth Term Of A Geometric Series Are 27 And 9 Respectively. If The Sum** of the first n terms of a geometric series is to be determined, understanding the fundamental properties of geometric progressions (GP) is essential. This problem provides specific terms of the series, and by analyzing these, we can derive the common ratio and the first term, which are critical for calculating the sum of the series. This article will walk you through the detailed steps involved in solving this type of problem, offering insights into the principles of geometric series, formulas involved, and the methods used to arrive at the solution.

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## Understanding Geometric Series

A geometric series is a sequence of numbers where each term after the first is found by multiplying the previous term by a constant called the common ratio, denoted as r.

### Definition of Geometric Series

A geometric series can be expressed as:

$$ a, ar, ar^2, ar^3, \ldots, ar^{n-1} $$

where:
- $a$ = the first term of the series
- $r$ = the common ratio between consecutive terms
- $n$ = the number of terms

### Key Formulas

The sum of the first n terms of a geometric series, denoted as $S_n$, is given by:

$$ S_n = a \frac{r^n - 1}{r - 1} \quad \text{for} \quad r \neq 1 $$

The nth term of the series, denoted as $T_n$, is:

$$ T_n = a r^{n-1} $$

These formulas are foundational for solving problems involving geometric series, especially when specific terms are known.

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## Given Data and What Is Being Asked

The problem states:

- The fifth term of the geometric series is 9.
- The seventh term of the geometric series is 27.
- The goal is to find the sum of the first n terms of this series, or related quantities, depending on the full

question.

While the question appears incomplete, in typical problems of this nature, once the terms are known, the steps involve:

1. Finding the first term $a$ and the common ratio $r$.
2. Using these to calculate the sum of the series up to a certain number of terms, or for all terms if an infinite series.

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## Step-by-Step Solution Approach

Let's analyze how to find $a$, $r$, and then compute the sum.

### Step 1: Express Known Terms

Given:
$$T_5 = a r^{4} = 9$$
$$T_7 = a r^{6} = 27$$

These two equations involve $a$ and $r$. Our goal is to find $r$ and $a$.

### Step 2: Find the Common Ratio $r$

Dividing $T_7$ by $T_5$:
$$\frac{T_7}{T_5} = \frac{a r^{6}}{a r^{4}} = r^{2}$$

Calculating:
$$\frac{27}{9} = 3$$

Therefore:
$$r^{2} = 3$$
$$r = \pm \sqrt{3}$$

Since geometric series often assume positive ratios unless specified otherwise, we consider $r = \sqrt{3}$.

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### Step 3: Find the First Term $a$

Using the value of $r$ in one of the known terms, for example $T_5$:
$$a r^{4} = 9$$
$$a (\sqrt{3})^{4} = 9$$

Calculating $(\sqrt{3})^{4}$:
$$(\sqrt{3})^{4} = (\sqrt{3})^{2 \times 2} = (\sqrt{3})^{2} \times (\sqrt{3})^{2} = 3 \times 3 = 9$$

Thus:
$$a \times 9 = 9$$
$$a = 1$$

The first term is $a = 1$.

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# Calculating the Sum of the Series

Depending on the problem's context, you might be asked to find:

- The sum of the first $n$ terms.
- The sum of the series up to a certain term.
- The sum to infinity if the series converges.

## Sum of First n Terms

Using the formula for the sum of the first $n$ terms:
$$S_n = a \frac{r^{n} - 1}{r - 1}$$

Suppose the question asks for the sum of the first $n$ terms, where $n$ is known or needs to be determined.

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# Practical Example: Calculating the Sum for a Specific Number of Terms

Let's assume the question asks for the sum of the first 10 terms.

Given:
- $a = 1$
- $r = \sqrt{3}$

Calculating:
$$S_{10} = 1 \times \frac{(\sqrt{3})^{10} - 1}{\sqrt{3} - 1}$$

Step-by-step:

1. Calculate $(\sqrt{3})^{10}$:

Since $(\sqrt{3})^{10} = ( (\sqrt{3})^{2} )^{5} = (3)^{5}$:

$$3^{5} = 243$$

2. Plug into the formula:

$$S_{10} = \frac{243 - 1}{\sqrt{3} - 1} = \frac{242}{\sqrt{3} - 1}$$

3. Rationalize the denominator:

$$\frac{242}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} = \frac{242 (\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$$

Recall:
$$(\sqrt{3} - 1)(\sqrt{3} + 1) = (\sqrt{3})^{2} - 1^{2} = 3 - 1 = 2$$

Therefore:

$$S_{10} = \frac{242 (\sqrt{3} + 1)}{2} = 121 (\sqrt{3} + 1)$$

This is the exact sum of the first 10 terms.

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## Additional Considerations and Variations

Depending on the problem's specifics, other aspects might be relevant:

- Sum to infinity: If $|r| < 1$, the series converges, and the sum to infinity is:

$$S_{\infty} = \frac{a}{1 - r}$$

- Negative ratio: If $r = -\sqrt{3}$, the calculations follow similarly, but with attention to signs.

- Different number of terms: Adjust calculations accordingly for different values of $n$.

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## Common Mistakes to Avoid

- Incorrect ratio calculation: Always verify the ratio by dividing two known terms.
- Ignoring the sign of $r$: Both positive and negative ratios are possible; interpret the context to select the appropriate value.
- Miscalculations in exponentiation: Carefully compute powers, especially fractional or irrational exponents.
- Forgetting to rationalize denominators: Always rationalize expressions when needed to simplify.

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## Summary and Final Remarks

This problem illustrates the importance of understanding the properties of geometric series to solve real-world and theoretical problems efficiently. By carefully analyzing known terms, calculating the common ratio, and then finding the first term, you can determine the sum of the series for any number of terms. Remember to double-check calculations, especially with powers and rationalizations, and consider the nature of the series (convergent or divergent) when dealing with infinite sums.

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## Conclusion

In conclusion, given the seventh and fifth terms of a geometric series, you can determine the common ratio and the first term, and subsequently compute the sum of the series up to any desired number of terms. Mastery of these fundamental formulas and methods enables solving a wide variety of problems involving geometric progressions, which are common in mathematics, physics, finance, and engineering. Practicing such problems enhances your understanding and ability to approach similar challenges with confidence.

# Frequently Asked Questions

## Given that the 7th term and 5th term of a geometric series are 27 and 9 respectively, how can we find the common ratio (r)?

Let the first term be a. The nth term of a geometric series is Tn = a r^(n-1). Using T5 = 9: a r^4 = 9, and T7 = 27: a r^6 = 27. Dividing the second by the first: (a r^6) / (a r^4) = r^2 = 27 / 9 = 3. Therefore, r = √3 or r = -√3.

## How do we determine the first term (a) of the geometric series given the terms 27 and 9?

Using T5 = 9: a r^4 = 9. Once we find r, we can substitute back to find a. For example, if r = √3, then a = 9 / (√3)^4 = 9 / (3^2) = 9 / 9 = 1. Similarly, if r = -√3, a = 1 as well.

## What is the sum of the first n terms of this geometric series if the common ratio is √3?

The sum of the first n terms is given by S_n = a (r^n - 1) / (r - 1). Substituting a = 1, r = √3, the sum becomes S_n = (√3)^n - 1 / (√3 - 1).

## How does the sum change if the common ratio is negative, say r = -√3?

If r = -√3, the sum of n terms is S_n = a (r^n - 1) / (r - 1). With a = 1, it becomes S_n = (-√3)^n - 1 / (-√3 - 1). The sum will alternate in sign depending on whether n is even or odd.

## Can we find the sum of the series up to the 7th term using the given information?

Yes. Once we determine a and r, we can compute S_7 = a (r^7 - 1) / (r - 1). For example, if r = √3 and a = 1, S_7 = (√3)^7 - 1 / (√3 - 1).

## What assumptions are made in solving this problem regarding the geometric series?

The primary assumptions are that the series is geometric with a common ratio r (which can be positive or negative) and that the given terms are correctly identified as T5 and T7. We also assume the series continues infinitely or up to the 7th term as required.

## How can we verify the correctness of the calculated sum for the series?

We verify by substituting the found values of a and r into the series terms to ensure T5 and T7 match the given 9 and 27. Then, calculate the sum using the formula and check if it aligns with the series terms' pattern.

## What is the significance of the given terms 27 and 9 in understanding the series?

These terms help us determine the common ratio and the first term, which are essential to defining the entire geometric series and calculating sums or other terms within the series.

# Additional Resources

Understanding Geometric Series: An In-Depth Analysis of the Given Problem

When delving into the world of sequences and series, geometric series hold a prominent place due to their widespread applications across mathematics, physics, finance, and computer science. The problem at hand—Given that the seventh term and fifth term of a geometric series are 27 and 9 respectively—presents an excellent opportunity to explore the fundamental properties, formulas, and problem-solving techniques related to geometric progressions (GP).

In this comprehensive review, we will systematically analyze the problem, derive the necessary formulas, interpret the given data, and explore methods to find the sum of the series. This discussion will be thorough, detailed, and aimed at fostering a deep understanding of geometric series concepts.

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# Fundamentals of Geometric Series

Before tackling the specific problem, it is essential to establish a solid understanding of geometric series fundamentals.

## What Is a Geometric Series?

A geometric series is the sum of the terms of a geometric progression (GP). A GP is a sequence where each term after the first is obtained by multiplying the previous term by a constant called the common ratio, denoted as $r$.

General form of a GP:

$$
a, ar, ar^2, ar^3, \dots
$$

- $a$: the first term
- $r$: the common ratio

Terms of the GP:

$$
T_n = a \times r^{n-1}
$$

where $T_n$ is the $n^\text{th}$ term.

Sum of the first $n$ terms:

$$
S_n = a \frac{r^n - 1}{r - 1}, \quad \text{for } r \neq 1
$$

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# Analyzing the Given Data

The problem states:

> "Given that the seventh term and fifth term of a geometric series are 27 and 9 respectively."

Mathematically:

$$
T_7 = a r^{6} = 27
$$

$$
T_5 = a r^{4} = 9
$$

From these, our goal is to find the common ratio $r$, the first term $a$, and then determine the sum of the series (though the exact sum is not fully specified in the prompt, but we will explore how to find it).

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# Step-by-Step Solution Approach

Let's break down the problem into manageable steps:

1. Establish Equations from the Given Data

From the terms:

$$
a r^{6} = 27 \quad (1)
$$

$$
a r^{4} = 9 \quad (2)
$$

2. Find the Common Ratio $r$

Dividing equation (1) by equation (2):

$$
\frac{a r^{6}}{a r^{4}} = \frac{27}{9}
$$

$$
r^{2} = 3
$$

$$
r = \pm \sqrt{3}
$$

Thus, the common ratio has two possible values:

$$
r = \sqrt{3} \quad \text{or} \quad r = - \sqrt{3}
$$

3. Find the First Term $a$

Using $a r^{4} = 9$, substitute the values of $r$:

- For $r = \sqrt{3}$:

$$
a (\sqrt{3})^{4} = 9
$$

Note:

$$
(\sqrt{3})^{4} = (\sqrt{3})^{2 \times 2} = ( (\sqrt{3})^2 )^2
$$
Since:
$$
(\sqrt{3})^2 = 3
$$
then:
$$
(\sqrt{3})^{4} = 3^2 = 9
$$
Therefore:
$$
a \times 9 = 9 \Rightarrow a = 1
$$

- For $r = - \sqrt{3}$:
$$
a (-\sqrt{3})^{4} = 9
$$
Again:
$$
(-\sqrt{3})^{4} = (\sqrt{3})^{4} = 9
$$
So:
$$
a \times 9 = 9 \Rightarrow a = 1
$$

Result:
$$
a = 1
$$
regardless of the sign of $r$.

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## Summary of Findings So Far

- First term $a = 1$
- Common ratio $r = \pm \sqrt{3}$

Note: The series could be either increasing or decreasing depending on the sign of $r$. Both are mathematically valid solutions based on the given data.

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## Implications of the Two Possible Ratios

The two possible values for $r$ lead to two different series:

Series with $r = \sqrt{3}$:
$$
T_n = 1 \times (\sqrt{3})^{n-1}
$$

$$
$$
which is an increasing geometric sequence.

Series with $r = -\sqrt{3}$:
$$
T_n = 1 \times (-\sqrt{3})^{n-1}
$$
which alternates signs, producing an oscillating sequence.

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## Calculating the Sum of the Series

The problem seems to hint at finding the sum, but the total number of terms or the series' nature (finite or infinite) isn't explicitly provided. Let's explore both possibilities.

1. Finite Series Sum

If the series is finite with $n$ terms, the sum is:
$$
S_n = a \frac{r^n - 1}{r - 1}
$$

To compute this, we need to know the total number of terms $n$. Since the problem initially references the seventh term, it suggests considering at least 7 terms.

2. Infinite Series Sum

If the series were infinite, the sum exists only when $|r| < 1$. However, since $r = \pm \sqrt{3}$, which is greater than 1 in magnitude, the infinite sum does not converge.

Therefore, the focus should be on the sum of the first $n$ terms, especially the first 7 terms, which relate directly to the given data.

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## Calculating the Sum of First 7 Terms

Using the formula:
$$
S_7 = a \frac{r^{7} - 1}{r - 1}
$$

We will analyze this for both values of $r$.

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## Case 1: $r = \sqrt{3}$

- First term: $a = 1$
- $r^{7} = (\sqrt{3})^{7}$

Calculate $(\sqrt{3})^{7}$:

$$
(\sqrt{3})^{7} = (\sqrt{3})^{4} \times (\sqrt{3})^{3}
$$

Recall:
$$
(\sqrt{3})^{4} = 9
$$
and
$$
(\sqrt{3})^{3} = (\sqrt{3})^{2} \times \sqrt{3} = 3 \times \sqrt{3}
$$

Thus:
$$
(\sqrt{3})^{7} = 9 \times 3 \times \sqrt{3} = 27 \sqrt{3}
$$

Now, compute $S_7$:

$$
S_7 = 1 \times \frac{27 \sqrt{3} - 1}{\sqrt{3} - 1}
$$

Simplify numerator and denominator:

- Numerator: $27 \sqrt{3} - 1$
- Denominator: $\sqrt{3} - 1$

To rationalize or simplify, sometimes it's convenient to rationalize the denominator:

$$
S_7 = \frac{27 \sqrt{3} - 1}{\sqrt{3} - 1}
$$

Multiply numerator and denominator by the conjugate $\sqrt{3} + 1$:

$$
S_7 = \frac{(27 \sqrt{3} - 1)(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)}
$$

Recall:
$$
(\sqrt{3} - 1)(\sqrt{3} + 1) = (\sqrt{3})^2 - 1^2 = 3 - 1 = 2
$$

Now, expand numerator:

$$
(27 \sqrt{3})(\sqrt{3} + 1) - 1 \times (\sqrt{3} + 1)
$$

which is:

$$
27 \sqrt{3} \times \sqrt{3} + 27 \sqrt{3} \times 1 - \sqrt{3} - 1
$$

Calculate each term:

- $27 \sqrt{3} \times \sqrt{3} = 27 \times 3 = 81$
- $27 \sqrt{3} \times 1 =$

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