Given The Inverse Demand Function P = 100-Q2 And Supply Function P = 15Q, Visually And Analytical Find

Given The Inverse Demand Function P = 100 - Q^2 And Supply Function P = 15Q, Visually And Analytical Find

Introduction

Understanding the interaction between demand and supply is fundamental in economics. When given specific demand and supply functions, analysts can determine key market characteristics such as equilibrium price and quantity. In this article, we will explore how to analyze the inverse demand function $P = 100 - Q^2$ and the supply function $P = 15Q$ both visually and analytically. By doing so, we aim to provide a comprehensive understanding of market equilibrium, how to find it, and interpret the results effectively.

Understanding the Functions

Inverse Demand Function: $P = 100 - Q^2$

- Definition: This function indicates the price consumers are willing to pay for a given quantity Q.
- Characteristics:
- The demand curve is downward sloping when viewed from the price perspective.
- As quantity increases, the maximum price consumers are willing to pay decreases quadratically.
- The quadratic nature introduces a nonlinear relationship, which can impact the shape of the demand curve.

Supply Function: $P = 15Q$

- Definition: This function shows the price at which producers are willing to supply a certain quantity Q.
- Characteristics:
- The supply curve is linear and upward sloping.
- As quantity increases, the price required to supply that quantity increases proportionally.

Graphical Representation of Demand and Supply

Plotting the Demand Curve

- To plot $P = 100 - Q^2$, select a range of Q values and compute corresponding prices:

Q	$P = 100 - Q^2$
0	100
5	100 - 25 = 75
8	100 - 64 = 36
9	100 - 81 = 19
10	100 - 100 = 0

- The demand curve starts at $P = 100$ when $Q = 0$ and curves downward, reaching zero at $Q = 10$.

Plotting the Supply Curve

- For the supply function $P = 15Q$:

Q	$P = 15Q$
0	0
1	15
2	30
3	45
4	60
5	75
6	90
7	105

- The supply curve is a straight line starting at the origin and rising linearly.

Visual Analysis

- When plotted together:
- The demand curve is a downward-opening parabola.
- The supply curve is a straight line.
- The intersection point(s) represent the market equilibrium.

Analytical Approach to Find Equilibrium

Setting Demand Equal to Supply

- To find the equilibrium quantity $Q^$, set the demand and supply functions equal:

$$
100 - Q^2 = 15Q
$$

- Rearranged to standard quadratic form:

$$
Q^2 + 15Q - 100 = 0
$$

Solving the Quadratic Equation

- Use the quadratic formula:

$$
Q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

where $a = 1$, $b = 15$, and $c = -100$.

- Calculate the discriminant:

$$
\Delta = 15^2 - 4(1)(-100) = 225 + 400 = 625
$$

- Find the roots:

$$
Q = \frac{-15 \pm \sqrt{625}}{2} = \frac{-15 \pm 25}{2}
$$

- Two solutions:

1. $Q = \frac{-15 + 25}{2} = \frac{10}{2} = 5$
2. $Q = \frac{-15 - 25}{2} = \frac{-40}{2} = -20$

- Since quantity cannot be negative in this context, discard $Q = -20$.

Therefore, the equilibrium quantity is $Q^ = 5$.

Finding the Equilibrium Price $P^{\wedge}$

- Substitute $Q = 5$ into either the demand or supply function:

$$
P = 15Q = 15 \times 5 = 75
$$

or

$$
P = 100 - Q^2 = 100 - 25 = 75
$$

Hence, the equilibrium price is $P^{\wedge} = 75$.

Interpreting the Results

Equilibrium Point

- The market reaches equilibrium at:
- Quantity: $Q^{\wedge} = 5$
- Price: $P^{\wedge} = 75$

Market Behavior at Equilibrium

- At this point, the quantity that consumers are willing to buy exactly matches what producers are willing to supply.
- The price of 75 balances demand and supply, ensuring no excess demand (shortage) or excess supply (surplus).

Other Possible Solutions

- The negative root $Q = -20$ is not economically meaningful, as quantity cannot be negative.
- The quadratic nature of the demand function indicates that the demand curve intersects the supply curve only at the positive root.

Implications of the Quadratic Demand Function

- Unlike linear demand functions, a quadratic demand curve introduces curvature, affecting how changes in quantity impact price.
- Market analysis must account for this non-linearity, especially in elasticity calculations.

Elasticity and Responsiveness

- Price elasticity of demand can be derived from the inverse demand function:

$$
E_d = \frac{dQ}{dP} \times \frac{P}{Q}
$$

- For the demand function $P = 100 - Q^2$, rearranged to $Q = \sqrt{100 - P}$, elasticity varies along the curve.
- Understanding elasticity helps in predicting how quantity demanded responds to price changes near equilibrium.

Conclusion

Analyzing demand and supply functions both visually and analytically provides a comprehensive understanding of market equilibrium. In this case, with the inverse demand function $P = 100 - Q^2$ and the supply function $P = 15Q$, the equilibrium occurs at a quantity of 5 units and a price of 75. Recognizing the nonlinear nature of the demand curve enhances the depth of market analysis and helps policymakers and businesses make informed decisions. Whether viewed graphically or through algebraic solutions, such analysis underscores the importance of mathematical tools in economic modeling and real-world market assessments.

Further Considerations

- Impact of External Factors: Changes in consumer preferences or production costs can shift demand or supply curves.
- Market Dynamics: Real markets often experience shifts, making ongoing analysis essential.
- Elasticity Analysis: Deepening elasticity studies can inform pricing strategies.

By mastering both visual and analytical methods, economists and analysts can better interpret complex market behaviors and anticipate future trends effectively.

Frequently Asked Questions

How do you find the equilibrium quantity and price from the given inverse demand and supply functions?

Set the demand function $P = 100 - Q^2$ equal to the supply function $P = 15Q$ and solve for Q to find the equilibrium quantity. Then substitute Q back into either function to find the equilibrium price.

What is the equilibrium point when $P = 100 - Q^2$ and $P = 15Q$?

Solve $100 - Q^2 = 15Q$. Rearranged, $Q^2 + 15Q - 100 = 0$. Solve this quadratic to find the equilibrium quantity Q, then plug Q into $P = 15Q$ to find the equilibrium price.

How do you solve the quadratic equation derived from setting demand equal to supply?

Use the quadratic formula $Q = [-b \pm \sqrt{b^2 - 4ac}] / 2a$, where $a = 1$, $b = 15$, and $c = -100$. Choose the positive root as the meaningful equilibrium quantity.

What is the significance of graphically analyzing the demand and supply functions in this context?

Graphically, the demand curve $P = 100 - Q^2$ is a downward-sloping curve, and the supply curve $P = 15Q$ is an upward-sloping straight line. Their intersection point visually shows the equilibrium price and quantity.

Can you determine the equilibrium price without solving the quadratic equation explicitly?

No, explicit solving of the quadratic is necessary to find the exact equilibrium quantity, which then allows calculation of the equilibrium price. Graphical methods approximate this point visually.

What are the steps to analyze the market visually for equilibrium using these functions?

Plot $P = 100 - Q^2$ as a downward-curving parabola and $P = 15Q$ as a straight line on the same graph. Identify their intersection point, which indicates the equilibrium price and quantity.

How does the analytical approach complement the visual analysis in this problem?

The analytical approach provides precise numerical values for equilibrium, while the visual analysis offers an intuitive understanding of how supply and demand interact and where equilibrium occurs.

What are potential challenges when solving for equilibrium algebraically in this case?

The main challenge is solving the quadratic equation accurately, especially when roots are complex or negative, which are not meaningful in this economic context. Ensuring the correct root is used is essential for meaningful results.

Additional Resources

Market Equilibrium Analysis: Inverse Demand and Supply Functions Explored

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Introduction

Understanding how markets reach equilibrium is foundational for economists, business strategists, and policymakers alike. By analyzing the intersection of demand and supply functions, one can uncover the market price and quantity at which buyers and sellers are in perfect harmony. In this feature, we delve into a specific scenario where the inverse demand function is given as $P = 100 - Q^2$, and the supply function as $P = 15Q$. Through both visual and analytical lenses, we will explore how these functions interact to establish equilibrium, what insights they provide, and what implications they carry for market behavior.

The Significance of Demand and Supply Functions

Before we analyze the specific functions, it’s essential to grasp the roles they play:

- Demand Function: Represents how much consumers are willing to purchase at various prices.
- Supply Function: Indicates the quantity producers are willing to supply at different prices.
- Market Equilibrium: The point where quantity demanded equals quantity supplied, determining the market price and quantity.

In our scenario, the inverse demand function $P = 100 - Q^2$ illustrates a nonlinear relationship, suggesting that as quantity increases, the maximum price consumers are willing to pay diminishes quadratically. The supply function $P = 15Q$ is linear, implying a direct proportionality between price and quantity supplied.

Understanding the Functions

Inverse Demand Function: $P = 100 - Q^2$

This quadratic form indicates a nonlinear demand curve. Key features include:

- Maximum Willingness to Pay: At zero quantity ($Q=0$), the price is $P=100$. This is the choke price where demand drops to zero.
- Demand Decline: As Q increases, the price consumers are willing to pay decreases quadratically, meaning the demand curve steeply drops as quantity expands.
- Domain: Since the price must be non-negative, the feasible range for Q is when $P \geq 0$, leading to $100 - Q^2 \geq 0 \Rightarrow Q \leq 10$.

Supply Function: $P = 15Q$

A simple linear supply curve with:

- Proportional Relationship: Price increases directly with quantity.
- Range: The supply is valid for $Q \geq 0$, with price starting at zero and increasing indefinitely as Q increases.

Analytical Approach to Find Equilibrium

Setting Demand Equal to Supply

To find the equilibrium point(s), equate the two functions:

$$
100 - Q^2 = 15Q
$$

Rearranged:

$$
Q^2 + 15Q - 100 = 0
$$

This quadratic equation can be solved using the quadratic formula:

$$
Q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

where $a=1$, $b=15$, and $c=-100$.

Calculating the discriminant:

$$
\Delta = b^2 - 4ac = (15)^2 - 4(1)(-100) = 225 + 400 = 625
$$

Square root:

$$
\sqrt{\Delta} = \sqrt{625} = 25
$$

Solutions:

$$
Q = \frac{-15 \pm 25}{2}
$$

- First solution:

$$
Q = \frac{-15 + 25}{2} = \frac{10}{2} = 5
$$

- Second solution:

$$
Q = \frac{-15 - 25}{2} = \frac{-40}{2} = -20
$$

Since negative quantities are not feasible in this context, the only valid equilibrium quantity is:

$$
Q^{} = 5
$$

Calculating the Equilibrium Price

Substitute $Q=5$ into either the demand or supply function:

- Demand:

$$
P = 100 - (5)^2 = 100 - 25 = 75
$$

- Supply:

$$
P = 15 \times 5 = 75
$$

Thus, the market equilibrium occurs at:

$$
\boxed{
Q^{} = 5,\quad P^{} = 75
}
$$

\]

Visualizing the Equilibrium

While a textual description can't replace actual graphs, envisioning the demand and supply curves provides valuable intuition:

- Demand Curve $P = 100 - Q^2$: A downward-opening parabola starting at $P=100$ at $Q=0$, decreasing rapidly as Q approaches 10.
- Supply Curve $P=15Q$: An upward-sloping straight line starting at the origin, increasing linearly.

The intersection at $Q=5$, $P=75$ signifies the point where consumers’ willingness to pay matches producers’ minimum acceptable price. This visual confirmation aligns with the algebraic solution, reinforcing the equilibrium's validity.

Deep Dive: Market Behavior and Implications

Nonlinear Demand and Its Effects

The quadratic demand function suggests that consumer willingness to pay drops sharply as quantity increases, indicating potential for:

- Strong consumer preference for smaller quantities at high prices.
- Rapid demand erosion as prices decline or quantities grow.

This shape may reflect markets with saturated demand or where consumer utility diminishes quickly with increasing consumption.

Linear Supply Dynamics

The supply function’s simplicity indicates that producers’ willingness to supply increases proportionally with price, typical for many commodities where costs are linear or proportional to output.

Equilibrium Characteristics

- The equilibrium quantity of 5 units indicates a moderate market size.
- The equilibrium price of 75 is relatively high compared to the maximum willingness to pay of 100 at $Q=0$, but well below the choke price at $Q=10$, where demand drops to zero.

Market insights include:

- Pricing Power: At equilibrium, producers can command a price of 75, which is substantial relative to the initial maximum.

- Consumer Surplus: The gap between the maximum willingness to pay (100) and the equilibrium price (75) represents consumer surplus, reflecting the benefit consumers receive at equilibrium.
- Producer Surplus: The area between the supply curve and the equilibrium price, up to the equilibrium quantity, indicates producer benefits.

Sensitivity and Scenario Analysis

Understanding how shifts in demand or supply could affect equilibrium is vital:

- Demand Shift: If the maximum willingness to pay decreases (say, due to market saturation or reduced consumer income), the demand curve shifts downward, lowering both equilibrium price and quantity.
- Supply Shift: An increase in supply (e.g., technological innovation reducing costs) shifts the supply curve outward, potentially lowering the equilibrium price and increasing quantity.
- Policy Implications: Taxation, subsidies, or regulation can shift these curves, affecting market balance.

Conclusion

The analytical and visual examination of the inverse demand function $P=100 - Q^2$ alongside the supply function $P=15Q$ reveals a clear and precise market equilibrium at $Q=5$, $P=75$. This equilibrium point exemplifies the interplay between nonlinear consumer preferences and linear producer willingness, highlighting the importance of both demand elasticity and supply linearity in market outcomes.

Such an in-depth understanding offers valuable insights into market dynamics, guiding strategic decisions for businesses, informing policy interventions, and enriching economic theory. Whether viewed through equations or mental imagery, the equilibrium uncovered here underscores the power of mathematical modeling in decoding the complex dance of supply and demand.

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