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Understanding multivariable calculus is essential for analyzing functions of two or more variables, especially when exploring the behavior of surfaces in three-dimensional space. When dealing with a given surface defined by a function Z = F(x, y), calculating partial derivatives allows us to examine how the surface changes with respect to one variable while keeping the other constant. In this article, we will delve deeply into the process of computing the partial derivatives of the surface Z = F(x, y) = x + x + 2xy + 5y, specifically focusing on how to evaluate the partial derivative with respect to x at the point (1, 2) and further analyzing related derivatives.

Understanding the Surface Function Z = F(x, y) = x + x + 2xy + 5y

Before we proceed to partial derivatives, it's crucial to understand the structure of the function itself.

Simplifying the Function

The given function is:

$$Z = F(x, y) = x + x + 2xy + 5y$$

Note that the terms $x + x$ can be combined:

$$Z = 2x + 2xy + 5y$$

This simplified form makes it easier to differentiate with respect to x and y.

Interpreting the Surface

This function describes a surface in three-dimensional space, where:
- x and y are independent variables.
- Z (or $F(x, y)$) gives the height of the surface at each point (x, y).

The surface's shape depends on the interplay of the linear and bilinear terms in x and y.

Calculating the Partial Derivative Fx at a Point

Partial derivatives measure how a function changes concerning one variable, holding others constant.

Definition of Partial Derivative with Respect to x

The partial derivative of F with respect to x is:

$$F_x(x, y) = \frac{\partial}{\partial x} F(x, y)$$

Applying this to our function:

$$F(x, y) = 2x + 2xy + 5y$$

We differentiate term-by-term:

1. $\frac{\partial}{\partial x} (2x) = 2$
2. $\frac{\partial}{\partial x} (2xy) = 2y$ (since y is treated as a constant)
3. $\frac{\partial}{\partial x} (5y) = 0$ (no x dependence)

Therefore:

$$F_x(x, y) = 2 + 2y$$

Evaluating F_x at the Point (1, 2)

To compute $F_x(1, 2)$:

Substitute $x = 1$ and $y = 2$:

$$F_x(1, 2) = 2 + 2 \times 2 = 2 + 4 = 6$$

Result:

$$
\boxed{
F_x(1, 2) = 6
}
$$

This value indicates the rate of change of the surface in the x-direction at the point $(1, 2)$.

Calculating the Partial Derivative Fy and Its Evaluation

The question also alludes to the second part of the derivative, which likely involves the partial derivative of F with respect to y.

Partial Derivative of F with Respect to y

Differentiate $F(x, y) = 2x + 2xy + 5y$:

1. $\frac{\partial}{\partial y} (2x) = 0$ (no y dependence)
2. $\frac{\partial}{\partial y} (2xy) = 2x$ (since x is constant)
3. $\frac{\partial}{\partial y} (5y) = 5$

Thus,

$$F_y(x, y) = 2x + 5$$

Evaluating F_y at (1, 2)

Substitute $x = 1$, $y = 2$:

$$F_y(1, 2) = 2 \times 1 + 5 = 2 + 5 = 7$$

Result:

$$
\boxed{
F_y(1, 2) = 7
}
$$

This derivative indicates how the surface height changes as y varies, at the specific point.

Applications of Partial Derivatives in Multivariable Calculus

Partial derivatives are fundamental tools in various applications across mathematics, physics, engineering, and economics.

Key Applications Include:

1. **Gradient Calculation:** Combining partial derivatives to find the gradient vector, which points in the direction of the steepest ascent of the surface.

2. **Surface Tangent Planes:** Using partial derivatives to find tangent planes at specific points on the surface.

3. **Optimization Problems:** Identifying local maxima, minima, and saddle points where derivatives are zero.

4. **Modeling and Simulation:** Applying partial derivatives to understand how small changes in variables influence complex systems.

Understanding the Importance of Partial Derivatives in Real-World Contexts

Analyzing the behavior of functions like $Z = F(x, y)$ is crucial in fields such as physics, economics, and engineering.

Examples Include:

- **Physics:** Calculating the rate of change of temperature or pressure with respect to position in a physical system.

- **Economics:** Assessing how demand or supply reacts to changes in price or income levels.

- **Engineering:** Designing systems where the response depends on multiple variables, such as stress and strain in materials.

Conclusion: Mastering Partial Derivatives for Surface Analysis

In this article, we explored how to compute the partial derivatives of the surface $Z = F(x, y) = 2x + 2xy + 5y$, focusing on the derivative with respect to x at the point $(1, 2)$, which is 6.

Additionally, we discussed how to calculate the partial derivative with respect to \(y \), which at that point is \(7 \). These derivatives provide vital insights into the surface's behavior, including how it slopes and reacts to changes in the variables.

Understanding and calculating partial derivatives is a fundamental skill in multivariable calculus, enabling professionals and students to analyze complex systems effectively. Whether used in academic research, engineering design, or economic modeling, mastery of these concepts is essential for interpreting the dynamics of multivariable functions.

Key Takeaways:
- Simplify the function before differentiating.
- Differentiate term-by-term for partial derivatives.
- Substitute specific points to evaluate derivatives.
- Use partial derivatives to analyze surface behavior comprehensively.

By mastering the calculation of partial derivatives, you gain powerful tools to explore and interpret the intricate behaviors of multivariable functions across various disciplines.

Keywords for SEO Optimization:
- Partial derivatives of multivariable functions
- Calculating Fx and Fy at specific points
- Surface analysis in calculus
- Gradient of a surface
- Multivariable calculus tutorial
- Derivative of Z = F(x, y) = 2x + 2xy + 5y
- Evaluating partial derivatives at (1, 2)
- Applications of partial derivatives in physics and engineering

Frequently Asked Questions

What is the partial derivative of Z = F(x,y) = x + x + 2xy + 5y with respect to x?

The partial derivative Fx is 2 + 2y.

How do you evaluate the partial derivative Fx at the point (1, 2) for the given surface?

Substitute x=1 and y=2 into Fx = 2 + 2y, resulting in Fx(1,2) = 2 + 22 = 6.

What is the partial derivative of Z with respect to y for the function Z = x + x + 2xy + 5y?

The partial derivative Fy is 2x + 5.

How do you compute the partial derivative Fy at the point (1, 2)?

Substitute x=1 into Fy = 2x + 5, giving Fy(1,2) = 21 + 5 = 7.

Why is the partial derivative Fx = 2 + 2y for the given function?

Because differentiating Z with respect to x, treating y as constant, yields d/dx of x as 1, d/dx of 2xy as 2y, and the constants as zero, resulting in Fx = 1 + 2y.

Can you clarify how to evaluate Fx at the point (1, 2)?

Yes, plug y=2 into Fx = 2 + 2y, which gives Fx(1,2) = 2 + 4 = 6.

What is the significance of the partial derivatives Fx and Fy in analyzing the surface?

They represent the slopes of the surface in the x and y directions at a given point, indicating how the surface changes locally.

Are there any errors in the given function Z = x + x + 2xy + 5y? Should it be simplified first?

Yes, since x + x simplifies to 2x, the function can be written as Z = 2x + 2xy + 5y for clarity before differentiation.

Additional Resources

Surface Analysis and Partial Derivatives: A Deep Dive into Z = F(x, y) = x + x + 2xy + 5y

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Introduction: Unveiling the Surface and Its Significance

When exploring multivariable calculus, understanding how a surface behaves in relation to its variables is fundamental. The surface described by the function:

$$ Z = F(x, y) = x + x + 2xy + 5y $$

may appear straightforward at first glance, but its intricate structure offers valuable insights into the behavior of functions in two variables. This article dives deep into this surface, examining its properties, particularly focusing on partial derivatives — essential tools for understanding how the surface changes along different directions.

From the initial glance, the function combines linear and bilinear terms, setting the stage for a comprehensive exploration of its derivatives and their implications.

Understanding the Function: The Surface in Context

Before computing derivatives, it's important to analyze the structure of the function:

$$Z = F(x, y) = x + x + 2xy + 5y$$

Simplification:

Notice that $x + x = 2x$, so the function simplifies to:

$$Z = 2x + 2xy + 5y$$

This simplified form reveals the surface's dependence on both variables, with terms that are linear in x and y, as well as a bilinear term $2xy$.

Interpretation:

- The term $2x$ indicates a linear increase in the surface as x increases.
- The $2xy$ term couples both variables, meaning the surface's behavior in the x-direction depends on y, and vice versa.
- The $5y$ term adds a linear component in y.

Visual Perspective:

Imagine a plane that tilts and curves depending on the values of x and y. The bilinear term introduces a tilt that varies with both variables, making the surface more complex than a simple plane.

Part (a): Calculating the Partial Derivative F_x at (1, 2)

What Is a Partial Derivative?

In multivariable calculus, the partial derivative of a function with respect to one variable measures how the function changes as that variable varies, while all other variables are held constant. For the surface $Z = F(x, y)$, the partial derivative F_x describes the slope of the surface in the x-direction at a specific point.

Mathematically:

$$F_x(x, y) = \frac{\partial}{\partial x} F(x, y)$$

Computing F_x: Step-by-Step

Given the simplified function:

$$F(x, y) = 2x + 2xy + 5y$$

Treat y as a constant during differentiation.

- The derivative of $2x$ with respect to x is 2.
- The derivative of $2xy$ with respect to x is $2y$ because y is treated as a constant.
- The derivative of $5y$ with respect to x is 0, as it doesn't depend on x.

Putting it together:

$$F_x(x, y) = 2 + 2y$$

Evaluating F_x at (1, 2)

Substitute $x = 1$ and $y = 2$:

$$F_x(1, 2) = 2 + 2 \times 2 = 2 + 4 = 6$$

Result: The partial derivative of Z with respect to x at the point $(1, 2)$ is 6.

Part (b): Calculating the Partial Derivative F_y at (1, 2)

Understanding F_y

Similarly, the partial derivative with respect to y, denoted F_y, measures how the surface changes as y varies, with x held constant.

$$F_y(x, y) = \frac{\partial}{\partial y} F(x, y)$$

Calculating F_y: Step-by-Step

Starting again from:

$$F(x, y) = 2x + 2xy + 5y$$

Treat x as a constant:

- The derivative of $2x$ with respect to y is 0.
- The derivative of $2xy$ with respect to y is $2x$, since x is constant.
- The derivative of $5y$ with respect to y is 5.

Putting it together:

$$F_y(x, y) = 2x + 5$$

Evaluating F_y at (1, 2)

Substitute $x = 1$:

$$F_y(1, 2) = 2 \times 1 + 5 = 2 + 5 = 7$$

Result: The partial derivative of Z with respect to y at the point $(1, 2)$ is 7.

Interpreting the Partial Derivatives: Insights into the Surface

Understanding the significance of these derivatives helps visualize the surface's behavior:

- $F_x(1, 2) = 6$: Indicates that moving along the x-direction at $(1, 2)$ causes the surface to increase at a rate of 6 units per unit increase in x. This relatively steep slope suggests that the surface is quite sensitive to changes in x at this point.

- $F_y(1, 2) = 7$: Suggests that at $(1, 2)$, the surface increases even more rapidly along the y-direction — at a rate of 7 units per unit increase in y, highlighting the influence of the linear $5y$ term and the coupling via $2xy$.

These derivatives offer a snapshot of the local behavior of the surface, vital for applications such as optimization, modeling, and understanding surface curvature.

Extending the Analysis: Second-Order Derivatives and the Gradient

While the primary focus was on first derivatives, exploring second derivatives and the gradient vector can provide deeper insights.

Second partial derivatives:

- $F_{xx} = \frac{\partial^2 F}{\partial x^2} = 0$, since $F_x = 2 + 2y$ does not depend on x.
- $F_{yy} = \frac{\partial^2 F}{\partial y^2} = 0$, as $F_y = 2x + 5$ does not depend on y.
- $F_{xy} = \frac{\partial^2 F}{\partial x \partial y} = 2$, indicating the mixed curvature.

Gradient vector:

$$
\nabla F = \left(F_x, F_y \right) = (2 + 2y, 2x + 5)
$$

At $(1, 2)$:

$$
\nabla F(1, 2) = (2 + 4, 2(1) + 5) = (6, 7)
$$

This vector points in the direction of the steepest ascent on the surface at the point $(1, 2)$, with magnitude indicating the rate of maximum increase.

Conclusion: The Power of Partial Derivatives in Surface Analysis

The detailed exploration of the surface $Z = F(x, y) = 2x + 2xy + 5y$ underscores the pivotal role of partial derivatives in understanding multivariable functions. The calculations of F_x and F_y at specific points provide immediate insight into how the surface responds to changes along each axis.

- The derivative $F_x(1,2) = 6$ reveals a significant sensitivity to x, implying that small increases in x at this point cause notable rises in Z.
- The derivative $F_y(1,2) = 7$ indicates an even greater responsiveness to changes in y, emphasizing the combined influence of the linear and bilinear terms.

By mastering these derivatives, mathematicians and engineers can predict surface behavior, optimize functions, and develop models with greater precision. Whether in physics, economics, or engineering, understanding the nuances of multivariable functions through their derivatives remains an essential skill.

In essence, the surface defined by $Z = F(x, y)$ exemplifies the rich interplay between variables, and the derivatives serve as vital tools for unraveling its secrets.

Given The Surface Z F X Y X X 2xy 5y A Enter The Partial Derivative Fx 1 2 B Enter The

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