Graph $M(x)=2x^2+20x+47$ On The Cartesian Plane To The Right. To Graph The Function You'll First Have To

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Understanding how to graph quadratic functions is an essential skill in mathematics, especially when working with parabolas on the Cartesian plane. The quadratic function $M(x)=2x^2+20x+47$ presents an excellent opportunity to learn the process of plotting a parabola step-by-step. In this article, we will explore how to graph this specific quadratic function accurately, focusing on identifying key features such as the vertex, axis of symmetry, y-intercept, and x-intercepts. By the end, you'll be equipped with a comprehensive approach to graphing quadratic functions like $M(x)=2x^2+20x+47$ on the coordinate plane.

Understanding the Quadratic Function $M(x)=2x^2+20x+47$

Before diving into the graphing process, it's crucial to understand the structure of the quadratic function in question.

Standard Form of a Quadratic Function

The quadratic function is given in the standard form:

$$M(x) = ax^2 + bx + c$$

For our function:

- $a = 2$
- $b = 20$
- $c = 47$

The leading coefficient $a = 2$ indicates the parabola opens upwards, and its width depends on the magnitude of a.

Key Features of the Parabola

- Vertex: The highest or lowest point of the parabola.
- Axis of Symmetry: The vertical line passing through the vertex.
- Y-intercept: The point where the parabola crosses the y-axis.

- X-intercepts: The points where the parabola crosses the x-axis (if any).

Step-by-Step Guide to Graph M(x)=2x^2+20x+47

To accurately graph the quadratic function, follow these systematic steps.

Step 1: Find the Vertex

The vertex of a parabola in standard form can be found using the vertex formula:

$$ x_{vertex} = -\frac{b}{2a} $$

Plugging in the values:

$$ x_{vertex} = -\frac{20}{2 \times 2} = -\frac{20}{4} = -5 $$

Next, compute the y-coordinate of the vertex by substituting $x = -5$ into the original function:

$$ M(-5) = 2(-5)^2 + 20(-5) + 47 $$

$$ M(-5) = 2(25) - 100 + 47 $$

$$ M(-5) = 50 - 100 + 47 = -3 $$

Vertex Coordinates: $(-5, -3)$

Step 2: Determine the Axis of Symmetry

The axis of symmetry is the vertical line through the vertex:

$$ x = -5 $$

This line divides the parabola into two mirror-image halves.

Step 3: Find the Y-intercept

Set $x = 0$ to find where the parabola crosses the y-axis:

$$ M(0) = 2(0)^2 + 20(0) + 47 = 47 $$

Y-intercept: $(0, 47)$

Step 4: Find the X-intercepts (if any)

To find the x-intercepts, set $M(x) = 0$:

$$2x^2 + 20x + 47 = 0$$

Use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Calculate the discriminant:

$$D = b^2 - 4ac = (20)^2 - 4 \times 2 \times 47 = 400 - 376 = 24$$

Since $D > 0$, there are two real x-intercepts:

$$x = \frac{-20 \pm \sqrt{24}}{2 \times 2}$$

Simplify:

$$x = \frac{-20 \pm 2\sqrt{6}}{4}$$

$$x = -5 \pm \frac{\sqrt{6}}{2}$$

Numerical approximations:

- $\sqrt{6} \approx 2.45$
- First intercept:

$$x \approx -5 + \frac{2.45}{2} = -5 + 1.225 = -3.775$$

- Second intercept:

$$x \approx -5 - 1.225 = -6.225$$

X-intercepts: approximately $(-3.78, 0)$ and $(-6.23, 0)$

Plotting the Key Points and Sketching the Parabola

Now that we have the critical points, proceed to plot and sketch the parabola.

Step 1: Plot the Vertex

- Mark the point $(-5, -3)$.

Step 2: Plot the Y-intercept

- Mark the point $(0, 47)$.

Step 3: Plot the X-intercepts

- Mark approximately $(-3.78, 0)$ and $(-6.23, 0)$.

Step 4: Draw the Parabola

Using the symmetry about the axis $x = -5$, reflect points around the vertex to get additional points and draw a smooth, upward-opening parabola passing through all these points.

Additional Tips for Accurate Graphing

- Choose more points: To ensure a precise shape, compute additional points to the left and right of the vertex.
- Use a table of values: Select x-values near the vertex, such as $-4, -6, -7, -3$, and calculate corresponding y-values.
- Check symmetry: Confirm that points equidistant from the axis of symmetry have the same y-coordinate.
- Label all points: Clearly mark key points on your graph for clarity.

Understanding the Graph's Features and Behavior

Shape and Width of the Parabola

- The positive coefficient $a=2$ indicates an upward-opening parabola.
- Since a is relatively large, the parabola is narrower compared to functions with a smaller a.

Direction of Opening

- The parabola opens upward because $a > 0$.

Vertex as the Minimum Point

- The vertex at \((-5, -3)\) is the lowest point on the graph.

Symmetry

- The parabola is symmetric about the line \(x = -5 \).

Real-World Applications of Graphing Quadratic Functions

Understanding how to graph quadratic functions like M(x)=2x^2+20x+47 has many practical applications:

- Projectile motion: Trajectory of objects under gravity.
- Economics: Cost and revenue modeling.
- Engineering: Design of structures involving parabolic shapes.
- Physics: Analyzing the path of particles.

Conclusion

Graphing the quadratic function M(x)=2x^2+20x+47 involves a systematic process of identifying key features such as the vertex, axis of symmetry, intercepts, and additional points for accuracy. By calculating the vertex \((-5, -3)\), y-intercept \((0, 47)\), and x-intercepts approximately at \((-3.78, 0)\) and \((-6.23, 0)\), you can plot these points and sketch the parabola with confidence. Remember to leverage symmetry to simplify the process and ensure your graph accurately reflects the function's shape and behavior. Mastering this approach enhances your ability to interpret quadratic functions in various mathematical and real-world contexts.

Keywords: quadratic graph, parabola, Cartesian plane, vertex, x-intercept, y-intercept, quadratic formula, graphing tips, math tutorial

Frequently Asked Questions

What is the first step to graph the quadratic function M(x) = 2x^2 + 20x + 47 on the Cartesian plane?

The first step is to find the vertex of the parabola by using the vertex formula or completing the square.

How do you find the vertex of the quadratic function M(x) = 2x^2 + 20x + 47?

Calculate the x-coordinate of the vertex using x = -b/(2a). For this function, a=2 and b=20, so x = -20/(22) = -20/4 = -5. Then, substitute x=-5 into the function to find y.

What is the y-coordinate of the vertex for M(x) = 2x^2 + 20x + 47?

Substitute x = -5 into the function: M(-5) = 2(-5)^2 + 20(-5) + 47 = 225 - 100 + 47 = 50 - 100 + 47 = -3. So, the vertex is at (-5, -3).

What does the coefficient 2 in M(x) = 2x^2 + 20x + 47 tell us about the parabola's shape?

The coefficient 2, which is positive, indicates that the parabola opens upward and is relatively narrow.

How can you find additional points on the graph of M(x) = 2x^2 + 20x + 47?

Choose other x-values, plug them into the function, and calculate the corresponding y-values to plot multiple points for an accurate graph.

Is there a way to rewrite M(x) in vertex form to aid in graphing?

Yes, by completing the square, you can rewrite M(x) as a vertex form: M(x) = a(x - h)^2 + k, where (h, k) is the vertex.

What is the importance of the axis of symmetry for this parabola?

The axis of symmetry is the vertical line x = -b/(2a), which passes through the vertex and divides the parabola into two mirror images. For this function, it's at x = -5.

How does understanding the vertex help in graphing M(x) = 2x^2 + 20x + 47?

Knowing the vertex provides the highest or lowest point of the parabola, serving as a key reference point for sketching the graph accurately.

What is the y-intercept of the function M(x) = 2x^2 + 20x + 47?

Set x=0 in the function: M(0) = 47. So, the y-intercept is at (0, 47).

Why is it necessary to find the vertex and other points when graphing this quadratic function?

Finding the vertex and additional points helps to accurately sketch the parabola, understanding its shape, position, and orientation on the Cartesian plane.

Additional Resources

Graph $M(x) = 2x^2 + 20x + 47$ on the Cartesian Plane: A Comprehensive Guide

Understanding how to graph quadratic functions is fundamental in algebra and calculus, providing insights into parabolas, their properties, and their applications. The quadratic function $M(x) = 2x^2 + 20x + 47$ is a classic example that reveals various attributes of parabolic graphs when plotted on the Cartesian plane. In this detailed review, we will explore the process of graphing this function step-by-step, uncovering its key features such as the vertex, axis of symmetry, intercepts, and its overall shape. You'll learn not only how to plot the function but also how to interpret its graph in relation to the mathematical principles it embodies.

Understanding the Structure of the Quadratic Function

Before diving into the graphing process, it’s important to understand the basic form and components of the quadratic function $M(x) = 2x^2 + 20x + 47$.

Standard Form of a Quadratic Equation

The quadratic function is typically expressed in the standard form:

$$ y = ax^2 + bx + c $$

where:

- a determines the opening direction and the "width" of the parabola.
- b influences the position of the vertex along the x-axis.
- c is the y-intercept, where the parabola crosses the y-axis.

In our case:

- $a = 2$
- $b = 20$
- $c = 47$

Key Characteristics of the Function

- Leading coefficient (a): Since $a = 2 > 0$, the parabola opens upward.
- Width of the parabola: Because $|a| = 2$ is greater than 1, the parabola is relatively narrow.
- Y-intercept: The point where the graph crosses the y-axis is at $(0, c) = (0, 47)$.

Step 1: Rewriting the Function in Vertex Form

To graph the parabola accurately, especially to locate its vertex, it's helpful to rewrite the quadratic in vertex form:

$$y = a(x - h)^2 + k$$

where (h, k) is the vertex of the parabola.

Completing the Square

Let's convert $M(x) = 2x^2 + 20x + 47$ into vertex form:

1. Factor out the coefficient of x^2:

$$
M(x) = 2(x^2 + 10x) + 47
$$

2. Complete the square inside the parentheses:

- Take half of the coefficient of x, which is 10:

$$
\frac{10}{2} = 5
$$

- Square it:

$$
5^2 = 25
$$

3. Add and subtract 25 inside the parentheses to complete the square:

$$
M(x) = 2(x^2 + 10x + 25 - 25) + 47
$$

$$
= 2[(x + 5)^2 - 25] + 47
$$

4. Distribute the 2:

$$
= 2(x + 5)^2 - 50 + 47
$$

5. Simplify constants:

$$
= 2(x + 5)^2 - 3
$$

Now, the function is in vertex form:

$$ M(x) = 2(x + 5)^2 - 3 $$

Vertex: The vertex is at $(-5, -3)$.

Step 2: Plotting the Key Features

Having rewritten the function, we can now identify and plot its essential features.

Vertex

- Located at (-5, -3).
- This point is the minimum of the parabola since it opens upward.

Axis of Symmetry

- The axis of symmetry passes through the vertex and is given by:

$$
x = h = -5
$$

- The parabola is symmetric with respect to this vertical line.

Y-Intercept

- Found by evaluating $M(0)$:

$$
M(0) = 2(0)^2 + 20(0) + 47 = 47
$$

- So, the y-intercept is at (0, 47).

X-Intercepts (Roots of the Function)

To find the x-intercepts, solve $M(x) = 0$:

$$
2x^2 + 20x + 47 = 0
$$

Using the quadratic formula:

$$
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

Plug in the values:

$$
x = \frac{-20 \pm \sqrt{(20)^2 - 4 \times 2 \times 47}}{2 \times 2}
$$

Calculate discriminant:

$$
\Delta = 400 - 4 \times 2 \times 47 = 400 - 376 = 24
$$

Since the discriminant is positive, there are two real roots:

$$
x = \frac{-20 \pm \sqrt{24}}{4}
$$

Simplify:

$$
\sqrt{24} = 2\sqrt{6} \approx 4.89898
$$

Thus,

$$
x = \frac{-20 \pm 2\sqrt{6}}{4} = \frac{-20}{4} \pm \frac{2\sqrt{6}}{4} = -5 \pm \frac{\sqrt{6}}{2}
$$

Numerically,

- First root:

$$
x \approx -5 + \frac{2.4495}{2} = -5 + 1.22475 \approx -3.775
$$

- Second root:

$$
x \approx -5 - 1.22475 \approx -6.225
$$

X-intercepts are approximately at:

- (-3.775, 0)
- (-6.225, 0)

Step 3: Sketching the Graph

Now, with all key features identified, let’s discuss how to accurately sketch the graph on the Cartesian plane.

Plotting Points

Begin by plotting:

- The vertex at (-5, -3).
- The y-intercept at (0, 47).
- The x-intercepts at approximately (-3.78, 0) and (-6.23, 0).

Additional points can be calculated for more accuracy:

- Pick x-values around the vertex, e.g., $x = -4, -5, -6$, and compute corresponding y-values.

For example:

- $x = -4$:

$$
M(-4) = 2(-4)^2 + 20(-4) + 47 = 2(16) - 80 + 47 = 32 - 80 + 47 = -1
$$

- $x = -6$:

$$
M(-6) = 2(36) + 20(-6) + 47 = 72 - 120 + 47 = -1
$$

These points, (-4, -1) and (-6, -1), are symmetric about the vertex and help shape the parabola.

Drawing the Parabola

- Use the plotted points to sketch a smooth, symmetric parabola opening upward.
- Ensure the curve passes through the key points, with the vertex at the lowest point.
- The parabola should be narrow, consistent with the value of $a = 2$.

Step 4: Understanding the Graph’s Properties and Behavior

Once the graph is plotted, analyzing its properties offers deeper insights.

Direction of Opening

- Since $a = 2 > 0$, the parabola opens upward.
- The minimum point is at the vertex (-5, -3).

Width and Shape

- The larger the absolute value of a, the narrower the parabola.
- Here, with $a=2$, the parabola is relatively narrow, emphasizing the steepness of the graph around the vertex.

Symmetry

- The parabola is symmetric with respect to the line $x = -5$.

- For each point \((x, y) \), there exists a corresponding point \((2 \times -5 - x, y) \).

Range and Domain

- Domain: All real numbers, since quadratic functions are defined everywhere.

\[
\text{Domain} = (-\infty, \infty)
\]

- Range: Since the parabola opens upward with a minimum at \(y = -3 \):

\[
\text{Range} = [-3, \infty)
\]

Step 5: Applications and Significance of the Graph