

Help Im Stuck On TheseConsider The Space Curve $F(t) = (2 \cos(t), 2 \sin(t), 5t)$. A. Find The Arc Length

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Are you struggling to compute the arc length of a space curve? If you've encountered the curve defined by the parametric functions $F(t) = (2 \cos(t), 2 \sin(t), 5t)$ and are unsure how to find its arc length, you're in the right place. Calculating the arc length of a space curve is a fundamental concept in calculus, especially in vector calculus, and mastering this process opens doors to understanding the geometry and physics of three-dimensional objects and paths. This comprehensive guide will walk you through the detailed steps to find the arc length of the given curve, explain the underlying principles, and provide tips to avoid common pitfalls.

Understanding the Space Curve $F(t) = (2 \cos(t), 2 \sin(t), 5t)$

Before diving into the calculations, it's essential to understand what the curve represents and how it behaves.

Parametric Components

The space curve is given by three functions of parameter t :

1. $x(t) = 2 \cos(t)$
2. $y(t) = 2 \sin(t)$
3. $z(t) = 5t$

These functions describe a path in three-dimensional space, with t typically ranging over an interval (e.g., $[0, T]$).

Geometric Interpretation

- The x and y components, $2 \cos(t)$ and $2 \sin(t)$, trace a circle of radius 2 in the xy -plane.
- The z component, $5t$, increases linearly with t , causing the curve to spiral upward or downward depending on the direction.

This results in a helical or spiral-like structure wrapping around the z-axis, which is typical in many physical systems like springs, DNA structures, or electromagnetic coils.

Mathematical Foundations for Arc Length Calculation

Calculating the length of a parametric space curve relies on understanding the fundamental formula derived from calculus.

Arc Length Formula for a Space Curve

For a space curve $F(t) = (x(t), y(t), z(t))$, the arc length L over an interval $[a, b]$ is given by:

$$L = \int_a^b \sqrt{[(dx/dt)^2 + (dy/dt)^2 + (dz/dt)^2]} dt$$

This formula computes the accumulated length by integrating the magnitude of the derivative vector over t .

Step-by-Step Approach

1. Find the derivatives dx/dt , dy/dt , dz/dt .
2. Compute the square of each derivative.
3. Sum these squares.
4. Take the square root of the sum to find the integrand.
5. Integrate this expression over the desired interval of t .

Understanding this process is crucial as it applies universally to parametric curves.

Step-by-Step Solution for the Given Curve

Let's now apply this framework to the specific curve $F(t) = (2 \cos(t), 2 \sin(t), 5t)$.

Step 1: Find the derivatives

Calculate the derivatives of each component with respect to t :

1. $dx/dt = d/dt [2 \cos(t)] = -2 \sin(t)$
2. $dy/dt = d/dt [2 \sin(t)] = 2 \cos(t)$

$$3. \frac{dz}{dt} = \frac{d}{dt} [5t] = 5$$

Step 2: Square each derivative

Determine the squares:

$$1. \left(\frac{dx}{dt}\right)^2 = (-2 \sin(t))^2 = 4 \sin^2(t)$$

$$2. \left(\frac{dy}{dt}\right)^2 = (2 \cos(t))^2 = 4 \cos^2(t)$$

$$3. \left(\frac{dz}{dt}\right)^2 = 5^2 = 25$$

Step 3: Sum the squares

Sum the expressions:

$$4 \sin^2(t) + 4 \cos^2(t) + 25$$

Recall the Pythagorean identity $\sin^2(t) + \cos^2(t) = 1$, so:

$$4 (\sin^2(t) + \cos^2(t)) + 25 = 4 \cdot 1 + 25 = 4 + 25 = 29$$

Step 4: Take the square root

The integrand becomes:

$$\sqrt{29}$$

Since $\sqrt{29}$ is a constant, the integral simplifies significantly.

Step 5: Integrate over the interval

Determine the limits of integration, which depend on the problem context. For example, if t varies from 0 to T , then:

$$L = \int_0^T \sqrt{29} \, dt = \sqrt{29} (T - 0) = \sqrt{29} T$$

Note: If the problem specifies a particular interval for t , substitute accordingly.

Final Expression for the Arc Length

The total arc length of the space curve from $t = a$ to $t = b$ is:

$$L = \sqrt{29} (b - a)$$

This elegant formula reflects the constant magnitude of the derivative vector's sum of squares, leading to a straightforward calculation.

Practical Applications of Arc Length Calculations

Understanding how to compute the arc length of space curves like $F(t) = (2 \cos(t), 2 \sin(t), 5t)$ has numerous practical applications:

- Designing helical structures such as springs or coils
- Calculating the distance traveled along a three-dimensional path in physics and engineering
- Modeling DNA double helix structures in biology
- Trajectory planning in robotics and aerospace navigation
- Analyzing electromagnetic coils and their magnetic field paths

Each of these applications relies on understanding the geometry and length of complex curves.

Tips and Common Pitfalls to Avoid

While calculating the arc length may seem straightforward, there are common mistakes to be aware of:

1. **Forgetting to differentiate correctly:** Ensure derivatives are computed accurately, especially when dealing with trigonometric functions.

2. **Misapplying identities:** Remember the Pythagorean identity $\sin^2(t) + \cos^2(t) = 1$ to simplify expressions.
3. **Neglecting the interval of integration:** Always specify and correctly compute the limits of t for the arc length integral.
4. **Assuming the integrand is variable:** Recognize when the integrand is constant, simplifying the integral significantly.
5. **Ignoring the physical context:** Understanding the physical or geometric interpretation can help verify whether the computed arc length makes sense.

Conclusion

Calculating the arc length of the space curve $F(t) = (2 \cos(t), 2 \sin(t), 5t)$ involves a systematic approach rooted in calculus fundamentals. By differentiating each component, summing the squares, simplifying using trigonometric identities, and integrating over the specified interval, you can derive a concise formula for the length. In this case, the constant magnitude of the derivatives simplifies the process, leading to a straightforward result: $L = \sqrt{29} (b - a)$.

Mastering this technique enhances your understanding of space curves, enabling you to analyze complex paths in various scientific and engineering disciplines. Whether designing mechanical components, analyzing physical systems, or exploring mathematical structures, the ability to compute arc lengths accurately is an essential skill.

If you encounter similar problems, remember to:

- Break down the parametric functions step-by-step.
- Use identities to simplify expressions.
- Recognize when integrals reduce to simple multiplications.
- Always verify your limits of integration.

With practice, calculating the arc length of space curves will become an intuitive and invaluable part of your mathematical toolkit.

Frequently Asked Questions

How do I find the arc length of the space curve $F(t) = (2 \cos(t), 2 \sin(t), 5t)$?

To find the arc length, first compute the derivative $F'(t)$, then find its magnitude $|F'(t)|$, and integrate this magnitude over the given interval.

What is the derivative of $F(t) = (2 \cos(t), 2 \sin(t), 5t)$?

$F'(t) = (-2 \sin(t), 2 \cos(t), 5)$.

How do I compute the magnitude of $F'(t)$ for this curve?

Calculate $|F'(t)| = \sqrt{(-2 \sin(t))^2 + (2 \cos(t))^2 + 5^2} = \sqrt{4 \sin^2(t) + 4 \cos^2(t) + 25}$.

What simplifies the magnitude expression for the derivative in this problem?

Since $\sin^2(t) + \cos^2(t) = 1$, the magnitude simplifies to $\sqrt{4(1) + 25} = \sqrt{4 + 25} = \sqrt{29}$.

What is the formula for the arc length of the curve from $t = a$ to $t = b$?

The arc length $L = \int[a \text{ to } b] |F'(t)| dt$. In this case, since $|F'(t)| = \sqrt{29}$, the integral simplifies to $L = \int[a \text{ to } b] \sqrt{29} dt$.

How do I evaluate the integral to find the total arc length over a specific interval?

If the magnitude is constant, the arc length over $[a, b]$ is simply $\sqrt{29} (b - a)$.

Given the space curve $F(t) = (2 \cos(t), 2 \sin(t), 5t)$, how long is the arc from $t = 0$ to $t = T$?

The length is $L = \sqrt{29} T$, since the magnitude of the derivative is constant at $\sqrt{29}$.

Additional Resources

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Introduction

If you've recently delved into the fascinating world of calculus and differential geometry, chances are you've encountered the concept of space curves — those elegant, multidimensional paths traced out by parametric functions. One common challenge students face is calculating the arc length of such curves, especially when they involve trigonometric functions intertwined with linear components.

Imagine you're exploring the space curve defined by $F(t) = (2 \cos(t), 2 \sin(t), 5t)$. At first glance, it might seem straightforward, but as you attempt to compute the arc length, questions often arise: Where do I start? How do I incorporate all parts of the curve? What formulas should I apply?

In this article, we'll take a detailed, step-by-step approach to solving this problem. We'll explore the fundamental concepts, derive the necessary formulas, and walk through the calculations methodically, making the process clear and accessible for learners at various levels. Whether you're a student struggling with parametric curves or someone seeking to deepen your understanding of arc length computation, this guide aims to clarify each step.

Understanding Space Curves and Arc Length

Before diving into calculations, it's essential to grasp what a space curve is and why calculating its length matters.

What Is a Space Curve?

A space curve is a curve that exists in three-dimensional space, typically represented parametrically as a vector function:

$$\mathbf{F}(t) = (x(t), y(t), z(t))$$

Here, t is a parameter, often representing time or another variable, and $x(t)$, $y(t)$, $z(t)$ are functions describing the position along each axis.

In our case:

$$\mathbf{F}(t) = (2 \cos t, 2 \sin t, 5t)$$

This describes a spiral wrapping around the z -axis, with a radius of 2 in the xy -plane and ascending along the z -axis as t increases.

Why Calculate Arc Length?

Arc length measures the distance traveled along the curve between two points—for example, from $t = a$ to $t = b$. It provides a sense of the "actual" length of the path, which is crucial in physics, engineering, and computer graphics.

The Arc Length Formula for Space Curves

To compute the arc length of a space curve $\mathbf{F}(t)$ over an interval $[a, b]$, we use the integral:

$$L = \int_a^b |\mathbf{F}'(t)| \, dt$$

where $|\mathbf{F}'(t)|$ is the magnitude (length) of the derivative of $\mathbf{F}(t)$.

Deriving the Formula

Given:

$$\mathbf{F}(t) = (x(t), y(t), z(t))$$

Its derivative:

$$\mathbf{F}'(t) = (x'(t), y'(t), z'(t))$$

The magnitude:

$$|\mathbf{F}'(t)| = \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}$$

Thus, the arc length becomes:

$$L = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2} \, dt$$

Applying the Formula to Our Curve

Let's now apply this to the specific space curve $\mathbf{F}(t) = (2 \cos t, 2 \sin t, 5t)$.

Step 1: Compute the Derivatives

Differentiate each component with respect to t :

- $x(t) = 2 \cos t \rightarrow x'(t) = -2 \sin t$
- $y(t) = 2 \sin t \rightarrow y'(t) = 2 \cos t$
- $z(t) = 5t \rightarrow z'(t) = 5$

Step 2: Find the Magnitude $|\mathbf{F}'(t)|$

Plugging the derivatives into the magnitude formula:

$$|\mathbf{F}'(t)| = \sqrt{(-2 \sin t)^2 + (2 \cos t)^2 + (5)^2}$$

Simplify inside the square root:

$$|\mathbf{F}'(t)| = \sqrt{4 \sin^2 t + 4 \cos^2 t + 25}$$

Recall that:

$$\sin^2 t + \cos^2 t = 1$$

So the expression simplifies to:

$$|\mathbf{F}'(t)| = \sqrt{4(1) + 25} = \sqrt{4 + 25} = \sqrt{29}$$

Remarkably, this magnitude is constant, independent of t .

Step 3: Compute the Arc Length Integral

Suppose we want the arc length from $t = a$ to $t = b$. The total length is:

$$L = \int_a^b |\mathbf{F}'(t)| \, dt = \int_a^b \sqrt{29} \, dt$$

Since $|\mathbf{F}'(t)|$ is constant, the integral simplifies:

$$L = \sqrt{29} \times (b - a)$$

Practical Example: Finding the Length Over a Specific Interval

Now, let's demonstrate how this works with a concrete example. Suppose we want to find the length of the curve from $t = 0$ to $t = 2\pi$. This interval corresponds to one full revolution around the circle in the xy -plane, with the spiral ascending along the z -axis.

Applying the formula:

$$L = \sqrt{29} \times (2\pi - 0) = 2\pi \sqrt{29}$$

Numerically, this approximates to:

$$L \approx 2 \times 3.1416 \times 5.3852 \approx 33.837$$

\]

Thus, the length of the space curve over one full period $[0, 2\pi]$ is approximately 33.837 units.

Interpreting the Results

The key takeaway from this example is the simplicity that arises due to the constant magnitude of the derivative $|\mathbf{F}'(t)|$. In many space curves involving trigonometric functions, the derivatives can be complex, and the magnitude may depend on t . However, in this case, the symmetry of the circle combined with the linear z -component leads to a constant speed along the curve, simplifying the arc length calculation.

This insight is invaluable for students and professionals alike, as recognizing when the derivative's magnitude is constant can drastically reduce computational effort.

Additional Considerations and Applications

When Is the Magnitude Not Constant?

In more intricate curves, the derivatives' magnitudes depend on t , requiring the evaluation of more complex integrals. For example:

$$\mathbf{F}(t) = (\sin t, t^2, e^t)$$

would involve variable derivatives, leading to non-constant $|\mathbf{F}'(t)|$ and more involved calculations.

Practical Applications

Understanding how to compute arc lengths of space curves has numerous real-world applications:

- Engineering: Designing paths for robotic arms or vehicle trajectories.
- Physics: Calculating the distance traveled along a particle's path.
- Computer Graphics: Generating smooth curves and paths in 3D modeling.
- Biology: Modeling the shape of DNA strands or other biological structures.

Summary and Key Takeaways

- The arc length of a space curve $\mathbf{F}(t)$ over an interval $[a, b]$ is given by:

$$L = \int_a^b |\mathbf{F}'(t)| \, dt$$

- To compute $|\mathbf{F}'(t)|$, differentiate each component and find the square root of the sum of squares.
- In our specific example, the derivative's magnitude was constant, simplifying the integral to a straightforward multiplication.
- Recognizing patterns, such as constant speed, can significantly ease the calculation process.
- Always check whether the derivatives simplify or if the magnitude depends on t , as this affects the method of integration.

Final Thoughts

Calculating the arc length of space curves is a fundamental skill in calculus, blending geometric intuition with analytical techniques. While some curves lead to complex integrals, others, like our example, showcase elegant symmetries that make the process straightforward. By mastering these techniques, you'll gain a deeper appreciation for the geometry of space and enhance your problem-solving toolkit for a variety of scientific and engineering challenges.

Remember, the key is to approach each problem methodically, understand the underlying concepts, and look for simplifications along the way. Happy calculating!

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