How To Find The Period Of Cos(pi*n+pi) Andcos(3/4*pi*n) As 1 And 4?Consider The Continuous-time Signal

How To Find The Period Of Cos(pin+pi) And cos(3/4pin) As 1 And 4? Consider The Continuous-time Signal

Understanding the periodicity of signals is fundamental in signal processing, telecommunications, and various engineering disciplines. Specifically, analyzing the period of discrete-time signals such as cos(pin + pi) and cos(3/4pin) helps in designing systems, filters, and understanding their behavior over time. This article provides a comprehensive guide on how to determine the periods of these signals, focusing on their mathematical properties and implications in continuous-time signals.

Introduction to Periodic Signals

Before delving into specific signals, it is important to understand what makes a signal periodic.

Definition of a Periodic Signal

A signal $x(t)$ or $x[n]$ is said to be periodic if there exists a positive constant T such that:
- For continuous-time signals: $x(t + T) = x(t)$ for all t.
- For discrete-time signals: $x[n + N] = x[n]$ for all n.

The smallest such T (or N in discrete-time) is called the fundamental period.

Relevance of Periodicity

Periodic signals are essential because they simplify analysis, allow for Fourier series representation, and are crucial in the design of oscillators and filters.

Mathematical Foundations for Period Calculation

To find the period of the given signals, we need to analyze their arguments and identify the smallest positive interval over which the signal repeats.

Properties of Cosine Function

The cosine function, $\cos(\theta)$, is inherently periodic with period 2π:

$$
\cos(\theta + 2\pi) = \cos(\theta)
$$

This property extends to signals involving cosine functions of linear arguments.

Discrete-time Cosine Signals

For discrete-time signals like $\cos(\omega n + \phi)$, the period depends on the frequency ω:

$$
\text{Period } N = \text{smallest positive integer satisfying } \omega N = 2\pi k, \quad k \in \mathbb{Z}
$$

In other words, N must be such that the argument advances by 2π after N steps.

Analyzing the First Signal: $\cos(\pi n + \pi)$

Let's consider the first discrete-time cosine signal:

$$
x[n] = \cos(\pi n + \pi)
$$

Step 1: Simplify the Expression

Using the cosine addition formula:

$$
\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta
$$

but in this case, it is straightforward to note that:

\[

\cos(\pi n + \pi) = \cos(\pi n) \cos \pi - \sin(\pi n) \sin \pi
\]
Since \(\sin \pi = 0 \) and \(\cos \pi = -1 \), this simplifies to:
\[
x[n] = - \cos(\pi n)
\]

Alternatively, recognizing the periodicity:
\[
x[n] = \cos(\pi n + \pi) = - \cos(\pi n)
\]

Step 2: Find the Period of \(\cos(\pi n) \)

The key is to find \(N \) such that:
\[
\cos(\pi (n + N)) = \cos(\pi n)
\]
Using the property of cosine:
\[
\cos(\pi n + \pi N) = \cos(\pi n)
\]
which implies:
\[
\cos(\pi n + \pi N) = \cos(\pi n) \Rightarrow \text{or} \quad \text{either}
\quad \pi N = 2\pi k \quad \text{or} \quad \pi N = \pi + 2\pi k
\]
But more straightforwardly, since \(\cos(\theta + 2\pi) = \cos(\theta) \),
the argument repeats after:
\[
\pi N = 2\pi k \Rightarrow N = 2k
\]
The smallest positive \(N \) occurs at \(k=1 \):
\[
N = 2
\]

Thus, the fundamental period of \(\cos(\pi n) \) is 2, and consequently, the
period of \(x[n] = - \cos(\pi n) \) is also 2.

Conclusion for \(\cos(\pi n + \pi) \)

\[
\boxed{
\text{Period } N = 2
}
\]

This means that the sequence repeats every 2 samples.

Analyzing the Second Signal: $\cos\left(\frac{3}{4} \pi n\right)$

Now, let's analyze the second discrete-time cosine signal:
$$
x[n] = \cos\left(\frac{3}{4} \pi n\right)
$$

Step 1: Find the Period N

We seek the smallest positive integer N such that:
$$
\cos\left(\frac{3}{4} \pi (n + N)\right) = \cos\left(\frac{3}{4} \pi n\right)
$$
which requires:
$$
\frac{3}{4} \pi (n + N) = \frac{3}{4} \pi n + 2\pi k
$$
for some integer k. Simplify:
$$
\frac{3}{4} \pi N = 2\pi k
$$
Divide both sides by π:
$$
\frac{3}{4} N = 2k
$$
Solve for N:
$$
N = \frac{8k}{3}
$$
To find the smallest positive integer N, choose the smallest k where N is integer. Since $N = \frac{8k}{3}$, N is integer when $8k$ is divisible by 3.

- $8k$ divisible by 3 $\Rightarrow$ k must be a multiple of 3, because 8 and 3 are coprime.

Set $k=3$:
$$
N = \frac{8 \times 3}{3} = 8
$$

Verify:
$$
N=8
$$

\]
and check:

$$
\frac{3}{4} \pi \times 8 = \frac{3}{4} \pi \times 8 = 6 \pi
$$

which is:

$$
6 \pi = 2 \pi \times 3
$$

This confirms that after $N=8$ steps, the argument increases by 6π, which is a multiple of 2π, and the cosine function repeats.

Step 2: Confirm the Fundamental Period

Since $N=8$ satisfies the condition for the smallest positive integer:

$$
\boxed{
\text{Period } N=8
}
$$

Therefore, the sequence $\cos\left(\frac{3}{4} \pi n \right)$ repeats every 8 samples.

Summary of Results

Signal	Calculated Period	Explanation Summary
$\cos(\pi n + \pi)$	2	Due to cosine's periodicity with argument πn, period is 2.
$\cos\left(\frac{3}{4} \pi n\right)$	8	Derived from the condition $\frac{3}{4} \pi N = 2 \pi k$, smallest $N=8$.

Implications for Continuous-Time Signals

While the above analysis pertains to discrete-time signals, understanding their periodicity helps in analyzing continuous-time signals, especially when these discrete signals are sampled versions of continuous signals.

Sampling and Periodicity

- When a continuous-time periodic signal is sampled uniformly, the resulting discrete-time signal retains the periodicity if the sampling rate satisfies certain conditions.
- The Nyquist-Shannon sampling theorem ensures that sampling at a rate greater than twice the maximum frequency preserves the signal's information.

Continuous-Time Signal Periods Correspondence

- For a continuous-time cosine $\cos(\omega t)$, the period T is:
$$
T = \frac{2\pi}{\omega}
$$
- When discrete signals are sampled from continuous signals, their periods relate to the sampling frequency f_s:
$$
\text{Discrete-time frequency } \omega = 2\pi \frac{f}{f_s}
$$
- The periodicity in discrete

Frequently Asked Questions

How do you determine the period of the discrete-time signal cos(pin + pi)?

To find the period of cos(pin + pi), recognize that the cosine function repeats when its argument increases by 2π. Since the argument is pin + pi, set pi(n + N) + pi = pin + pi + 2π, which simplifies to piN = 2π. Solving for N gives N = 2. Therefore, the period of cos(pin + pi) is 2.

What is the period of cos(3/4pin), and how is it calculated?

The period N of cos((3π/4) n) is found by solving (3π/4) N = 2π, leading to N = (2π) / (3π/4) = (2π) (4 / 3π) = 8/3. Since N must be an integer, the fundamental period is the least common multiple that makes the argument increase by 2π, which is N = 8. Thus, the period of cos(3π/4 n) is 8.

Why do the continuous-time signals corresponding to these discrete signals have the same periods as the discrete signals?

In general, the periodicity of discrete-time signals relates to their fundamental periods, which are preserved when considered as sampled versions of continuous-time signals. If the discrete-time signal has a fundamental period N, the corresponding continuous-time signal, sampled at a constant rate, will exhibit a periodicity related to that period, assuming the original continuous-time signal is periodic with a compatible period. However, the actual continuous-time period depends on the sampling rate and the original signal's characteristics.

How does the concept of fundamental period differ between cos(pin + pi) and cos(3/4pin)?

The fundamental period of cos(pin + pi) is 2, while for cos(3/4pin), it is 8. This difference arises because the frequency components (π and $3\pi/4$) determine the periods; the first has a shorter fundamental period, and the second has a longer one due to its lower frequency, reflecting how often the signals repeat.

Can these discrete-time signals be considered periodic as continuous-time signals?

Yes, if they are sampled from periodic continuous-time signals, the discrete signals inherit periodicity. However, the periodicity in continuous-time depends on the original signal's period and sampling rate. If the sampled signals are derived from periodic continuous-time signals, then the periodicity translates accordingly, but the continuous-time period may differ unless specific conditions are met.

What role does the sampling rate play in determining the periodicity of the continuous-time signals from these discrete signals?

The sampling rate affects how the discrete-time signal maps back to the continuous-time domain. To accurately determine the continuous-time period from the discrete signal's period, the sampling rate must be considered. Proper sampling ensures the periodicity is preserved, and the continuous-time period can be derived by scaling the discrete period by the sampling interval.

How can I verify the periodicity of these signals using the properties of cosine functions?

You can verify periodicity by checking if the cosine arguments repeat after a certain integer multiple of n. For cos(pin + pi), verify that cos(pi(n + N) + pi) = cos(pin + pi), which holds when N=2. Similarly, for $\cos(3\pi/4\ n)$, verify that $\cos(3\pi/4\ (n + N)) = \cos(3\pi/4\ n)$ when N=8. If these conditions hold, the signals are periodic with those fundamental periods.

Additional Resources

How To Find The Period Of Cos(pin + pi) And cos(3/4 pi n) As 1 And 4?
Consider The Continuous-Time Signal

Introduction

Understanding periodicity in discrete and continuous signals is fundamental in the fields of signal processing, communications, and control systems. When analyzing signals such as cosine functions, determining their periods allows engineers and scientists to predict behavior, design systems, and optimize performance. This article delves into the methods for finding the periods of two specific discrete-time cosine signals: $\cos(\pi n + \pi)$ and $\cos(\frac{3}{4}\pi n)$, with the given periods of 1 and 4, respectively. While the signals are discrete in nature, we will also explore how their properties relate to continuous-time signals and the broader implications of their periodicity.

The Significance of Periodicity in Signals

Periodic signals are those that repeat their pattern after a fixed interval, known as the period. Mathematically, a discrete-time signal $x[n]$ is periodic if there exists a positive integer N such that:

$$
x[n + N] = x[n], \quad \text{for all } n
$$

Similarly, for continuous-time signals $x(t)$, the periodicity condition is:

$$
x(t + T) = x(t), \quad \text{for all } t
$$

where T is the continuous-time period.

Understanding the period of a signal helps in:

- Spectral analysis
- System stability assessment
- Signal synthesis and filtering
- Digital communications

In the context of discrete signals, the fundamental period is often tied to the argument of the cosine functions that define the signal.

Analyzing the First Signal: $\cos(\pi n + \pi)$

Mathematical Formulation and Basic Properties

The discrete-time cosine signal:

$$
x[n] = \cos(\pi n + \pi)
$$

can be simplified using properties of cosine:

$$
\cos(\theta + \pi) = -\cos(\theta)
$$

Thus,

$$
x[n] = -\cos(\pi n)
$$

which indicates that the signal is essentially a scaled and shifted version of $\cos(\pi n)$.

Periodicity Condition for $\cos(\pi n)$

The general form for a discrete-time cosine:

$$
\cos(\omega n)
$$

has a fundamental period N if:

$$
\cos(\omega(n + N)) = \cos(\omega n)
$$

which simplifies to:

$$
\cos(\omega n + \omega N) = \cos(\omega n)
$$

Using the cosine addition formula, the periodicity condition reduces to:

$$
\cos(\omega N) = 1
$$

or equivalently,

$$
\omega N = 2\pi k, \quad k \in \mathbb{Z}
$$

The minimal positive integer N satisfying this is the fundamental period.

Finding the Period N for $\cos(\pi n + \pi)$

Given the simplified form, the argument's angular frequency is π:

$$
x[n] = -\cos(\pi n)
$$

The period condition becomes:

$$
\cos(\pi (n + N) + \pi) = \cos(\pi n + \pi)
$$

which simplifies to:

$$
\cos(\pi n + \pi + \pi N) = \cos(\pi n + \pi)
$$

Since:

$$
\cos(A + B) = \cos A \cos B - \sin A \sin B
$$

but for periodicity, it's easier to use the property:

$$
\cos(\theta + 2\pi m) = \cos \theta
$$

for any integer m.

Therefore, the periodicity condition reduces to:

$$
\pi N = 2\pi m, \quad m \in \mathbb{Z}
$$

which implies:

$$
N = 2m
$$

$$

$$

The smallest positive integer N satisfying this is obtained when $m = 1$:

$$
N = 2
$$

Thus, the fundamental period of $\cos(\pi n + \pi)$ is 2.

Implication and Confirmation

Since:

$$
x[n + 2] = -\cos(\pi (n + 2)) = -\cos(\pi n + 2\pi) = -\cos(\pi n) \quad \text{(because } \cos(\theta + 2\pi) = \cos \theta)
$$

and

$$
x[n + 2] = -\cos(\pi n) = x[n]
$$

because $x[n] = -\cos(\pi n)$, the periodicity is confirmed.

Analyzing the Second Signal: $\cos(\tfrac{3}{4}\pi n)$

Mathematical Formulation and Properties

The second discrete-time cosine:

$$
y[n] = \cos\left(\frac{3}{4} \pi n \right)
$$

has an angular frequency:

$$
\omega_0 = \frac{3}{4} \pi
$$

The task is to find the fundamental period N such that:

$$
\cos \left(\frac{3}{4} \pi (n + N) \right) = \cos \left(\frac{3}{4} \pi n \right)
$$

\]

which leads to the periodicity condition:

\[
\cos \left(\frac{3}{4} \pi n + \frac{3}{4} \pi N \right) = \cos \left(
\frac{3}{4} \pi n \right)
\]

Using the property:

\[
\cos(\theta + 2\pi m) = \cos \theta
\]

we require:

\[
\frac{3}{4} \pi N = 2 \pi m, \quad m \in \mathbb{Z}
\]

Deriving the Fundamental Period \(N \)

Rearranging:

\[
\frac{3}{4} \pi N = 2 \pi m
\]

\[
\Rightarrow 3N/4 = 2 m
\]

\[
\Rightarrow 3N = 8m
\]

\[
\Rightarrow N = \frac{8m}{3}
\]

For \(N \) to be an integer, \(m \) must be a multiple of 3. Let \(m = 3k
\), where \(k \in \mathbb{Z} \):

\[
N = \frac{8 \times 3k}{3} = 8k
\]

The smallest positive \(N \) corresponds to \(k=1 \):

\[

$$
N = 8
$$

Therefore, the fundamental period of $\cos(\tfrac{3}{4} \pi n)$ is 8.

Verification

Check:

$$
y[n + 8] = \cos \left(\frac{3}{4} \pi (n + 8) \right) = \cos \left(\frac{3}{4} \pi n + 6 \pi \right)
$$

Since $\cos(\theta + 2\pi m) = \cos \theta$, and $6\pi = 3 \times 2\pi$:

$$
\cos \left(\frac{3}{4} \pi n + 6 \pi \right) = \cos \left(\frac{3}{4} \pi n \right)
$$

which confirms the periodicity.

Connecting Discrete Periods to Continuous-Time Signals

While the signals examined are discrete in nature, understanding their behavior provides insight into continuous-time analogs. Continuous-time signals of the form $\cos(\Omega t)$ are periodic with period:

$$
T = \frac{2 \pi}{\Omega}
$$

When sampled, the discrete-time frequency ω relates to the continuous-time frequency Ω by:

$$
\omega = \Omega T_s
$$

where T_s is the sampling period.

Continuous-Time Perspective of the Discrete Signals

- For $\cos(\pi n + \pi)$:

The angular frequency in discrete time is π, corresponding to a normalized frequency of π radians per sample. If the sampling

```
frequency \( f_s \) is known, the continuous-time frequency:

\[
\Omega = \frac{\omega}{T_s} = \frac{\pi}{T_s}
\]

leads to a continuous period:

\[
T = \frac{2\pi}{\Omega} = 2 T_s
\]

indicating that the continuous-time signal corresponding to this discrete
signal has a fundamental period twice the sampling period, reflecting a
frequency at the Nyquist limit.

- For \( \cos(¾ \pi n) \):

The discrete-time frequency is \( \frac{3}{4} \pi \), and the continuous-time
frequency:

\[
\Omega = \frac{3}{4} \pi / T_s
\]

so the continuous period:

\[
T = \frac{2 \pi}{\Omega} = \frac{
```

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