If (1 To X) F(t)dt = 20x/sqrt Of (4x2 + 21) - 4, Then (1 To [infinity]) F(t)dt Is?A. 6B. 1C. -3D. -4E.

Understanding the Integral Problem: A Step-by-Step Guide to Solving for $\int_{1}^{\infty} F(t) dt$

If (1 To X) F(t) dt = $\frac{20x}{\sqrt{4x^2 + 21}} - 4$, Then (1 To [infinity]) F(t) dt Is? A. 6 B. 1 C. -3 D. -4 E.

This intriguing problem involves evaluating an improper integral based on a given definite integral expression. Our goal is to find the value of $\int_{1}^{\infty} F(t) dt$, which requires understanding the behavior of the function $F(t)$ and applying integral calculus techniques.

Breaking Down the Problem: From a Known Integral to an Unknown One

The problem provides a specific relationship:

$$
\int_{1}^{x} F(t) dt = \frac{20x}{\sqrt{4x^2 + 21}} - 4
$$

for $x \geq 1$.

Our task is to evaluate:

$$
\int_{1}^{\infty} F(t) dt
$$

which can be expressed as the limit:

$$
\lim_{x \to \infty} \int_{1}^{x} F(t) dt
$$

Given the known integral, this reduces to:

\[
\lim_{x \to \infty} \left(\frac{20x}{\sqrt{4x^2 + 21}} - 4 \right)
\]

The core of the problem is to evaluate this limit.

Step 1: Understanding the Behavior of the Given Expression as \(x \to \infty\)

Let's analyze:

\[
I(x) = \frac{20x}{\sqrt{4x^2 + 21}} - 4
\]

As \(x \to \infty\):

- The numerator is linear in \(x\): \(20x\).
- The denominator is \(\sqrt{4x^2 + 21}\).

To evaluate the limit, focus on the dominant terms:

\[
\sqrt{4x^2 + 21} \approx \sqrt{4x^2} = 2|x|
\]

Since \(x \to \infty\), \(x\) is positive, so:

\[
\sqrt{4x^2 + 21} \approx 2x
\]

Thus,

\[
I(x) \approx \frac{20x}{2x} - 4 = 10 - 4 = 6
\]

Therefore,

\[
\boxed{
\lim_{x \to \infty} I(x) = 6
}
\]

which implies:

$$
\int_{1}^{\infty} F(t) dt = 6
$$

Matching the multiple-choice options, the answer is A. 6.

Step 2: Confirming the Limit with Rigorous Calculation

While the approximation suggests the limit is 6, let's verify more rigorously.

Rewrite:

$$
I(x) = \frac{20x}{\sqrt{4x^2 + 21}} - 4
$$

Factor x^2 inside the square root:

$$
\sqrt{4x^2 + 21} = \sqrt{4x^2 \left(1 + \frac{21}{4x^2}\right)} = 2x \sqrt{1 + \frac{21}{4x^2}}
$$

Substituting back:

$$
I(x) = \frac{20x}{2x \sqrt{1 + \frac{21}{4x^2}}} - 4 = \frac{10}{\sqrt{1 + \frac{21}{4x^2}}} - 4
$$

As $x \to \infty$, $\frac{21}{4x^2} \to 0$, so:

$$
I(x) \to 10 / 1 - 4 = 6
$$

This confirms our earlier conclusion. The integral's value from 1 to infinity is 6.

Understanding the Significance of the Result

The integral $\int_{1}^{\infty} F(t) dt$ equates to 6, indicating the total accumulated value of $F(t)$ from $t=1$ to infinity.

- The key insight stems from the limit behavior of the given expression.
- The process demonstrates the importance of asymptotic analysis in improper integrals.
- It also emphasizes how knowing the integral from a finite point can help evaluate it over an infinite domain.

Additional Insights: Differentiating to Find $F(t)$

While the problem only asks for the value of the integral, understanding the nature of $F(t)$ is valuable.

Given:

$$
\int_{1}^{x} F(t) dt = \frac{20x}{\sqrt{4x^2 + 21}} - 4
$$

Differentiate both sides with respect to x:

$$
F(x) = \frac{d}{dx} \left(\frac{20x}{\sqrt{4x^2 + 21}} - 4 \right)
$$

Calculating:

$$
F(x) = \frac{d}{dx} \left(\frac{20x}{\sqrt{4x^2 + 21}} \right)
$$

since the derivative of a constant (-4) is zero.

Using the quotient rule or chain rule:

$$
F(x) = \frac{d}{dx} \left(20x \cdot (4x^2 + 21)^{-1/2} \right)
$$

Set:

$$
u = 20x, \quad v = (4x^2 + 21)^{-1/2}
$$

Then,

$$
F(x) = u' v + u v'
$$

\]

Calculate:

$$
u' = 20
$$

$$
v' = -\frac{1}{2} (4x^2 + 21)^{-3/2} \cdot 8x = -4x (4x^2 + 21)^{-3/2}
$$

Therefore,

$$
F(x) = 20 \cdot (4x^2 + 21)^{-1/2} + 20x \cdot \left(-4x (4x^2 + 21)^{-3/2} \right)
$$

Simplify:

$$
F(x) = \frac{20}{\sqrt{4x^2 + 21}} - 80x^2 (4x^2 + 21)^{-3/2}
$$

This explicit form of $F(x)$ confirms its behavior at infinity and supports the earlier integral analysis.

Conclusion: Final Answer and Recap

By analyzing the behavior of the given integral expression as $x \to \infty$, applying asymptotic techniques, and confirming through rigorous calculation, we find:

$$
\boxed{
\int_{1}^{\infty} F(t) dt = 6
}
$$

which corresponds to option A. 6.

Summary of Key Takeaways:

- The limit of the integral expression as $x \to \infty$ determines the value of the improper integral.
- Asymptotic analysis simplifies evaluation of limits involving radicals and rational functions.
- Differentiating the known integral helps find the explicit form of

$F(t)$.
- Proper understanding of limits and calculus techniques is crucial for solving integral problems involving infinity.

This problem exemplifies how advanced calculus concepts intertwine to solve real-world and theoretical integral questions, emphasizing the importance of limit evaluation and function behavior analysis.

Frequently Asked Questions

What is the value of the definite integral from 1 to X of F(t) dt if it equals 20x divided by the square root of $4x^2$ plus 21 minus 4?

The integral is given by $20x / \sqrt{4x^2 + 21} - 4$.

How do we find the value of the integral of F(t) from 1 to infinity based on the given expression?

We first analyze the limit of the integral as X approaches infinity, considering the behavior of $20x / \sqrt{4x^2 + 21} - 4$.

What is the limit of $20x / \sqrt{4x^2 + 21}$ as x approaches infinity?

As x approaches infinity, $20x / \sqrt{4x^2 + 21}$ approaches $20x / (2x) = 10$.

Does the integral from 1 to infinity converge or diverge based on the given expression?

The integral converges because the expression approaches a finite limit as x approaches infinity.

What is the value of the indefinite integral of F(t) from 1 to infinity?

It is the limit as X approaches infinity of the given integral expression, which simplifies to a finite value.

How do we evaluate the limit of $20x / \sqrt{4x^2 + 21} - 4$ as x approaches infinity?

We examine the dominant terms: $20x / (2x) = 10$; thus, the entire expression tends toward 10 minus 4, which is 6.

What is the correct value of the integral from 1 to infinity of F(t) dt based on the options provided?

Based on the limit calculation, the value is 6, which corresponds to option A.

Why does the integral from 1 to infinity of F(t) dt equal 6?

Because the limit of the integral expression as X approaches infinity is 6, indicating the total accumulated value converges to 6.

Which option correctly represents the value of the integral from 1 to infinity of F(t) dt?

Option A: 6.

Additional Resources

Understanding the Integral: If (1 To X) F(t) dt = 20x / sqrt($4x^2$ + 21) - 4, Then (1 To [infinity]) F(t) dt Is?

The question involving the integral of a function $F(t)$ from 1 to a variable upper limit X, and then extending that to infinity, is a classic problem in calculus, especially within the realm of improper integrals and limits. When presented with an expression such as

$$
\int_{1}^{X} F(t) \, dt = \frac{20X}{\sqrt{4X^2 + 21}} - 4,
$$

the key challenge is to determine the value of the integral

$$
\int_{1}^{\infty} F(t) \, dt,
$$

which involves understanding the behavior of $F(t)$ as $t \to \infty$ and applying fundamental concepts of limits and improper integrals.

In this comprehensive guide, we will dissect the problem step-by-step, explore relevant calculus principles, and arrive at the correct answer with clarity and confidence.

Breakdown of the Problem

What is Given?

- The definite integral of $F(t)$ from 1 to X:

$$
\int_{1}^{X} F(t) \, dt = \frac{20X}{\sqrt{4X^2 + 21}} - 4
$$

- The goal: Find the value of

$$
\int_{1}^{\infty} F(t) \, dt
$$

Why Is This Important?

This problem exemplifies how an integral's indefinite form can be expressed explicitly and then extended to infinity — a key concept in analyzing the convergence of improper integrals. Also, it emphasizes the importance of understanding limit behavior for large bounds, which is essential in many applications across physics, engineering, and mathematics.

Step-by-Step Approach to the Solution

Step 1: Recognize the Fundamental Relationship

The integral from 1 to X is given explicitly. To find the integral from 1 to infinity, we need to evaluate the limit as $X \to \infty$:

$$
\int_{1}^{\infty} F(t) \, dt = \lim_{X \to \infty} \int_{1}^{X} F(t) \, dt
$$

Using the given expression:

$$
\int_{1}^{X} F(t) \, dt = \frac{20X}{\sqrt{4X^2 + 21}} - 4
$$

Our task reduces to evaluating:

$$
\lim_{X \to \infty} \left(\frac{20X}{\sqrt{4X^2 + 21}} - 4 \right)
$$

Step 2: Simplify the Limit Expression

Focus on the dominant behavior of the fraction as $X \to \infty$:

$$
\lim_{X \to \infty} \frac{20X}{\sqrt{4X^2 + 21}}
$$

Notice that for large X, the $+21$ becomes negligible compared to $4X^2$, so:

$$
\sqrt{4X^2 + 21} \sim \sqrt{4X^2} = 2X
$$

Therefore, the fraction simplifies to:

$$
\frac{20X}{2X} = 10
$$

But to be rigorous, let's perform the limit precisely.

Step 3: Rigorous Limit Calculation

Express the denominator to factor out X^2:

$$
\sqrt{4X^2 + 21} = \sqrt{4X^2 \left(1 + \frac{21}{4X^2}\right)} = 2X \sqrt{1 + \frac{21}{4X^2}}
$$

Thus, the fraction becomes:

$$
\frac{20X}{2X \sqrt{1 + \frac{21}{4X^2}}} = \frac{20X}{2X} \times \frac{1}{\sqrt{1 + \frac{21}{4X^2}}} = 10 \times \frac{1}{\sqrt{1 + \frac{21}{4X^2}}}
$$

As $X \to \infty$, $\frac{21}{4X^2} \to 0$, so:

$$
\lim_{X \to \infty} \frac{20X}{\sqrt{4X^2 + 21}} = 10 \times \frac{1}{\sqrt{1 + 0}} = 10 \times 1 = 10
$$

Step 4: Compute the Final Limit for the Integral

Recall the entire expression:

$$
\lim_{X \to \infty} \left(\frac{20X}{\sqrt{4X^2 + 21}} - 4 \right) = 10 - 4 = 6
$$

Hence,

$$
\boxed{
\int_{1}^{\infty} F(t) \, dt = 6
}
$$

which matches choice A.

Additional Insights and Key Concepts

1. Asymptotic Behavior of Rational Functions

The method used hinges on understanding the dominant behavior of the function as $t \to \infty$. Recognizing that the leading terms dictate the limit allows for simplified and accurate evaluation, especially when dealing with radical expressions involving quadratic terms.

2. Improper Integrals and Limits

The integral from 1 to infinity is an improper integral. Its convergence is determined by the limit of the integral as the upper bound approaches infinity. If this limit exists (is finite), the integral converges; otherwise, it diverges.

3. Physical and Mathematical Interpretations

In physics, such integrals could represent accumulated quantities over an infinite domain, such as total energy or charge. Mathematically, the problem illustrates how explicit integral expressions facilitate analyzing the behavior at infinity, crucial in fields like analysis and differential equations.

Summary of the Step-by-Step Process

Step	Description	Key Takeaway
1	Recognize the given integral expression and the goal	Set the problem as a limit as $X \to \infty$
2	Simplify the limit expression	Focus on dominant terms for large X
3	Factor out X^2 inside the radical	Use algebra to evaluate the limit precisely
4	Compute the limit and subtract the constant	Find that the integral converges to 6

Final Answer and Explanation

Based on the detailed analysis, the value of the improper integral is:

$$
\boxed{
\textbf{A. } 6
}
$$

This result demonstrates the power of limit evaluation and asymptotic analysis in solving integral problems involving infinite bounds.

Additional Practice and Extensions

Interested readers can further explore:

- Different forms of $F(t)$ and their integrals
- Convergence criteria for improper integrals
- Application of substitutions to evaluate more complex integrals
- Real-world problems where such integral limits are relevant

This problem highlights the importance of understanding the behavior of functions at infinity and mastering techniques for evaluating limits — skills essential for advanced calculus, mathematical analysis, and applied sciences.

If 1 To X F T Dt 20x Sqrt Of 4x2 21 4 Then 1 To Infinity F T Dt Is A 6b 1c 3d 4e

If 1 To X F T Dt 20x Sqrt Of 4x2 21 4 Then 1 To Infinity F T Dt Is A 6b 1c 3d 4e

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