

IF A CERTAIN NUMBER IS INCREASED BY 5, ONE-HALF OF THE RESULT IS THREE FIFTHS OF THE EXCESS OF 61 OVER

IF A CERTAIN NUMBER IS INCREASED BY 5, ONE-HALF OF THE RESULT IS THREE FIFTHS OF THE EXCESS OF 61 OVER IS A CLASSIC ALGEBRAIC PROBLEM THAT CHALLENGES STUDENTS AND MATH ENTHUSIASTS TO TRANSLATE WORDS INTO EQUATIONS, SOLVE FOR AN UNKNOWN, AND UNDERSTAND THE RELATIONSHIPS BETWEEN DIFFERENT QUANTITIES. THIS TYPE OF PROBLEM EXEMPLIFIES THE IMPORTANCE OF ALGEBRAIC THINKING, LOGICAL REASONING, AND PROBLEM-SOLVING SKILLS. IN THIS COMPREHENSIVE ARTICLE, WE WILL EXPLORE THE PROBLEM IN DEPTH, EXPLAIN THE STEPS TO SOLVE IT, DISCUSS RELATED CONCEPTS, AND PROVIDE TIPS FOR MASTERING SIMILAR ALGEBRAIC WORD PROBLEMS.

UNDERSTANDING THE PROBLEM: BREAKING DOWN THE STATEMENT

BEFORE JUMPING INTO CALCULATIONS, IT'S CRUCIAL TO CAREFULLY ANALYZE THE PROBLEM STATEMENT AND IDENTIFY ITS KEY COMPONENTS:

- THE PHRASE "A CERTAIN NUMBER" INDICATES AN UNKNOWN QUANTITY, OFTEN REPRESENTED BY A VARIABLE, SUCH AS x .
- "INCREASED BY 5" SUGGESTS ADDING 5 TO THIS UNKNOWN NUMBER.
- "ONE-HALF OF THE RESULT" REFERS TO HALF OF THE SUM OBTAINED AFTER INCREASING THE NUMBER BY 5.
- "THREE FIFTHS OF THE EXCESS OF 61 OVER" POINTS TO A FRACTIONAL PART INVOLVING THE NUMBER 61 AND SOME RELATED QUANTITY.

TRANSLATING THIS INTO AN ALGEBRAIC EXPRESSION INVOLVES IDENTIFYING THESE PARTS AND TRANSLATING WORDS INTO SYMBOLS.

STEP-BY-STEP BREAKDOWN OF THE PROBLEM

1. DEFINE THE UNKNOWN

- LET x BE THE CERTAIN NUMBER.

2. EXPRESS THE INCREASED NUMBER

- INCREASED BY 5: $x + 5$

3. FIND ONE-HALF OF THE RESULT

- HALF OF $x + 5$: $\frac{1}{2}(x + 5)$

4. UNDERSTAND "THE EXCESS OF 61 OVER"

- "OVER" INDICATES A COMPARISON OR DIFFERENCE.
- "THE EXCESS OF 61 OVER" SOME QUANTITY, WHICH IS PROBABLY x OR AN EXPRESSION INVOLVING x .
- SINCE THE PHRASE IS "THE EXCESS OF 61 OVER," IT SUGGESTS $(61 - \text{SOMETHING})$.

5. CONNECT THE PARTS

- THE STATEMENT SAYS: “ONE-HALF OF THE RESULT” EQUALS “THREE FIFTHS OF THE EXCESS OF 61 OVER” SOME QUANTITY.

PUTTING IT ALL TOGETHER, THE KEY RELATIONSHIP IS:

$$\frac{1}{2}(x + 5) = \frac{3}{5}(61 - x)$$

FORMULATING THE ALGEBRAIC EQUATION

BASED ON THE ABOVE REASONING, THE ALGEBRAIC EQUATION BECOMES:

$$\frac{1}{2}(x + 5) = \frac{3}{5}(61 - x)$$

OUR GOAL IS TO SOLVE FOR X, THE UNKNOWN NUMBER.

SOLVING THE EQUATION: STEP-BY-STEP GUIDE

STEP 1: ELIMINATE FRACTIONS

- TO MAKE CALCULATIONS EASIER, MULTIPLY BOTH SIDES OF THE EQUATION BY THE LEAST COMMON DENOMINATOR (LCD). THE DENOMINATORS ARE 2 AND 5; LCD = 10.

MULTIPLY BOTH SIDES BY 10:

$$10 \times \frac{1}{2}(x + 5) = 10 \times \frac{3}{5}(61 - x)$$

SIMPLIFY:

$$5(x + 5) = 6(61 - x)$$

STEP 2: EXPAND BOTH SIDES

- LEFT SIDE:

$$5x + 25$$

- RIGHT SIDE:

$$6 \times 61 - 6x = 366 - 6x$$

NOW, THE EQUATION IS:

$$5x + 25 = 366 - 6x$$

STEP 3: COLLECT LIKE TERMS

BRING ALL X TERMS TO ONE SIDE AND CONSTANTS TO THE OTHER:

$$5x + 6x = 366 - 25$$

$$11x = 341$$

STEP 4: SOLVE FOR X

$$x = \frac{341}{11}$$

$$x = 31 \frac{0}{11} \text{ \texttt{\text{or}} } x \approx 31.00$$

INTERPRETING THE SOLUTION

THE SOLUTION INDICATES THAT THE CERTAIN NUMBER X IS $(\frac{341}{11})$, APPROXIMATELY 31.00. TO VERIFY THE ANSWER, SUBSTITUTE X BACK INTO THE ORIGINAL RELATIONSHIP:

$$\frac{1}{2}(x + 5) \stackrel{?}{=} \frac{3}{5}(61 - x)$$

PLUGGING IN $(x = \frac{341}{11})$:

- LEFT SIDE:

$$\frac{1}{2} \left(\frac{341}{11} + 5 \right) = \frac{1}{2} \left(\frac{341}{11} + \frac{55}{11} \right) = \frac{1}{2} \times \frac{396}{11} = \frac{396}{22} = \frac{198}{11}$$

- RIGHT SIDE:

$$\left[\frac{3}{5} \left(61 - \frac{341}{11} \right) \right]$$

EXPRESS 61 AS $\left(\frac{671}{11} \right)$:

$$\left[\frac{3}{5} \left(\frac{671}{11} - \frac{341}{11} \right) \right] = \frac{3}{5} \times \frac{330}{11} = \frac{3 \times 330}{5 \times 11} = \frac{990}{55} = \frac{198}{11}$$

BOTH SIDES EQUAL $\left(\frac{198}{11} \right)$, CONFIRMING THE CORRECTNESS OF THE SOLUTION.

KEY CONCEPTS AND TIPS FOR SOLVING SIMILAR ALGEBRAIC WORD PROBLEMS

UNDERSTANDING WORD PROBLEMS

- CAREFULLY READ THE PROBLEM STATEMENT.
- IDENTIFY KEY QUANTITIES AND RELATIONSHIPS.
- TRANSLATE WORDS INTO ALGEBRAIC EXPRESSIONS STEP-BY-STEP.

SETTING UP EQUATIONS

- ASSIGN VARIABLES TO UNKNOWN QUANTITIES.
- EXPRESS ALL PARTS OF THE PROBLEM IN TERMS OF THE VARIABLES.
- DOUBLE-CHECK THE TRANSLATION FOR ACCURACY.

SOLVING EQUATIONS

- CLEAR FRACTIONS BY MULTIPLYING THROUGH BY LCD.
- EXPAND AND COLLECT LIKE TERMS.
- ISOLATE THE VARIABLE AND SOLVE.
- VERIFY SOLUTIONS BY SUBSTITUTION.

COMMON PITFALLS TO AVOID

- MISINTERPRETING THE WORDING (E.G., “OVER,” “EXCESS,” “INCREASED BY”).
- FORGETTING TO VERIFY SOLUTIONS.
- MAKING ARITHMETIC ERRORS DURING CALCULATIONS.

RELATED TOPICS AND FURTHER READING

- **LINEAR EQUATIONS AND INEQUALITIES:** UNDERSTANDING HOW TO SOLVE FOR UNKNOWN.
- **WORD PROBLEM STRATEGIES:** BREAKING DOWN COMPLEX PROBLEMS INTO MANAGEABLE PARTS.
- **ALGEBRAIC MANIPULATION:** TECHNIQUES FOR SIMPLIFYING EXPRESSIONS AND EQUATIONS.
- **MATHEMATICAL TRANSLATION:** CONVERTING VERBAL DESCRIPTIONS INTO ALGEBRAIC FORMULAS.
- **PROBLEM-SOLVING SKILLS:** DEVELOPING LOGICAL REASONING AND CRITICAL THINKING.

CONCLUSION: MASTERING ALGEBRAIC WORD PROBLEMS

THE PROBLEM “IF A CERTAIN NUMBER IS INCREASED BY 5, ONE-HALF OF THE RESULT IS THREE FIFTHS OF THE EXCESS OF 61 OVER” IS A PERFECT EXAMPLE OF HOW ALGEBRA TRANSFORMS WORDS INTO EQUATIONS, ENABLING PRECISE SOLUTIONS. BY CAREFULLY ANALYZING THE STATEMENT, TRANSLATING EACH PART INTO ALGEBRAIC EXPRESSIONS, AND FOLLOWING SYSTEMATIC SOLVING STEPS, YOU CAN CONFIDENTLY SOLVE SIMILAR PROBLEMS. PRACTICE IS KEY—TRY CREATING AND SOLVING YOUR OWN WORD PROBLEMS TO SHARPEN YOUR SKILLS. REMEMBER, UNDERSTANDING THE RELATIONSHIPS BETWEEN QUANTITIES IS AT THE HEART OF ALGEBRA, AND MASTERING THESE CONCEPTS OPENS THE DOOR TO A WIDE RANGE OF MATHEMATICAL CHALLENGES.

KEYWORDS FOR SEO OPTIMIZATION:

- ALGEBRA WORD PROBLEMS
- SOLVING FOR AN UNKNOWN
- ALGEBRAIC EQUATIONS
- TRANSLATING WORDS INTO MATH
- STEP-BY-STEP ALGEBRA SOLUTIONS
- HOW TO SOLVE ALGEBRA PROBLEMS
- MATH PROBLEM SOLVING TIPS
- ALGEBRA PRACTICE PROBLEMS
- BASIC ALGEBRA CONCEPTS
- UNDERSTANDING WORD PROBLEMS IN MATH

FREQUENTLY ASKED QUESTIONS

WHAT IS THE ALGEBRAIC EXPRESSION FOR ‘A CERTAIN NUMBER’ IN THE PROBLEM?

LET THE CERTAIN NUMBER BE REPRESENTED BY x .

HOW DO YOU TRANSLATE ‘IF A CERTAIN NUMBER IS INCREASED BY 5’ INTO AN EQUATION?

IT BECOMES $x + 5$.

WHAT DOES ‘ONE-HALF OF THE RESULT’ REFER TO IN THE PROBLEM?

IT REFERS TO HALF OF $(x + 5)$, SO $(x + 5)/2$.

How is 'Three fifths of the excess of 61 over' interpreted mathematically?

It is $(\frac{3}{5})(61 - x)$.

What is the step-by-step method to solve for x in this problem?

Set up the equation $(x + 5)/2 = (\frac{3}{5})(61 - x)$, then solve for x by cross-multiplying and simplifying.

What is the value of the certain number after solving the equation?

The certain number x equals 16 after solving the equation.

Additional Resources

Mathematical Problem Analysis: A Deep Dive into the Equation and Its Implications

Solving algebraic word problems often requires careful interpretation and logical reasoning. The problem under consideration is:

"If a certain number is increased by 5, one-half of the result is three-fifths of the excess of 61 over [that number]."

This statement, while seemingly straightforward, involves multiple layers of abstraction and algebraic translation. In this comprehensive review, we will dissect the problem, translate it into algebraic expressions, analyze each component, and explore solution strategies, all while emphasizing the importance of precise language in mathematical communication.

Understanding the Problem Statement

Before jumping into calculations, it is crucial to interpret the problem correctly. The statement contains several key phrases:

- "A certain number": an unknown variable, which we will denote as (x) .
- "Increased by 5": adding 5 to this unknown, giving $(x + 5)$.
- "One-half of the result": half of $(x + 5)$, i.e., $(\frac{1}{2})(x + 5)$.
- "Three-fifths of the excess of 61 over [that number]": this is the more nuanced part. The phrase "excess of 61 over [that number]" suggests the difference $(61 - x)$. Then, three-fifths of this difference is $(\frac{3}{5})(61 - x)$.

The core of the problem hinges on equating these two quantities, as implied by the statement: "one-half of the result is three-fifths of the excess of 61 over [that number]."

Translating the Word Problem into an Algebraic Equation

The first step in solving such a problem is to convert the language into a formal algebraic equation.

STEP-BY-STEP TRANSLATION:

1. IDENTIFY THE VARIABLES:

- LET (x) BE THE UNKNOWN NUMBER.

2. EXPRESS "A CERTAIN NUMBER INCREASED BY 5":

- $(x + 5)$.

3. EXPRESS "ONE-HALF OF THE RESULT":

- $(\frac{1}{2}(x + 5))$.

4. EXPRESS "THE EXCESS OF 61 OVER THE NUMBER":

- $(61 - x)$.

5. EXPRESS "THREE-FIFTHS OF THE EXCESS":

- $(\frac{3}{5}(61 - x))$.

6. SET UP THE EQUATION BASED ON THE STATEMENT:

- THE PHRASE "ONE-HALF OF THE RESULT IS THREE-FIFTHS OF THE EXCESS" INDICATES EQUALITY:

$$\left[\frac{1}{2}(x + 5) = \frac{3}{5}(61 - x) \right]$$

THIS ALGEBRAIC EQUATION ENCAPSULATES THE ENTIRE PROBLEM.

SOLVING THE ALGEBRAIC EQUATION

WITH THE EQUATION ESTABLISHED, THE NEXT STEP IS TO SOLVE FOR (x) . THE PROCESS INVOLVES ALGEBRAIC MANIPULATION, INCLUDING CLEARING FRACTIONS, EXPANDING, AND ISOLATING THE VARIABLE.

STEP 1: CLEAR DENOMINATORS

MULTIPLY BOTH SIDES OF THE EQUATION BY THE LEAST COMMON MULTIPLE OF DENOMINATORS, WHICH IS 10:

$$\left[10 \times \left(\frac{1}{2}(x + 5) \right) = 10 \times \left(\frac{3}{5}(61 - x) \right) \right]$$

CALCULATING EACH SIDE:

$$\left[5(x + 5) = 6(61 - x) \right]$$

STEP 2: EXPAND BOTH SIDES

$$\left[5x + 25 = 6 \times 61 - 6x \right]$$

$$5x + 25 = 366 - 6x$$

STEP 3: BRING ALL TERMS TO ONE SIDE

ADD $(6x)$ TO BOTH SIDES AND SUBTRACT 25:

$$5x + 6x = 366 - 25$$

$$11x = 341$$

STEP 4: SOLVE FOR (x)

$$x = \frac{341}{11}$$

$$x \approx 31 \frac{0}{11}$$

EXPRESSED AS AN IMPROPER FRACTION OR DECIMAL:

$$x \approx 31.0 \text{ (SINCE } 11 \times 31 = 341)$$

THUS, THE EXACT VALUE OF THE NUMBER IS:

$$\boxed{x = \frac{341}{11}}$$

OR APPROXIMATELY:

$$x \approx 31.0$$

VERIFICATION OF THE SOLUTION

VERIFYING THE SOLUTION ENSURES THE ALGEBRAIC MANIPULATIONS ARE CORRECT AND THE INTERPRETATION ALIGNS WITH THE ORIGINAL PROBLEM.

PLUGGING $\left(x = \frac{341}{11}\right)$ INTO THE ORIGINAL EXPRESSIONS:

- CALCULATE $(x + 5)$:

$$\left[x + 5 = \frac{341}{11} + 5 = \frac{341}{11} + \frac{55}{11} = \frac{396}{11} \right]$$

- HALF OF THE RESULT:

$$\left[\frac{1}{2} \times \frac{396}{11} = \frac{396}{22} = \frac{198}{11} \right]$$

- CALCULATE $(61 - x)$:

$$\left[61 - \frac{341}{11} = \frac{671}{11} - \frac{341}{11} = \frac{330}{11} = 30 \right]$$

- THREE-FIFTHS OF THE EXCESS:

$$\left[\frac{3}{5} \times 30 = \frac{3}{5} \times 30 = 3 \times 6 = 18 \right]$$

- COMPARE THE TWO QUANTITIES:

$$\left[\text{HALF OF THE RESULT} = \frac{198}{11} \approx 18 \right]$$

WHICH SIMPLIFIES EXACTLY TO 18:

$$\left[\frac{198}{11} = 18 \right]$$

SINCE BOTH ARE EQUAL TO 18, THE SOLUTION SATISFIES THE ORIGINAL CONDITION, CONFIRMING THE CORRECTNESS OF THE ANSWER.

EXPLORATION OF VARIATIONS AND RELATED PROBLEMS

THE PROBLEM EXEMPLIFIES THE TYPICAL STRUCTURE OF ALGEBRAIC WORD PROBLEMS. VARIATIONS OF THIS PROBLEM CAN INTRODUCE ADDITIONAL COMPLEXITIES OR DIFFERENT RELATIONSHIPS.

POTENTIAL VARIATIONS INCLUDE:

- CHANGING THE COEFFICIENTS: FOR EXAMPLE, REPLACING “ONE-HALF” WITH “ONE-THIRD” OR “TWO-THIRDS”.
- USING DIFFERENT CONSTANTS INSTEAD OF 5 OR 61.
- CONSIDERING INEQUALITIES RATHER THAN EQUATIONS.
- INTRODUCING MULTIPLE UNKNOWNNS AND CREATING SYSTEMS OF EQUATIONS.

EACH VARIATION REQUIRES CAREFUL TRANSLATION AND ALGEBRAIC MANIPULATION, EMPHASIZING THE IMPORTANCE OF UNDERSTANDING THE LANGUAGE AND STRUCTURE OF THE PROBLEM.

MATHEMATICAL CONCEPTS HIGHLIGHTED IN THE PROBLEM

THIS PROBLEM TOUCHES UPON SEVERAL FUNDAMENTAL ALGEBRAIC PRINCIPLES:

- TRANSLATION OF WORDS TO EQUATIONS: INTERPRETING LANGUAGE PRECISELY.
- HANDLING FRACTIONS: CLEARING DENOMINATORS AND SIMPLIFYING.
- SOLVING LINEAR EQUATIONS: ISOLATING THE VARIABLE.
- VERIFICATION: SUBSTITUTING SOLUTIONS BACK INTO ORIGINAL EXPRESSIONS.
- UNDERSTANDING PROPORTIONAL RELATIONSHIPS: INTERPRETING “HALF OF” AND “THREE-FIFTHS OF” AS SCALAR MULTIPLES.

FURTHERMORE, THE PROBLEM ILLUSTRATES THE IMPORTANCE OF LOGICAL REASONING WHEN DEALING WITH WORDED STATEMENTS, ILLUSTRATING HOW CAREFUL INTERPRETATION CAN PREVENT ERRORS.

PRACTICAL APPLICATIONS OF SIMILAR PROBLEMS

WHILE THIS PROBLEM IS MATHEMATICAL IN NATURE, SIMILAR REASONING IS APPLICABLE IN REAL-WORLD CONTEXTS:

- FINANCIAL CALCULATIONS: DETERMINING SAVINGS, INTEREST, OR DISCOUNTS INVOLVING PROPORTIONS.
- PHYSICS: RELATING QUANTITIES PROPORTIONAL TO EACH OTHER.
- ENGINEERING: DESIGNING SYSTEMS WITH PROPORTIONAL PARAMETERS.

UNDERSTANDING HOW TO TRANSLATE DESCRIPTIVE LANGUAGE INTO ALGEBRAIC EXPRESSIONS IS A VITAL SKILL ACROSS NUMEROUS DISCIPLINES.

CONCLUSION: THE SIGNIFICANCE OF PRECISE PROBLEM SOLVING

THIS IN-DEPTH ANALYSIS UNDERSCORES THE VALUE OF METICULOUS INTERPRETATION, ALGEBRAIC TRANSLATION, AND VERIFICATION. PROBLEMS LIKE “IF A CERTAIN NUMBER IS INCREASED BY 5, ONE-HALF OF THE RESULT IS THREE FIFTHS OF THE EXCESS OF 61 OVER” SERVE AS EXCELLENT EXERCISES FOR HONING THESE SKILLS.

BY SYSTEMATICALLY BREAKING DOWN THE PROBLEM, TRANSLATING IT INTO AN ALGEBRAIC FRAMEWORK, SOLVING STEP-BY-STEP, AND VERIFYING THE SOLUTION, WE NOT ONLY ARRIVE AT THE CORRECT ANSWER BUT ALSO DEEPEN OUR UNDERSTANDING OF ALGEBRAIC REASONING. SUCH PROBLEM-SOLVING STRATEGIES ARE FOUNDATIONAL FOR ADVANCED MATHEMATICS AND ESSENTIAL FOR CRITICAL THINKING IN NUMEROUS SCIENTIFIC AND PRACTICAL DOMAINS.

IN SUMMARY:

- THE UNKNOWN NUMBER (x) IS EXACTLY $(\frac{341}{11})$.
- THE PROBLEM DEMONSTRATES THE IMPORTANCE OF CAREFUL LANGUAGE INTERPRETATION.
- IT HIGHLIGHTS ALGEBRAIC TECHNIQUES SUCH AS CLEARING FRACTIONS AND VERIFYING SOLUTIONS.
- THE PROCESS EXEMPLIFIES RIGOROUS MATHEMATICAL REASONING APPLICABLE TO A WIDE ARRAY OF PROBLEMS.

MASTERING THESE SKILLS ENSURES PROFICIENCY IN TACKLING SIMILAR PROBLEMS AND ENHANCES OVERALL MATHEMATICAL LITERACY, WHICH IS INVALUABLE IN EDUCATION AND REAL-WORLD APPLICATIONS ALIKE.

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