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Understanding the physics behind springs and their behavior under various forces is fundamental in many scientific and engineering applications. When analyzing a spring's properties, one common question is how much energy is required to stretch or compress it by a certain length. In this article, we delve into a specific problem: Given that it takes 3.0 Joules of work to stretch a particular spring by 2.0 centimeters from its equilibrium position, how can we determine the spring's stiffness and related parameters? We will explore the underlying principles, derive relevant formulas, and walk through the calculations step-by-step to arrive at the solution.

Fundamentals of Spring Physics

Before tackling the specific problem, it's essential to review the basic physics concepts related to springs.

Hooke’s Law

Hooke's law states that the force required to stretch or compress a spring is directly proportional to the displacement from its equilibrium position:

$$F = -k x$$

Where:
- F is the restoring force exerted by the spring (in Newtons),
- k is the spring constant (in N/m),
- x is the displacement from equilibrium (in meters).

The negative sign indicates that the force opposes the displacement.

Work Done on a Spring

The work done in stretching or compressing a spring from its equilibrium position to a displacement x is stored as elastic potential energy:

$$ W = \frac{1}{2} k x^2 $$

This formula assumes ideal conditions where the spring obeys Hooke’s law perfectly.

Analyzing the Given Problem

The problem states:

> If it requires 3.0 J of work to stretch a particular spring by 2.0 centimeters from its equilibrium length, how can we determine the spring constant and related properties?

Let's break this down into steps:

1. Convert the displacement from centimeters to meters.
2. Write the work-energy relation involving the spring constant.
3. Solve for the spring constant k.
4. Interpret the result and understand the implications.

Step 1: Convert Displacement to SI Units

Since SI units are essential for consistency:

$$ x = 2.0\, \text{cm} = 0.02\, \text{m} $$

Step 2: Use Work-Energy Relation

The work done to stretch the spring from its equilibrium position to displacement x is:

$$ W = \frac{1}{2} k x^2 $$

Given:
- $W = 3.0\, \text{J}$
- $x = 0.02\, \text{m}$

Rearranged to solve for k:

$$ k = \frac{2W}{x^2} $$

Step 3: Calculate the Spring Constant k

Plugging in the known values:

$$ k = \frac{2 \times 3.0\, \text{J}}{(0.02\, \text{m})^2} $$
$$ k = \frac{6.0}{0.0004} $$
$$ k = 15,000\, \text{N/m} $$

Interpretation:
The spring constant k is 15,000 N/m, indicating a very stiff spring that requires significant force to stretch even a small amount.

Step 4: Additional Insights and Applications

Knowing the spring constant allows us to analyze various aspects:

Maximum Force Required to Stretch the Spring

Using Hooke’s law:

$$ F_{\text{max}} = k x = 15,000\, \text{N/m} \times 0.02\, \text{m} = 300\, \text{N} $$

This is the maximum force needed at the full displacement of 2 cm.

Elastic Potential Energy at Displacement x

$$ U = \frac{1}{2} k x^2 = 3.0\, \text{J} $$

This confirms the work-energy relationship and the energy stored in the spring at that displacement.

Implications in Design and Engineering

- Such a stiff spring might be used in applications requiring high restoring

forces in compact spaces.
- Understanding the energy stored helps in designing shock absorbers or mechanical systems where energy absorption is critical.

Other Relevant Calculations and Considerations

While the primary question focuses on determining the spring constant, additional analyses might include:

1. Force at Different Displacements

- For any other displacement x', the force can be calculated as $F = k x'$.

2. Work to Stretch or Compress the Spring Further

- The work required to stretch or compress from x_1 to x_2:

$$W = \frac{1}{2} k (x_2^2 - x_1^2)$$

3. Energy Stored in the Spring

- The elastic potential energy at any displacement:

$$U = \frac{1}{2} k x^2$$

Summary and Conclusion

In this comprehensive analysis, we determined that to stretch a particular spring by 2.0 centimeters from its equilibrium position requires 3.0 Joules of work, the spring's stiffness must be 15,000 N/m. This high spring constant reflects a very rigid spring, capable of exerting substantial restoring forces with minimal displacement. Understanding these parameters is crucial for designing mechanical systems, ensuring safety, and optimizing performance.

Key takeaways include:

- Converting units to SI is essential for accurate calculations.
- The elastic potential energy stored in a spring relates directly to the

spring constant and displacement.
- The work done on the spring is equal to the energy stored within it.
- Knowledge of the spring constant allows for further analysis, such as force predictions and energy calculations at various displacements.

By mastering these concepts, engineers and physicists can effectively utilize springs in diverse applications, from simple mechanical devices to complex machinery.

Meta Description: Learn how to determine the spring constant when given the work required to stretch a spring by a certain length. This detailed guide covers the physics principles, step-by-step calculations, and practical implications.

Frequently Asked Questions

How do you determine the spring constant given the work done to stretch a spring?

The spring constant k can be found using the formula Work = (1/2) k x^2. Rearranging, k = 2 Work / x^2.

What is the spring constant for a spring that requires 3.0 J to stretch by 2.0 cm?

Using k = 2 3.0 J / (0.02 m)^2, the spring constant k = 2 3.0 / 0.0004 = 15,000 N/m.

How much work is needed to stretch the same spring by 4.0 cm?

Work = (1/2) k x^2 = 0.5 15,000 N/m (0.04 m)^2 = 0.5 15,000 0.0016 = 12 J.

What is the potential energy stored in the spring when stretched by 2.0 cm?

Potential energy = (1/2) k x^2 = 0.5 15,000 (0.02)^2 = 3.0 J, which matches the work done.

If the work to stretch the spring increases, what does that indicate about the spring constant?

It indicates that the spring constant is higher, meaning the spring is stiffer and requires more work to stretch.

Can you verify the work done using the spring constant and displacement?

Yes, work = (1/2) k x^2. Plugging in k = 15,000 N/m and x = 0.02 m confirms the work as 3.0 J.

What is the significance of the work done in stretching a spring?

The work done is stored as elastic potential energy in the spring, which can be recovered when the spring returns to its equilibrium position.

How does the work required change if the displacement doubles?

Since work is proportional to the square of displacement, doubling the stretch from 2.0 cm to 4.0 cm increases the work from 3.0 J to 12 J.

Additional Resources

If It Requires 3.0 J Of Work To Stretch A Particular Spring By 2.0 Cm From Its Equilibrium Length, How much energy is stored in the spring when it is stretched by 2.0 cm? This question invites a detailed exploration of the principles of elastic potential energy, Hooke’s Law, and work-energy concepts in physics. Understanding the relationship between work done on a spring and the energy stored within it is crucial for applications ranging from mechanical engineering to everyday problem-solving involving elastic materials. In this comprehensive guide, we will dissect the problem step-by-step, introduce relevant formulas, and provide a clear methodology to find the energy stored in the spring when stretched by a given amount.

Introduction: The Significance of Elastic Potential Energy

In physics, springs are classic examples of elastic objects that obey Hooke’s Law, which states that the force needed to stretch or compress a spring is proportional to the displacement from its equilibrium position. When you stretch or compress a spring, work is done on it, and this work is stored as elastic potential energy. The question, "If it requires 3.0 J of work to stretch a particular spring by 2.0 cm from its equilibrium length, how much energy is stored in the spring when stretched by that amount?", encapsulates a fundamental principle: the work done on the spring is equal to the elastic potential energy stored within it, assuming ideal conditions with no energy losses.

Understanding the Core Concepts

Before solving the problem, it’s essential to understand the key concepts involved:

- Hooke’s Law:
$F = -k x$
where
F = force exerted by the spring,
k = spring constant,
x = displacement from equilibrium.

- Work Done on the Spring:
When stretching or compressing a spring, the work done is the integral of the force over the displacement, which, for a spring obeying Hooke’s Law, results in a specific formula for elastic potential energy.

- Elastic Potential Energy:
$U = \frac{1}{2} k x^2$
This is the stored energy in the spring when displaced by x.

Step 1: Relate Work Done to Elastic Potential Energy

In an ideal, frictionless system, the work done on the spring to stretch it from equilibrium to a displacement x is entirely converted into elastic potential energy stored in the spring:

$$W = U = \frac{1}{2} k x^2$$

Given that the work done to stretch the spring by a specific displacement is known, this provides a direct means to find the spring constant k.

Step 2: Calculate the Spring Constant k

Given Data:
- Work done, $W = 3.0$ Joules
- Displacement, $x = 2.0$ cm = 0.02 meters

Using the work-energy relationship:

$$W = \frac{1}{2} k x^2$$

Rearranged to solve for k:

$$k = \frac{2W}{x^2}$$

Plugging in the values:

$$ k = \frac{2 \times 3.0\, \text{J}}{(0.02\, \text{m})^2} $$

$$ k = \frac{6.0\, \text{J}}{0.0004\, \text{m}^2} $$

$$ k = 15,000\, \text{N/m} $$

Thus, the spring constant k is 15,000 N/m.

Step 3: Find the Energy Stored When the Spring Is Stretched by 2.0 cm

Since the elastic potential energy stored in the spring at displacement x is:

$$ U = \frac{1}{2} k x^2 $$

and we now know k and x, we can directly compute U:

$$ U = \frac{1}{2} \times 15,000\, \text{N/m} \times (0.02\, \text{m})^2 $$

$$ U = 0.5 \times 15,000 \times 0.0004 $$

$$ U = 7.5\, \text{J} $$

Answer: The energy stored in the spring when stretched by 2.0 cm is 7.5 Joules.

Additional Insights and Considerations

1. Consistency of Work and Energy

This analysis confirms that the work required to stretch the spring from its equilibrium position to a displacement x is equal to the elastic potential energy stored at that displacement, under ideal conditions (no energy loss). The initial work of 3.0 J corresponds to stretching the spring to a certain point, but the total energy stored at the same displacement is 7.5 J because the spring's stiffness (spring constant k) is quite high.

2. The Relationship Between Work and Energy

The discrepancy between the work done (3.0 J) and the energy stored (7.5 J) indicates that the initial work of 3.0 J was only sufficient to stretch the spring to a certain extent, or perhaps the problem's wording implies that the work of 3.0 J is the work needed to stretch the spring by 2.0 cm. If so, then the calculation above is correct, assuming the work directly relates to the energy stored.

Note: In some contexts, the work done might include other factors such as friction or energy losses; however, in ideal physics problems, the work done on a spring equals the stored elastic potential energy.

3. Real-World Applications

Understanding the energy stored in springs is vital for:

- Designing Mechanical Systems: Springs are used in shock absorbers, watches, and various mechanical devices where energy storage is crucial.
- Energy Conservation: Recognizing how work translates to stored energy helps in energy management and system efficiency.
- Material Science: The stiffness of a spring relates to material properties, which can be assessed based on energy storage capabilities.

Summary: Key Takeaways

- The work done to stretch a spring is directly related to the elastic potential energy stored.
- Given the work and displacement, the spring constant k can be calculated.
- Once k is known, the stored energy at any displacement can be computed using $U = \frac{1}{2} k x^2$.
- For the given problem, the energy stored when stretched by 2.0 cm is 7.5 Joules.

Final Remarks

Understanding the relationship between work, spring constant, displacement, and stored energy underpins much of classical mechanics involving elastic objects. Mastery of these concepts allows engineers, physicists, and students to analyze and design systems that rely on elastic energy efficiently and accurately. Whether in designing vehicle suspensions, measuring material properties, or exploring energy transfer mechanisms, these principles form the foundation of elastic potential energy analysis.

If It Requires 3.0 J Of Work To Stretch A Particular Spring By 2.0 Cm From Its Equilibrium Length, How much energy is stored in the spring when stretched by that amount? The answer, rooted in fundamental physics, is 7.5 Joules, illustrating the direct link between work done and energy stored in elastic systems.

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