If Point A, Havingcoordinates (p, 3) Andpoint B, With Coordinates(6, P) Lie On A Line Havinga Slope Of

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Understanding the Problem

To analyze the relationship between two points and the slope of the line passing through them, we first need to understand the given information:

- Point A has coordinates $(p, 3)$
- Point B has coordinates $(6, p)$
- The slope of the line passing through these points is m (a given value, or to be determined)

The goal is to determine the value(s) of p based on the given slope, or vice versa, and to understand the geometric relationship between the points and the line.

Fundamentals of Slope and Coordinates

Definition of Slope

The slope m of a line passing through two points (x_1, y_1) and (x_2, y_2) is given by:

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

This formula indicates how steep the line is, representing the ratio of the change in y to the change in x.

Applying the Slope Formula to the Given Points

Given points:

- $A = (p, 3)$
- $B = (6, p)$

The slope m is:

$$
m = \frac{p - 3}{6 - p}
$$

This expression relates the variable p to the slope m.

Deriving Conditions for p

Expressing p in Terms of the Slope

Rearranging the formula:

$$
m = \frac{p - 3}{6 - p}
$$

Cross-multiplied:

$$
m (6 - p) = p - 3
$$

Expanding:

$$
6m - p m = p - 3
$$

Grouping p terms:

$$
- p m - p = - 3 - 6m
$$

Factor p:

$$
p (- m - 1) = - 3 - 6m
$$

Solve for p:

$$
p = \frac{- 3 - 6m}{- m - 1}
$$

Simplify numerator and denominator:

$$
p = \frac{3 + 6m}{m + 1}
$$

This equation expresses p explicitly in terms of the slope m.

Special Cases and Constraints

- When $m = -1$, the denominator becomes zero, and p is undefined. This indicates that for $m = -1$, the points do not define a unique p, and the line may be vertical or degenerate.
- For other values of m, the value of p is well-defined and can be computed from the formula.

Analyzing Specific Scenarios

Case 1: Known Slope m

Suppose the slope m is provided; then, the value of p can be directly calculated using:

$$
p = \frac{3 + 6m}{m + 1}
$$

Example:

- If $m = 2$:

$$
p = \frac{3 + 12}{2 + 1} = \frac{15}{3} = 5

$$

- The points are:

$$
A = (5, 3), \quad B = (6, 5)
$$

And the slope calculation:

$$
m = \frac{5 - 3}{6 - 5} = \frac{2}{1} = 2
$$

which matches the initial assumption.

Case 2: p is known

If p is known, then the slope m between the points is:

$$
m = \frac{p - 3}{6 - p}
$$

Example:

- If $p = 4$:

$$
m = \frac{4 - 3}{6 - 4} = \frac{1}{2}
$$

- The line passing through $(4, 3)$ and $(6, 4)$ has slope 0.5.

Geometric Interpretations and Constraints

Line Equation Passing Through the Points

Using point $A=(p, 3)$ and the slope m:

$$

\text{Equation: } y - 3 = m(x - p)
\]

Similarly, using point \(B=(6, p)\):

\[
y - p = m(x - 6)
\]

These form the basis for understanding the line's equation in various scenarios.

Conditions for the Points to Lie on the Same Line

For the two points to be on the same line with slope \(m\), the following must hold:

\[
\text{Both points satisfy } y - 3 = m(x - p)
\]

\[
\text{and } y - p = m(x - 6)
\]

Substituting the coordinates and verifying:

- For \(A\):

\[
3 - 3 = m(p - p) \Rightarrow 0 = 0
\]

- For \(B\):

\[
p - p = m(6 - 6) \Rightarrow 0 = 0
\]

This confirms that the points are consistent with the line's equation if the earlier conditions on \(p\) and \(m\) are satisfied.

Implications and Applications

Determining p for a Given Slope

Knowing the slope m, the formula:

$$
p = \frac{3 + 6m}{m + 1}
$$

can be used to find the specific p that makes the points $(p, 3)$ and $(6, p)$ lie on a line with slope m.

This is particularly useful in:

- Geometric problem-solving
- Coordinate geometry exercises
- Designing graphs with specified properties
- Real-world applications where points must satisfy specific linear relationships

Finding the Slope for a Given p

Conversely, if p is given, the slope is:

$$
m = \frac{p - 3}{6 - p}
$$

which can be used to analyze the steepness of the line or to verify if the line meets certain criteria.

Conclusion

The relationship between the points $(p, 3)$ and $(6, p)$ and the slope m offers a rich avenue for exploration in coordinate geometry. By deriving the formula:

$$
p = \frac{3 + 6m}{m + 1}
$$

we establish a direct link between the variable p and the slope m. This relationship allows for flexible problem-solving, enabling calculations of p for known slopes, or vice versa. Furthermore, understanding the special cases, such as when $m = -1$, provides deeper insights into the geometric configurations and potential degeneracies.

The analysis of such points and slopes finds applications across mathematics, physics, engineering, and computer science, wherever the properties of lines and points in a plane are relevant. Mastery of these relationships enhances problem-solving skills and fosters a more profound understanding of the fundamental principles of coordinate geometry.

Frequently Asked Questions

How can we find the value of P if points A(p, 3) and B(6, P) lie on a line with a given slope?

You can use the slope formula (m = (y2 - y1) / (x2 - x1)) and set it equal to the given slope to solve for P.

What is the formula to determine if two points lie on a line with a specific slope?

Calculate the slope between the two points and compare it to the given slope. If they match, the points lie on that line.

If the slope of the line is known, how do we find the coordinate P for point B?

Use the slope formula: m = (P - 3) / (6 - p), then solve for P by substituting the known slope value.

Can P be any value if points A and B are on the same line with a certain slope?

No, P must satisfy the slope condition between A and B. Only specific P values will satisfy this condition depending on the slope.

What steps are involved in verifying that points A and B lie on the same line with slope m?

Calculate the slope between A and B and check if it equals m. If yes, both points lie on the same line with that slope.

How does changing P affect whether points A and B are collinear on a line with a fixed slope?

Adjusting P changes the y-coordinate of B, which affects the slope calculation. Only the P value that matches the slope condition ensures collinearity.

If the slope of the line is 2, what is the value of P for points A(p, 3) and B(6, P)?

Using the slope formula: 2 = (P - 3) / (6 - p). Solving for P gives P = 2(6 - p) + 3.

Additional Resources

If Point A, Having Coordinates (p, 3) and Point B, With Coordinates (6, p) Lie on a Line Having a Slope of

Understanding the relationship between points and the slopes of lines connecting them is a fundamental concept in coordinate geometry. When given specific points, such as Point A with coordinates (p, 3) and Point B with coordinates (6, p), and a condition that these points lie on a line with a certain slope, it opens the door to exploring algebraic relationships and constraints on the variables involved. In this guide, we will thoroughly analyze the scenario where these two points are on a line with a known slope, and how to determine the value or range of the variable p accordingly.

Introduction to Coordinate Geometry and Slope

Before delving into the specific problem, it's important to review some foundational concepts:

- Coordinates of a Point: A point in the Cartesian plane is represented as (x, y).
- Slope of a Line: The measure of the steepness or incline of a line, calculated as the change in y over the change in x between two points.

The slope formula for any two points, A(x_1, y_1) and B(x_2, y_2), is:

\[m = \frac{y_2 - y_1}{x_2 - x_1} \]

where m is the slope.

The Problem Setup

Given:

- Point A: (p, 3)
- Point B: (6, p)
- The line passing through these points has a slope of m (a known or

specified value).

Our goal is to analyze the conditions under which these points lie on such a line, and how the variable p is constrained based on the slope.

Step-by-Step Breakdown

1. Express the Slope Using the Coordinates

Using the slope formula between points A and B:

$$ m = \frac{p - 3}{6 - p} $$

This is a key relationship because it links p with the slope m.

2. Conditions for the Points to Lie on the Line

Since both points are on the same line with slope m, the above equation must hold true for the specific p and m.

- If m is given, then:

$$ m = \frac{p - 3}{6 - p} $$

- If m is unknown, but the points are to lie on such a line, then p must satisfy this relationship for some m.

Analyzing Different Scenarios

Depending on whether m is specified or not, different analyses are possible.

Case 1: The Slope m Is Known

Suppose the problem states that the line has a specific slope, say, m = k, where k is a known constant.

1. Solve for p

Given:

$$ k = \frac{p - 3}{6 - p} $$

Cross-multiplied:

$$ k(6 - p) = p - 3 $$

which simplifies to:

$$ 6k - kp = p - 3 $$

Rearranged:

$$ -kp - p = -6k - 3 $$

Factor out p:

$$ p(-k - 1) = -6k - 3 $$

Solve for p:

$$ p = \frac{-6k - 3}{-k - 1} $$

or equivalently:

$$ p = \frac{6k + 3}{k + 1} $$

This expression provides the value of p in terms of the known slope k.

2. Constraints and Validity

- Denominator Check: The expression is valid as long as $k + 1 \neq 0$, i.e., $k \neq -1$.
- Special Cases:
- If $k = -1$, then the denominator is zero, and the calculation is invalid. In this case, analyze separately to see if the points are collinear or if the line is vertical.

Case 2: The Slope m Is Unknown, but Both Points Lie on the Same Line

In the absence of a specified slope, the key is to understand the relationship between p and the geometry of the points.

- The points are collinear if the slope between them is consistent.
- The only condition is that the slope calculated from the two points must be a real number (not undefined or infinite).

From the earlier formula:

$$ m = \frac{p - 3}{6 - p} $$

- To avoid division by zero, the denominator must not be zero:

$$ 6 - p \neq 0 \Rightarrow p \neq 6 $$

- The slope m is real and finite for all p except $p = 6$.

Special Cases and Geometric Interpretations

1. Vertical Line Scenario

- When the denominator $6 - p = 0$, i.e., $p = 6$, the slope becomes undefined (vertical line).
- The points are then:

- Point A: (6, 3)
- Point B: (6, 6)

- These points lie on a vertical line at $x = 6$.

2. Horizontal Line Scenario

- The slope is zero when:

$$ m = 0 \Rightarrow \frac{p - 3}{6 - p} = 0 $$

- Numerator must be zero:

$$ p - 3 = 0 \Rightarrow p = 3 $$

- Points:

- Point A: (3, 3)
- Point B: (6, 3)

- These points lie on a horizontal line at $y = 3$.

Graphical Representation and Intuitive Understanding

Visualizing these points and the line helps deepen understanding:

- The points are symmetric in some cases, depending on p, and the slope

reflects the angle of the line connecting them.
- When p varies, the slope changes, illustrating different inclinations.

Practical Applications and Examples

Let's consider some concrete examples to solidify understanding.

Example 1: Given $m = 1$

Find p such that the points lie on a line with slope 1.

Solution:

$$p = \frac{6(1) + 3}{1 + 1} = \frac{6 + 3}{2} = \frac{9}{2} = 4.5$$

- Points:

- $A(4.5, 3)$
- $B(6, 4.5)$

- Slope:

$$m = 1$$

Example 2: Find p when the line is vertical

- When $p = 6$, points are:

- $A(6, 3)$
- $B(6, 6)$

- Slope is undefined, and points are on the vertical line $x = 6$.

Example 3: Find p when the line is horizontal

- Set $p = 3$:

- Points:

- $A(3, 3)$

- $B(6, 3)$

- Slope:

$$m = 0$$

Summary of Key Findings

- The relationship between p and the slope m is:

$$p = \frac{6m + 3}{m + 1} \quad \text{(for } m \neq -1)$$

- When m is known, p can be directly calculated.
- When m is unknown, p can vary, but specific values correspond to particular slopes (e.g., horizontal, vertical).
- The points are collinear for any p except when $p = 6$, which makes the line vertical.

Final Thoughts: Applying the Concepts

This exploration underscores the importance of understanding how variables interact in coordinate geometry. Whether working with known slopes or determining possible positions of points based on variable parameters, the algebraic relationships provide vital insights into the geometry of the problem.

By mastering these relationships, students and professionals can solve complex problems involving points, lines, and slopes with confidence, making this a fundamental skill in mathematics, physics, engineering, and related fields.

Remember: Always analyze the domain restrictions, special cases like vertical and horizontal lines, and interpret the geometric meaning behind the algebra to develop a comprehensive understanding of the problem at hand.

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