If (x,y) Satisfies The Simultaneous Equations \begin{align*} 3xy - 4x^2 - 36y + 48x &= 0, \\ X^2

If (x,y) satisfies the simultaneous equations \begin{align} 3xy - 4x^2 - 36y + 48x &= 0, \\ x^2 &= \text{(second equation not provided)} \end{align}, it indicates a set of solutions that simultaneously satisfy both algebraic conditions. In this article, we will focus on analyzing the first equation in detail, exploring its possible solutions, and understanding how they relate to the second equation once clarified. We aim to develop a comprehensive understanding of the solution set, employing algebraic techniques, graphical interpretations, and logical reasoning. Although the second equation appears incomplete, we will proceed with the analysis of the first equation and discuss potential contexts for the second, assuming it might involve x^2 or another relation involving x and y.

Understanding the First Equation

Given Equation Analysis

The first equation is:

\[
3xy - 4x^2 - 36y + 48x = 0.
\]

This is a quadratic equation involving two variables, x and y, which suggests it might represent a conic section (circle, ellipse, hyperbola, or parabola). To analyze it systematically, we should attempt to express it in a form that reveals its geometric nature.

Rearranging the Equation

Let's rewrite the equation to group similar terms:

\[
3xy - 36y = 4x^2 - 48x.
\]

Factor the left and right sides where possible:

$$
y(3x - 36) = 4x^2 - 48x.
$$

Expressed as:

$$
y(3x - 36) = 4x(x - 12).
$$

Provided that $3x - 36 \neq 0$, we can solve for y:

$$
y = \frac{4x(x - 12)}{3x - 36}.
$$

Note that if $3x - 36 = 0$, i.e., $x = 12$, the original equation reduces to:

$$
3 \times 12 \times y - 4 \times 12^2 - 36 y + 48 \times 12 = 0,
$$

which simplifies to:

$$
36 y - 4 \times 144 - 36 y + 576 = 0,
$$

$$
36 y - 576 - 36 y + 576 = 0,
$$

$$
0 = 0,
$$

which is always true, indicating that when $x=12$, the entire line $x=12$ is part of the solution set.

Graphical Interpretation

Nature of the Equation

The expression:

$$
y = \frac{4x(x - 12)}{3x - 36},
$$

suggests a rational function, with a potential vertical asymptote at $x=12$, where the denominator zeroes out.

- For $x \neq 12$, y is defined by the rational expression.
- At $x=12$, the entire line $x=12$ belongs to the solution set, as shown earlier.

This indicates the equation describes a conic section with a line of solutions where the denominator vanishes, hinting at a hyperbola or other rational curve with an asymptote.

Plotting the Function

To visualize the solutions:

1. Identify key points:

- When $x=0$:

$$
y = \frac{4 \times 0 \times (-12)}{3 \times 0 - 36} = 0,
$$

so the point $(0,0)$ lies on the graph.

- When $x=6$:

$$
y = \frac{4 \times 6 \times (-6)}{18 - 36} = \frac{4 \times 6 \times -6}{-18} = \frac{-144}{-18} = 8,
$$

point $(6,8)$ is on the graph.

- When $x=18$:

$$
y= \frac{4 \times 18 \times 6}{54 - 36} = \frac{4 \times 18 \times 6}{18} = 4 \times 6 = 24,
$$

point $(18,24)$.

2. Asymptotic behavior:

- As $x \to 12^+$, $3x -36 \to 0^+$, so $y \to +\infty$.

- As $x \to 12^-$, $3x -36 \to 0^-$, so $y \to -\infty$.

These behaviors confirm the presence of a vertical asymptote at $x=12$.

Summary of the Graphical Features

- The equation describes a hyperbola with a vertical asymptote at $x=12$.
- The solution set includes all points on the line $x=12$ (since the original equation reduces to an identity there).
- The rational function is continuous elsewhere, with the shape characteristic of hyperbolas.

Finding Specific Solutions

Method 1: Substituting Specific Values of x

One straightforward approach is to select specific x values and compute corresponding y values:

- For $x=0$, $y=0$.
- For $x=6$, $y=8$.
- For $x=18$, $y=24$.
- For $x=12$, the entire line $x=12$.

Note: Since the second equation is not completely provided, we will assume that it involves the variable x^2 in some way, perhaps as $x^2 = y$ or similar.

Method 2: Solving for x in terms of y

Alternatively, rearranged the original equation to solve for x:

$$
3xy - 4x^2 - 36 y + 48 x=0,
$$

which can be viewed as quadratic in x:

$$
-4x^2 + (3 y + 48) x - 36 y=0.
$$

Expressed as:

$$
4x^2 - (3 y + 48) x + 36 y=0.
$$

Applying quadratic formula:

$$
x= \frac{(3 y + 48) \pm \sqrt{(3 y + 48)^2 - 4 \times 4 \times 36 y}}{2 \times 4}.
$$

Simplify discriminant:

$$
D = (3 y + 48)^2 - 576 y,
$$

$$
D= 9 y^2 + 288 y + 2304 - 576 y = 9 y^2 - 288 y + 2304.
$$

Solutions exist where $D \ge 0$:

$$
9 y^2 - 288 y + 2304 \ge 0.
$$

Divide through by 9:

$$
y^2 - 32 y + 256 \ge 0.
$$

This quadratic in y factors as:

$$
(y - 16)^2 \ge 0,
$$

which is always true, with equality when $y=16$. Therefore, for all real y, x is real, indicating the solution set spans a wide range.

Implications for the Second Equation

Although the second equation is incomplete, the mention of x^2 suggests possible relationships such as:

- $y = x^2$,
- or some other quadratic relation involving x^2.

If we assume the second equation is $y = x^2$, then the solutions to the system are the intersection points of the hyperbola $3xy - 4x^2 - 36 y + 48 x=0$ and the parabola $y = x^2$.

Finding Intersection Points

Substitute $y = x^2$ into the first equation:

$$
3x(x^2) - 4x^2 - 36 x^2 + 48 x=0,
$$
$$
3 x^3 - 4 x^2 - 36 x^2 + 48 x=0,
$$
$$
3 x^3 - 40 x^2 + 48 x=0,
$$
factor out x:

$$
x(3 x^2 - 40 x + 48)=0.

\]

Solutions:

- \(x=0\),
- or solving quadratic:

\[
3 x^2 - 40 x + 48=0,
\]

discriminant:

\[
D= (-40)^2 - 4 \times 3 \times 48= 1600 - 576=1024,
\]
\[
\sqrt{D} =

Frequently Asked Questions

What is the first step to solve the simultaneous equations involving 3xy - $4x^2$ - 36y + 48x = 0 and another equation starting with X^2?

The initial step is to analyze and simplify the first equation, possibly expressing one variable in terms of the other to facilitate substitution once the second equation is fully provided.

How can we express y in terms of x from the equation 3xy - $4x^2$ - 36y + 48x = 0?

Rearrange the equation to group y terms: (3x - 36)y = $4x^2$ - 48x, then solve for y: y = ($4x^2$ - 48x) / (3x - 36), provided the denominator is not zero.

What conditions should be checked when solving for y in the given equation?

Check for values of x that make the denominator 3x - 36 equal to zero, i.e., x = 12, and analyze the behavior of y at those points separately.

Once y is expressed in terms of x, how do we find the solutions to the system?

Substitute the expression for y into the second equation (starting with X^2), then solve the resulting equation in x to find possible solutions, and back-substitute to find y.

Are there any special cases or restrictions when solving the system of equations involving these two equations?

Yes, values of x that make the denominator zero in the y expression need special attention, as they might lead to extraneous solutions or require separate analysis.

What are the potential types of solutions for the system of equations?

Solutions could be real or complex, depending on the nature of the resulting equations after substitution. Also, some solutions might be invalid if they cause division by zero in intermediate steps.

How does the incomplete second equation affect solving the system?

Since the second equation is incomplete ('X^2'), it suggests it involves a quadratic expression, so once fully provided, it likely involves quadratic methods or substitution to find solutions.

What methods are recommended for solving such systems involving quadratic and linear parts?

Methods include substitution, elimination, and using the quadratic formula, especially after expressing one variable in terms of the other and reducing the system to a single-variable equation.

Additional Resources

Comprehensive Analysis of the System of Equations: Exploring Solutions for \((x, y)\)

Introduction

In the realm of algebra and analytical geometry, systems of equations serve as foundational tools for understanding the relationships between variables. The particular system under scrutiny is:

\[
\begin{cases}

3xy - 4x^2 - 36y + 48x = 0, \\
x^2 = \text{(incomplete in the prompt, but for the purposes of this analysis, we will assume it to be } x^2 = y \text{ or similar, as per common forms)}.
\end{cases}
\]

This review aims to dissect the problem thoroughly, exploring solution techniques, algebraic manipulations, geometric interpretations, and implications. Our goal is to provide an in-depth understanding that transcends mere calculation, delving into the structure and nature of solutions, their intersections, and their significance.

Clarifying the System of Equations

Given the prompt, the equations are:

\[
\begin{cases}
3xy - 4x^2 - 36y + 48x = 0, \\
x^2 = \text{(some expression)}.
\end{cases}
\]

However, the second equation appears incomplete. Typically, in systems involving \(x\) and \(y\), common forms include:

- \(y = x^2 \),
- \(y = \) some function involving \(x\),
- or other polynomial relations.

Assumption for the analysis: We'll assume the second equation to be:

\[
y = x^2,
\]

which is a common quadratic relation, often used in conjunction with the first to analyze intersection points.

Thus, our system becomes:

\[

\begin{cases}
3xy - 4x^2 - 36y + 48x = 0, \\
y = x^2.
\end{cases}
\]

This assumption allows us to proceed with detailed algebraic and geometric analysis. Should the actual second equation differ, the methodology remains similar.

Algebraic Approach: Substitution and Simplification

Step 1: Substituting \(y = x^2 \)

Substituting into the first equation:

\[
3x(x^2) - 4x^2 - 36(x^2) + 48x = 0,
\]

which simplifies to:

\[
3x^3 - 4x^2 - 36x^2 + 48x = 0.
\]

Combine like terms:

\[
3x^3 - 40x^2 + 48x = 0.
\]

Step 2: Factoring the resulting polynomial

Factor out the common term \(x \):

\[
x(3x^2 - 40x + 48) = 0.
\]

This yields two possibilities:

1. $x = 0$,
2. $3x^2 - 40x + 48 = 0$.

Let's analyze each case:

Case 1: $x = 0$

Substitute back into $y = x^2$:

$$
y = 0^2 = 0.
$$

Solution point:

$$
(0, 0).
$$

Verification:

Insert into the original first equation:

$$
3 \times 0 \times 0 - 4 \times 0^2 - 36 \times 0 + 48 \times 0 = 0,
$$

which simplifies to 0, confirming this solution.

Case 2: Solving $3x^2 - 40x + 48 = 0$

This is a quadratic in x. Use the quadratic formula:

$$
x = \frac{40 \pm \sqrt{(-40)^2 - 4 \times 3 \times 48}}{2 \times 3}.
$$

Compute discriminant:

$$
\Delta = 1600 - 4 \times 3 \times 48 = 1600 - 576 = 1024.
$$

Square root:

$$
\sqrt{1024} = 32.
$$

Calculate roots:

$$
x = \frac{40 \pm 32}{6}.
$$

- For the positive root:

$$
x = \frac{40 + 32}{6} = \frac{72}{6} = 12.
$$

- For the negative root:

$$
x = \frac{40 - 32}{6} = \frac{8}{6} = \frac{4}{3}.
$$

Corresponding y values:

Using $y = x^2$:

- When $x = 12$:

$$
y = 12^2 = 144,
$$

- When $x = \frac{4}{3}$:

$$
y = \left(\frac{4}{3}\right)^2 = \frac{16}{9}.
$$

Solutions:

$$
(12, 144), \quad \left(\frac{4}{3}, \frac{16}{9}\right).
$$

Summary of Solutions

Under the assumption $y = x^2$, the solutions to the system are:

$$
\boxed{
\begin{cases}
(0, 0), \\
(12, 144), \\
\left(\frac{4}{3}, \frac{16}{9}\right).
\end{cases}
}
$$

Geometric Interpretation of Solutions

The Parabola: $y = x^2$

This is a standard parabola opening upwards, symmetric about the y-axis.

The First Equation as a Curve

Expressed as:

$$
3xy - 4x^2 - 36y + 48x = 0,
$$

which can be rewritten as:

$$
3xy - 36y = 4x^2 - 48x,
$$
or

$$
y(3x - 36) = 4x^2 - 48x.
$$

Factor numerator and denominator:

$$
y = \frac{4x^2 - 48x}{3x - 36},
$$
with the condition $3x - 36 \neq 0$, i.e., $x \neq 12$.

This is a rational function describing a curve in the plane.

Geometric Analysis of the Curves and Their Intersections

The solutions derived are points of intersection between the parabola $y = x^2$ and the rational curve given by:

$$
y = \frac{4x^2 - 48x}{3x - 36}.
$$

Critical Observations:

- The point $(0, 0)$:

- Substituting $x=0$:

$$
y = \frac{0 - 0}{-36} = 0,
$$

confirming the intersection at the origin.

- The point $\left(12, 144\right)$:

- At $x=12$, denominator $3 \times 12 - 36 = 0$:

- The rational function is undefined (vertical asymptote).

- However, the algebraic solution indicates an intersection at $x=12$, $y=144$. The apparent contradiction suggests that at $x=12$, the rational function approaches infinity, but the parabola $y = x^2$ passes through $(12, 144)$.

- The point $\left(\frac{4}{3}, \frac{16}{9}\right)$:

- Check numerator:

$$
4 \times \left(\frac{4}{3}\right)^2 - 48 \times \frac{4}{3} = 4 \times \frac{16}{9} - 48 \times \frac{4}{3} = \frac{64}{9} - 48 \times \frac{4}{3}.
$$

- Calculate:

$$
48 \times \frac{4}{3} = 48 \times \frac{4}{3} = 16 \times 4 = 64,
$$

- So numerator:

$$
\frac{64}{9} - 64 = \frac{64}{9} - \frac{576}{9} = -\frac{512}{9}.
$$

- Denominator:

$$
3 \times \frac{4}{3} - 36 = 4 - 36 = -32.
$$

- Rational function value:

$$
y = \frac{-\frac{512}{9}}{-32} = \frac{512/9}{32} = \frac{512}{9} \times \frac{1}{32} = \frac{512}{9 \times 32} = \frac{512}{288} = \frac{64}{36} = \frac{16}{9}.
$$

- Confirmed with the quadratic substitution.

Conclusion: The algebraic and geometric analyses corroborate the solutions.

Deeper Insights: Nature of the Solutions and Their Significance

1. Number of Intersection Points

- The algebraic derivation yields exactly three real solutions, indicating the

If X Y Satisfies The Simultaneous Equations Begin Align 3xy 4x 2 36y 48x Amp 0 X 2

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